41413

國立政治大學115學年度學士班轉學生招生考試

系所組別：統計學系二年級

科目名稱：統計學

考試時間：07月01日（10:50-12:10）第 三、2 節

※考試開始鈴響前，不得翻閱本試題

注意事項：

1. 請檢查座位號碼及試卷(卡)之准考證號碼是否正確。
2. 請檢查試題、卷(卡)之系所組別、科目名稱是否正確。
3. 請務必作答於試卷上（或依題目規定作答於答案卡上），否則不予計分。
4. 不得要求增加試卷用紙。
5. 本試題隨試卷(卡)繳交。

考試科目	統計學	系所別	統計學系 二年級	考試時間	7月 1日(三)第二節

第一大題：選擇題(4pt for each, please write down the item numbers and answers in order. For example: 1 (C) 2 (B) 3(A).....)

1. A certain population follows a normal distribution with mean μ and standard deviation $\sigma = 2.5$. You collect data and test the hypotheses; H_0: $\mu = 1$, H_a: $\mu \neq 1$. You obtain a p-value of 0.022. Which of the following is true?

(A) A 99% confidence interval for μ will include the value 0.

(B) A 95% confidence interval for μ will include the value 1.

(C) A 95% confidence interval for μ will include the value 0.

(D) A 99% confidence interval for μ will include the value 1.

2. In order to determine whether or not the means of three populations are equal,

(A) A t-test must be performed

(B) An analysis of variance must be performed

(C) Either a t-test or an analysis of variance can be performed

(D) A chi-square test must be performed

3. The ability of an interval estimate to contain the value of the population parameter is described by the

(A) Confidence level

(B) Degrees of freedom

(C) Precise value of the population mean

(D) Degrees of freedom minus 1

The next 4-5 questions will be based on the following

Source of Variation	Sum of Squares	D.F.	Mean Square	F
Between Treatments	2,073.6	4		
Between Blocks	6,000	5	1,200	
Error		20	288	
Total		29		

4. The null hypothesis for this ANOVA problem is

(A). $\mu_1=\mu_2=\mu_3=\mu_4$

(B) $\mu_1=\mu_2=\mu_3=\mu_4=\mu_5$

(C) $\mu_1=\mu_2=\mu_3=\mu_4=\mu_5=\mu_6$

(D) $\mu_1=\mu_2= \ldots =\mu_2$

考試科目	統計學	系所別	統計學系 二年級	考試時間	7月 1日(三)第二節

5. The test statistic to test the null hypothesis equals

(A) 0.432

(B) 1.8

(C) 4.17

(D) 28.8

6. A study compares three fertilizing methods (A, B, C) on crop yield (in kg). Each fertilizing group consists of five observations, and the summary statistics are given below.

Method	Average	Variance
A	78	47.5
B	84	50.0
C	89	54.5

The following regression model is applied to analyze the data:

$$Y = \beta_0 + \beta_1 Z_1 + \beta_2 Z_2 + \varepsilon ,$$

where Z_1 and Z_2 are dummy variables indicating methods A and B, and method C serves as the reference group:

$$Z_1 = \begin{cases} 1, & \text{if method A} \\ 0, & \text{otherwise} \end{cases}, \qquad Z_2 = \begin{cases} 1, & \text{if method B} \\ 0, & \text{otherwise} \end{cases},$$

Which of the following statement is correct?

(A) The estimate of β_1 is positive

(B) The estimate of β_2 is positive

(C) $\mu_A = \beta_1$, where μ_A is the mean crop yield of method A

(D) $\mu_C = \beta_0$, where μ_C is the mean crop yield of method C

7. If a test of hypothesis has a Type I error probability of .05, this means that:

(A) If the null hypothesis is true, we don't reject if 5% of the time.

(B) If the null hypothesis is false, we reject it 5% of the time.

(C) If the null hypothesis is false, we don't reject it 5% of the time.

(D) If the null hypothesis is true, we reject it 5% of the time.

8. Consider the following partial ANOVA table:

Source of Variation	*SS*	*df*	*MS*	*F*
Treatments	75	*	25	6.67
Error	60	*	3.75	
Total	135	19		

考試科目	統計學	系所別	統計學系二年級	考試時間	7月1日(三)第二節

The numerator and denominator degrees of freedom for the F-test (identified by asterisks) are

(A) 4 and 15

(B) 3 and 16

(C) 15 and 4

(D) 16 and 3

以下計算題必須提供詳細過程

第二大題: (18 pt)

Let X be a discrete random variable with the support $\{1, 2, \dots, 10\}$. Suppose that $p_i = P(X = i)$ and $\lambda_i = P(X = i|X \geq i)$. Given $\lambda_i = \frac{1}{(11-i)}$ for $i = 1, 2, \dots, 10$,

a) (10 pt) Find the value of p_i, $i = 1, 2, .., 10$.

b) (8 pt) Someone has 10 keys but only one key can open the door. Each time he will randomly select one key and try to open the door; if he fails to open the door, he will remove the selected key and choose another key until the door is open. According to this experiment, define which outcome has the same distribution function of X? Hint: explain your answer by verifying p_i and λ_i

第三大題: (10 pt)

Suppose that $X_1, X_2, \dots, X_n$ are taken from a distribution of the discrete type with probability function $f(x; \theta) = \frac{1}{\theta}, x = 1, 2, .., \theta$, zero elsewhere, where θ is a positive integer. Let $Y_n = max\{X_1, X_2, \dots, X_n\}$. Find the probability function of Y_n.

