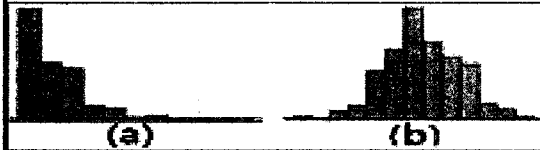


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| 考 試 科 目 | 統計學 | 系 所 別 | 統計系 | 考 試 時 間 | 7 月 2 日(三) 第二節 |
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**第一大題：選擇題(4pt for each, please write down the item numbers and answers in order)**

1. In each of the following histograms : the areas of the blocks represent the frequencies of observations for a quantitative variable and the total area is equal to 1. The horizontal scales and measurement units used are identical for both histograms.



Which of the following statements about the mean and median and the spread of the observations is (or are) TRUE?

- (a) For histogram (a), median = mean
- (b) For histogram (a), median > mean
- (c) The observations of histogram (a) have the same spread as those of (b).
- (d) The observations of histogram (a) are more spread out than those of (b).
- (e) None of the above

**Use the following to answer the questions 2--4**

The United Nations wants to determine the percentage of women in the U.S. labor force. Therefore, a representative from the United States Department of Labor selected a random sample of employment records, and show that a 95% confidence interval for the proportion of female employees is  $44\% \pm 10\%$ .

2. Which of the following statement is true?

- (a) The sample proportion is 50%
- (b) The margin of error is 20%
- (c) The critical value is 1
- (d) The standard error is 5.10%
- (e) None of the above

3. Which of the conclusions is correct?

- (a) Between 34% and 54% of all employees are female
- (b) The United Nation is 95% sure that interval contains the true proportion
- (c) The United Nation is 95% sure that between 34% and 54% of surveyed employees are female
- (d) The true proportion is between 34% and 54%
- (e) None of the above

4. If the Department of Labor wishes to shorten its interval, they should

- (a) Increase the confidence level
- (b) Increase the sample size
- (c) Decrease the sample size
- (d) Both a and b
- (e) Both a and c

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5. Which measure of central tendency can be used for both numerical and categorical variables?

- (a) Sample mean
- (b) Median
- (c) Mode
- (d)  $Q_1$
- (e) None of the above

6. Consider the following model  $Y = \beta_0 + \beta_1 X + \varepsilon$ , whether  $Y$  is the daily rate of return of a stock, and  $X$  is the daily rate of return of the stock market as a whole, measured by the daily rate of return of Standard & Poor's(S&P) 500 Composite Index. Using a random sample of  $n = 12$  days from 1980, the least squares lines shown in the table below were obtained for four firms. The estimated standard error of  $\hat{\beta}_1$  is shown to the right of each least squares prediction equation. For which of the following stocks, is there evidence (at  $\alpha = 0.05$ ) of a positive linear relationship between  $Y$  and  $X$ ?

| Firm | Estimated Model      | Estimated Standard Error of $\hat{\beta}_1$ |
|------|----------------------|---------------------------------------------|
| A    | $y = 0.0010 + 1.40x$ | 0.03                                        |
| B    | $y = 0.0005 - 1.21x$ | 0.06                                        |
| C    | $y = 0.0010 + 1.62x$ | 1.34                                        |
| D    | $y = 0.0013 + 0.76x$ | 0.15                                        |

- (a) Firms A and D
- (b) Firms B and D
- (c) Firms A, C and D
- (d) Firm C
- (e) Firms A, B, and D

7. One-way ANOVA was applied to three independent samples having means 10, 13, and 18, respectively. If each observation in the third sample were increased by 30, the value of the F statistics would

- (a) increase
- (b) decrease
- (c) remain unchanged
- (d) increase by 30

第二大題：簡答題(25%, 5pts for each)(不需提供計算過程)

(a) There are five urns and they are numbered 1 to 5. Each urn contains 10 balls. Urn  $i$  has  $(10-i)$  defective balls and  $i$  non-defective balls, for  $i=1,2,3,4,5$ . For instance urn 3 has 7 defective balls and 3 non-defective balls.

