

考試科目	統計學	系別	統計系	考試時間	7月9日 星期五	第二節
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1. Let X_i be a random variable with mean μ and variance σ^2 , $i=1,2,\dots,n$. Show that $E(\bar{X})=\mu$ and $\text{Var}(\bar{X})=\sigma^2/n$, where \bar{X} is a sample mean. (10%)

2. Let X_1 and X_2 be two discrete random variables with joint p.m.f given in the followings: (18%)

		X_1		
		1	2	3
X_2	1	$\frac{1}{9}$	0	$\frac{1}{6}$
	2	0	$\frac{1}{3}$	0
	3	$\frac{2}{9}$	0	$\frac{1}{6}$

- (a) Calculate the marginal probability distribution of X_1 and X_2 .
- (b) Calculate $E(X_1)$ and $E(X_2)$.
- (c) Calculate $\text{Var}(X_1)$ and $\text{Var}(X_2)$.
- (d) Compute $\text{Cov}(X_1, X_2)$ and $\text{Cov}(3X_1+3, X_2-1)$.
- (e) Compute $\text{Var}(3X_1-2)$ and $\text{Var}(2X_1-X_2)$.
- (f) Are X_1 and X_2 independent?

3. Please answer the following questions:

- (a) Write down the simple linear regression model. (3%)
- (b) Write down the assumptions of simple linear regression model. (5%)
- (c) Find the least squares estimators of simple linear regression model. (12%)

4. At the 0.025 significance level, do the data provide sufficient evidence to conclude that a difference exists between the population means of the four different brands? The sample data are given below.

Brand A	Brand B	Brand C	Brand D
15	20	21	15
25	17	22	15
21	22	20	14
23	23	19	23
22		18	22
20			28
			28

(12%)

備 考試 題 隨 卷 繳 交

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考 試 科 目	統計學	系 別	統計系	考 試 時 間	7 月 9 日 星期五	第 二 節
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5. Let a urn contain 12 balls and the distribution of the number of balls is $P(X=1)=\frac{1}{3}$, $P(X=4)=\frac{1}{4}$ and $P(X=6)=\frac{5}{12}$. We draw two balls from the urn with replacement. (12%)

- (a) Find the sampling distribution of \bar{X} .
- (b) Construct the random interval of $\bar{X} \pm 2$.
- (c) Calculate the confident coefficient of $\bar{X} \pm 2$.
- (d) What is the probability of $\mu \in [\bar{X} - 2, \bar{X} + 2]$ when $\bar{X} = 2$.

選擇題答案請作答於答案卷

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question. (28%)

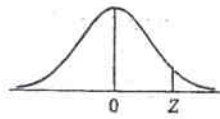
- 1) Let x represent the number which shows up when a balanced die is rolled. Then x is a random variable with a uniform distribution. Let \bar{x} denote the mean of the numbers obtained when the die is rolled 32 times. For samples of size 32, which of the following statements concerning the sampling distribution of the mean is true?
- A) \bar{x} is approximately normally distributed.
- B) \bar{x} is normally distributed.
- C) The distribution of \bar{x} is uniform.
- D) None of the above statements is true.
- 2) The amount of Jen's monthly phone bill is normally distributed with a mean of \$59 and a standard deviation of \$9. Find the first quartile, Q_1 .
- A) \$52.97 B) \$54.5 C) \$65.03 D) \$61.25
- 3) The number of calls received by a car towing service in an hour has a Poisson distribution with parameter $\lambda = 1.16$. Find the probability that in a randomly selected hour the number of calls is between 2 and 4 inclusive.
- A) 0.292 B) 0.316 C) 0.322 D) 0.516
- 4) A hypothesis test is to be performed to determine whether the mean hematocrit (percentage by volume of the blood occupied by red blood cells) for women differs from the mean hematocrit for men which is known to be 47%. Preliminary data analyses indicate that it is reasonable to apply a z-test. The hypotheses are
- $H_0: \mu = 47\%$
- $H_a: \mu \neq 47\%$.