第四大題: (20 pt)

The accompanying data on the alcohol content of wine is representative of that reported in a study in which wines were randomly selected and the actual content was determined by laboratory analysis.

Wine	1	2	3	4	5	6
Actual	14.2	14.5	14	14.9	13.6	12.6
Label	14.0	14.0	13.5	15.0	13.0	12.5

a) (10 pt) A person reported that the two-sample t test gives a test statistic value of 0.62 and a two-tailed p-value of 0.55. Please explain how to obtain these two statistics

考試科目	統計學	系所別	統計學系 二3組	考試時間	7月1日(三)第二節

b) (10 pt) Does this convince you that there is no significant difference between true average actual alcohol content and true average content stated on the label? Give a reason (5pt) and correct it if you have other better method (5 pt).

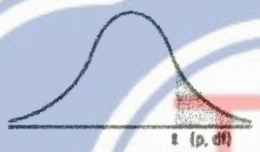
第五大題: (20 pt)

A coin is tossed independently 30 times to test the hypothesis that the probability of heads is 1/3 against the alternative that the probability is not 1/3. The test rejects if $\{X \leq 4 \text{ or } X \geq 15\}$, where X is the number of observing heads.

a) (10 pt) Find the approximated Type I error probability based on this rejection rule?

b) (10 pt) Approximate the probability of rejecting the null hypothesis if in fact the probability of heads is 0.36.

t table



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728

考試科目	統計學	系所別	統計學系 二年級	考試時間	7月 1 日(三)第二節

Standard Normal Probabilities

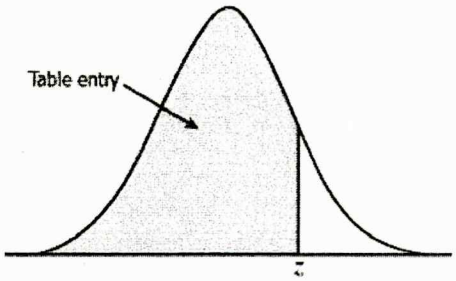


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

41414

國立政治大學115學年度學士班轉學生招生考試

系所組別：統計學系二年級

科目名稱：微積分

考試時間：07月01日（15:20-16:40）第 三、4 節

※考試開始鈴響前，不得翻閱本試題

注意事項：

1. 請檢查座位號碼及試卷(卡)之准考證號碼是否正確。
2. 請檢查試題、卷(卡)之系所組別、科目名稱是否正確。
3. 請務必作答於試卷上（或依題目規定作答於答案卡上），否則不予計分。
4. 不得要求增加試卷用紙。
5. 本試題隨試卷(卡)繳交。

考試科目	微積分	系所別	國貿系、統計系、財管系、風管系、經濟系 二年級	考試時間	07 月 01 日(三) 第 四 節

Part A Calculus

1. (10%) Suppose f is differentiable on $\mathbb{R}$ and $f'(x) = 0$ for all $x \in \mathbb{R}$. Prove that f is constant on $\mathbb{R}$.
2. (10%) Find the power series expansion of

$$\frac{1}{(1-x)^2}$$

centered at $x = 0$, and use it to evaluate

$$\sum_{n=1}^{\infty} \frac{n}{3^n}.$$

3. (10%) Prove that

$$\frac{1}{n}\sum_{k=0}^{n}\left(1-\frac{k}{n}\right)^n \le \frac{e}{n(e-1)}.$$

4. (10%) Evaluate the following limits:
   (a)

$$\lim_{x\to 0}\frac{\tan x - \sin x}{x^3};$$

   (b)

$$\lim_{x\to 0}\frac{e^x - 1 - x}{x^2}.$$

5. (10%) Find

$$\lim_{n\to\infty}\frac{1^n + 2^n + \cdots + n^n}{n^{n+1}}.$$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	微積分	系所別	國貿系、統計系、財管系、風管系、經濟系 二年級	考試時間	7月1日(三) 第四節

(B)

1. (10%) For which values of the real parameter p does the following improper integral converge?

$$\int_1^{\infty} \frac{(\ln x)^2}{x^p}\, dx$$

(A) $p > 0$, (B) $p \geq 1$, (C) $p > 1$, (D) The integral diverges for all real values of p.

2. (10%) Consider the power series

$$\sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)} (x-2)^2$$

What is its interval of convergence?
(A) $[0,4]$, (B) $(0,4)$, (C) $(0,4]$, (D) $[-2,2]$.

3. (10%) For the polar curve $r = 2 + \cos\theta$, the slope dy/dx of the tangent line at $\theta = \pi/3$ is _____.

4. (10%) Let R be the region in the xy-plane that lies inside the circle $x^2 + y^2 = 2x$, and outside the circle $x^2 + y^2 = 1$. Find the volume of the solid bounded below by R and above by the surface $z = x^2 + y^2$.

5. (a) (5%) Let n be a positive integer. Find the limit of $\lim_{x\to\infty} \frac{x^n}{e^x}$.

(b) (5%) Use part (a) to compare the growth rate of exponential function $G(x) = e^x$ with that of an arbitrary polynomial function with a degree m

$$P(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0, \quad a_m \neq 0.$$

Determine which function grows faster as $x \to \infty$, and justify your conclusion.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。