Consider the following random experiment with 2 steps: First, an urn is selected at random (that means each urn has equal probability of being selected). Second, then a ball is selected at random from the selected urn. What is the probability that a non-defective ball is selected?

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(b) A man with 10 keys wants to open his door and tries the keys at random. Suppose there is exactly one key will open the door. If unsuccessful keys are **eliminated**(淘汰) from further selections. Let  $X$  be the number of trials to find the right key. Which model can describe  $X$ ?

(c) The average score of all golfers for a particular course has a mean of 73 and a standard deviation of 3.5. Suppose 49 golfers played the course today. Find the probability that the average score of the 49 golfers exceeded 74.

(d) On a multiple-choice exam with 3 possible answers for each of the 5 questions, what is the probability that a student will get 4 or more correct answers just by guessing?

(e) A manufacture suspects that the lives of the different brands of lamps are different. There are a large number of brands. Four of these are randomly selected for study. 11 lamps of each brand are tested. Part of the ANOVA table is shown below. What is the value of the  $F$  statistic?

| Source of Variation | SS   | d.f. | MS | F |
|---------------------|------|------|----|---|
| Treatments          | ?    | ?    | ?  | ? |
| Errors              | 2000 | ?    | ?  |   |
| Total               | 5000 |      |    |   |

### 第三大題:計算題(16%)

Let  $X$  equal the number of female children in a three-child family. We shall test the null hypothesis that the chance of getting a boy is equal to girl. So a sample of 52 students came from families with three children, the results are the following:

|       |   |    |    |   |
|-------|---|----|----|---|
| X     | 0 | 1  | 2  | 3 |
| count | 6 | 16 | 24 | 6 |

Use  $\alpha = 0.05$  to test the null hypothesis.

### 第四大題:計算題(16%)

We have been asked to study the difference in systolic blood pressure (BP) between men ( $\mu_1$ ) and women ( $\mu_2$ ) with heart trouble. The BP measurements are normally distributed. To investigate this research, two groups of the subjects were sampled and their results were the following

Men      165, 147, 150, 180, 175, 125, 130.  
 Women    135, 145, 140, 160, 150, 150, 145.

Conduct a hypothesis testing to find whether or not there is a difference between men and women. Use  $\alpha = 0.05$  and report the  $p$ -value as well.

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## 第五大題:計算題(15%)

Suppose gamblers playing roulette at a certain casino are betting on either red or black. At each spin of the wheel, a gambler bet 1\$ on either red or black. There are 38 numbers on the wheel, 18 are red and 18 are black (2 are neither red nor black) and each number has an equal probability of coming up. If a gambler's color comes up, he or she wins 1\$; if not he or she loses 1\$. Now there is a gambler who is going to play 100 times continuously. Apply the CLT to find the probability that he will lose \$3 at least.

## 附錄表格

Example:  
If  $z = 1.96$ , then  
 $P(0 \text{ to } z) = 0.4750$ .



| z   | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |

考試科目 統計學

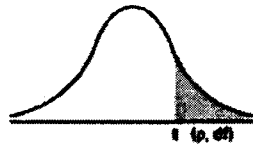
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t table