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Areas of the Standard Normal Distribution



The entries in this table are the probabilities that a standard normal random variable is between 0 and Z (the shaded area).

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998									

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		$\alpha = .025$								
		Numerator Degrees of Freedom								
v_2	v_1	1	2	3	4	5	6	7	8	9
1	1	647.79	799.48	864.15	899.60	921.83	937.11	948.20	956.64	963.28
2	1	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
3	1	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
4	1	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
5	1	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
6	1	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
7	1	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
8	1	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
9	1	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
10	1	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
11	1	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
12	1	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
13	1	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31
14	1	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21
15	1	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
16	1	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05
17	1	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98
18	1	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93
19	1	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88
20	1	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84

附表

Values of t for one-tailed test and $\alpha/2$ for two-tailed test				
df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$
1	3.078	6.314	12.706	31.821
2	1.886	2.920	4.303	6.965
3	1.638	2.353	3.182	4.541
4	1.533	2.132	2.776	3.747
5	1.476	2.015	2.571	3.365
6	1.440	1.943	2.447	3.143
7	1.415	1.895	2.365	2.998
8	1.397	1.860	2.306	2.896
9	1.383	1.833	2.262	2.821
10	1.372	1.812	2.228	2.764
11	1.363	1.796	2.201	2.718
12	1.356	1.782	2.179	2.681
13	1.350	1.771	2.160	2.650
14	1.345	1.761	2.145	2.624
15	1.341	1.753	2.131	2.602
16	1.337	1.746	2.120	2.583
17	1.333	1.740	2.110	2.567
18	1.330	1.734	2.101	2.552
19	1.328	1.729	2.093	2.539

附表

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考 試 科 目	微積分	系 別	統計系	考 試 時 間	7 月 9 日 星期五	第 四 節
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1. (10%) Determine whether each of the following limits exists. If so, find the limit. If not, explain why.

(a) $\lim_{(x,y) \rightarrow (0,0)} \sin(x^2 + y^2)/(x^2 + y^2)$.

(b) $\lim_{(x,y) \rightarrow (0,0)} \sin(x + y)/(x^2 + 2y^2)$.

2. (10%) Let $f(x) = \int_0^x (t-1)e^{-t} dt$ for $0 \leq x \leq 2$. Where does f attain its minimum on the interval $[0, 2]$?

3. (10%) For each of the following series, determine whether it is convergent. Justify your answers.

(a) $\sum_{k=2}^{\infty} \frac{k^2+1}{k^2 \ln(k)}$.

(b) $\sum_{k=1}^{\infty} (-1)^k / \sqrt{k}$.

4. (10%) Suppose that $a_k > 0$ and $b_k > 0$ for all $k \geq 1$. Prove or disprove the following statement:

$$\sum_{k=1}^{\infty} a_k < \infty \text{ and } \sum_{k=1}^{\infty} b_k < \infty \Rightarrow \sum_{k=1}^{\infty} a_k b_k < \infty.$$

5. (20%) Find the following integrals.

(a) $\int_0^{\infty} x^2 e^{-2x} dx$.

(b) $\int_0^1 1/(x^2 + 2x + 5) dx$.

6. (20%) Let $D_1 = \{(x, y) : -1 \leq x \leq 1 \text{ and } -2 \leq y \leq 2\}$, $D_2 = \{(x, y) : -1 < x < 1 \text{ and } -2 < y < 2\}$ and $f(x, y) = x^2 - y^2$.

(a) Find the maximum of f on D_1 .

(b) Is there any point $x \in D_2$ at which f has a relative maximum?

7. (20%) Let $D = \{(x, y) : 0 \leq x^2 + 4y^2 \leq 1\}$ and

$$f(x, y) = \begin{cases} \sqrt{1 - x^2 - 4y^2} & \text{if } x \geq 0 \text{ and } y \geq 0; \\ 1 & \text{otherwise.} \end{cases}$$

Find $\iint_D f(x, y) d(x, y)$.

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