| df/p | 0.40     | 0.25     | 0.10     | 0.05     | 0.025    | 0.01     | 0.005    | 0.0005   |
|------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1    | 0.324920 | 1.000000 | 3.077684 | 6.313752 | 12.70620 | 31.82052 | 63.65674 | 636.6192 |
| 2    | 0.288675 | 0.816497 | 1.885618 | 2.919986 | 4.30265  | 6.96456  | 9.92484  | 31.5991  |
| 3    | 0.276671 | 0.764892 | 1.637744 | 2.353363 | 3.18245  | 4.54070  | 5.84091  | 12.9240  |
| 4    | 0.270722 | 0.740697 | 1.533206 | 2.131847 | 2.77645  | 3.74695  | 4.60409  | 8.6103   |
| 5    | 0.267181 | 0.726687 | 1.475884 | 2.015048 | 2.57058  | 3.36493  | 4.03214  | 6.8688   |
| 6    | 0.264835 | 0.717558 | 1.439756 | 1.943180 | 2.44691  | 3.14267  | 3.70743  | 5.9588   |
| 7    | 0.263167 | 0.711142 | 1.414924 | 1.894579 | 2.36462  | 2.99795  | 3.49948  | 5.4079   |
| 8    | 0.261921 | 0.706387 | 1.396815 | 1.859548 | 2.30600  | 2.89646  | 3.35539  | 5.0413   |
| 9    | 0.260955 | 0.702722 | 1.383029 | 1.833113 | 2.26216  | 2.82144  | 3.24984  | 4.7809   |
| 10   | 0.260185 | 0.699812 | 1.372184 | 1.812461 | 2.22814  | 2.76377  | 3.18927  | 4.5869   |
| 11   | 0.259556 | 0.697445 | 1.363430 | 1.795885 | 2.20099  | 2.71808  | 3.10581  | 4.4370   |
| 12   | 0.259033 | 0.695483 | 1.356217 | 1.782288 | 2.17881  | 2.68100  | 3.05454  | 4.3178   |
| 13   | 0.258591 | 0.693829 | 1.350171 | 1.770833 | 2.16037  | 2.65031  | 3.01228  | 4.2208   |
| 14   | 0.258213 | 0.692417 | 1.345030 | 1.761310 | 2.14479  | 2.62449  | 2.97684  | 4.1405   |
| 15   | 0.257885 | 0.691197 | 1.340606 | 1.753050 | 2.13145  | 2.60248  | 2.94671  | 4.0728   |

Table I: Chi-square table

| Upper tail | 0.3   | 0.2   | 0.1   | 0.05  | 0.02  | 0.01  | 0.005 | 0.001 |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|
| df         |       |       |       |       |       |       |       |       |
| 2          | 2.41  | 3.22  | 4.61  | 5.99  | 7.82  | 9.21  | 10.60 | 13.82 |
| 3          | 3.06  | 4.04  | 6.25  | 7.81  | 9.84  | 11.34 | 12.84 | 16.27 |
| 4          | 4.88  | 5.99  | 7.78  | 9.49  | 11.67 | 13.28 | 14.86 | 18.47 |
| 5          | 6.06  | 7.29  | 9.24  | 11.07 | 13.39 | 15.09 | 16.75 | 20.52 |
| 6          | 7.23  | 8.56  | 10.64 | 12.59 | 15.03 | 16.81 | 18.55 | 22.46 |
| 7          | 8.38  | 9.80  | 12.02 | 14.07 | 16.82 | 18.48 | 20.28 | 24.32 |
| 8          | 9.52  | 11.03 | 13.36 | 15.51 | 18.17 | 20.09 | 21.95 | 26.12 |
| 9          | 10.66 | 12.24 | 14.68 | 16.92 | 19.68 | 21.67 | 23.59 | 27.88 |
| 10         | 11.78 | 13.44 | 15.99 | 18.31 | 21.16 | 23.21 | 25.19 | 29.59 |
| 11         | 12.90 | 14.63 | 17.28 | 19.68 | 22.62 | 24.72 | 26.76 | 31.26 |
| 12         | 14.01 | 15.81 | 18.55 | 21.03 | 24.05 | 26.22 | 28.30 | 32.91 |
| 13         | 15.12 | 16.98 | 19.81 | 22.36 | 25.47 | 27.69 | 29.82 | 34.53 |
| 14         | 16.22 | 18.15 | 21.06 | 23.68 | 26.87 | 29.14 | 31.32 | 36.12 |
| 15         | 17.32 | 19.31 | 22.31 | 25.00 | 28.26 | 30.58 | 32.80 | 37.70 |

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Table III: F table with  $\alpha = 0.05$ 

| Denominator DF | Numerator DF |         |         |         |         |         |         |         |         |         |
|----------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|                | 1            | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9       | 10      |
| 1              | 161.448      | 199.500 | 215.707 | 224.583 | 230.162 | 233.986 | 236.768 | 238.883 | 240.543 | 241.882 |
| 2              | 18.513       | 19.000  | 19.164  | 19.247  | 19.296  | 19.330  | 19.353  | 19.371  | 19.385  | 19.396  |
| 3              | 10.128       | 9.552   | 9.277   | 9.117   | 9.013   | 8.941   | 8.887   | 8.845   | 8.812   | 8.786   |
| 4              | 7.709        | 6.944   | 6.591   | 6.388   | 6.256   | 6.163   | 6.094   | 6.041   | 5.999   | 5.964   |
| 5              | 6.608        | 5.786   | 5.409   | 5.192   | 5.050   | 4.950   | 4.876   | 4.818   | 4.772   | 4.735   |
| 6              | 5.987        | 5.143   | 4.757   | 4.534   | 4.387   | 4.284   | 4.207   | 4.147   | 4.099   | 4.060   |
| 7              | 5.591        | 4.737   | 4.347   | 4.120   | 3.972   | 3.866   | 3.787   | 3.726   | 3.677   | 3.637   |
| 8              | 5.318        | 4.459   | 4.066   | 3.838   | 3.687   | 3.581   | 3.500   | 3.438   | 3.388   | 3.347   |
| 9              | 5.117        | 4.256   | 3.863   | 3.633   | 3.482   | 3.374   | 3.293   | 3.230   | 3.179   | 3.137   |
| 10             | 4.965        | 4.103   | 3.708   | 3.478   | 3.326   | 3.217   | 3.135   | 3.072   | 3.020   | 2.978   |
| 11             | 4.844        | 3.982   | 3.587   | 3.357   | 3.204   | 3.095   | 3.012   | 2.948   | 2.896   | 2.854   |
| 12             | 4.747        | 3.885   | 3.490   | 3.259   | 3.106   | 2.996   | 2.913   | 2.849   | 2.796   | 2.753   |
| 13             | 4.667        | 3.806   | 3.411   | 3.179   | 3.025   | 2.915   | 2.832   | 2.767   | 2.714   | 2.671   |
| 14             | 4.600        | 3.739   | 3.344   | 3.112   | 2.958   | 2.848   | 2.764   | 2.699   | 2.646   | 2.602   |
| 15             | 4.543        | 3.682   | 3.287   | 3.056   | 2.901   | 2.790   | 2.707   | 2.641   | 2.588   | 2.544   |
| 16             | 4.494        | 3.634   | 3.239   | 3.007   | 2.852   | 2.741   | 2.657   | 2.591   | 2.538   | 2.494   |
| 17             | 4.451        | 3.592   | 3.197   | 2.965   | 2.810   | 2.699   | 2.614   | 2.548   | 2.494   | 2.450   |
| 18             | 4.414        | 3.555   | 3.160   | 2.928   | 2.773   | 2.661   | 2.577   | 2.510   | 2.456   | 2.412   |
| 19             | 4.381        | 3.522   | 3.127   | 2.895   | 2.740   | 2.628   | 2.544   | 2.477   | 2.423   | 2.378   |
| 20             | 4.351        | 3.493   | 3.098   | 2.866   | 2.711   | 2.599   | 2.514   | 2.447   | 2.393   | 2.348   |
| 21             | 4.325        | 3.467   | 3.072   | 2.840   | 2.685   | 2.573   | 2.488   | 2.420   | 2.366   | 2.321   |
| 22             | 4.301        | 3.443   | 3.049   | 2.817   | 2.661   | 2.549   | 2.464   | 2.397   | 2.342   | 2.297   |
| 23             | 4.279        | 3.422   | 3.028   | 2.796   | 2.640   | 2.528   | 2.442   | 2.375   | 2.320   | 2.275   |
| 24             | 4.260        | 3.403   | 3.009   | 2.776   | 2.621   | 2.508   | 2.423   | 2.355   | 2.300   | 2.255   |
| 25             | 4.242        | 3.385   | 2.991   | 2.759   | 2.603   | 2.490   | 2.405   | 2.337   | 2.282   | 2.236   |
| 26             | 4.225        | 3.369   | 2.975   | 2.743   | 2.587   | 2.474   | 2.388   | 2.321   | 2.265   | 2.220   |
| 27             | 4.210        | 3.354   | 2.960   | 2.728   | 2.572   | 2.459   | 2.373   | 2.305   | 2.250   | 2.204   |
| 28             | 4.196        | 3.340   | 2.947   | 2.714   | 2.558   | 2.445   | 2.359   | 2.291   | 2.236   | 2.190   |
| 29             | 4.183        | 3.328   | 2.934   | 2.701   | 2.545   | 2.432   | 2.346   | 2.278   | 2.223   | 2.177   |
| 30             | 4.171        | 3.316   | 2.922   | 2.690   | 2.534   | 2.421   | 2.334   | 2.266   | 2.211   | 2.165   |

備

註

- 一、作答於試題上者，不予計分。  
二、試題請隨卷繳交。

|         |     |       |                             |         |                 |
|---------|-----|-------|-----------------------------|---------|-----------------|
| 考 試 科 目 | 微積分 | 系 所 別 | 經濟系、國貿系、統計系、<br>資管系、財管系、風管系 | 考 試 時 間 | 7 月 2 日(三) 第四 節 |
|---------|-----|-------|-----------------------------|---------|-----------------|

(A)

Question 1. Let  $f(x) = \frac{3 \sin(x)}{2 + \sin(x)}$ . Calculate the following limits (show your work to get the points):

(a) (5%)  $\liminf_{x \rightarrow +\infty} f(x)$ ;

(b) (5%)  $\limsup_{x \rightarrow +\infty} f(x)$ .

Question 2. (10%) Prove that for all  $0 < x < \frac{\pi}{6}$ ,  $\frac{\sin(x)}{x} > \frac{3}{\pi}$ .

Question 3. (10%) Assume a real function  $b: \mathbb{R} \rightarrow \mathbb{R}$  is differentiable. Prove that

$$\lim_{\epsilon \rightarrow 0^+} \frac{b(x + b(x)\sqrt{\epsilon}) - b(x)}{\sqrt{\epsilon}} = b(x)b'(x),$$

where  $b'(x) = \frac{d}{dx} b(x)$ .

Question 4. Show your work to get the points.

(a) (5%) Find the Taylor series of  $\ln(1-x)$  at  $x=0$ , and determine its interval of convergence.

(b) (5%) Use (a) to prove

$$\ln 2 = \sum_{n=1}^{\infty} \frac{1}{n2^n}.$$

Question 5. (10%) Find the maximum value of

$$f(x) = 2\sqrt{x} - \sqrt{x+1} - \sqrt{x-1}, \quad \text{for } x \in [1, \infty).$$

Show your work to get the points.

備

註

- 一、作答於試題上者，不予計分。  
二、試題請隨卷繳交。

|         |     |       |                             |         |                 |
|---------|-----|-------|-----------------------------|---------|-----------------|
| 考 試 科 目 | 微積分 | 系 所 別 | 經濟系、國貿系、統計系、<br>資管系、財管系、風管系 | 考 試 時 間 | 7 月 2 日(三) 第四 節 |
|---------|-----|-------|-----------------------------|---------|-----------------|

(B)

- (10%)  $\int_0^{\pi/6} \int_{\sin x}^{1/2} xy^2 dy dx = \underline{\hspace{2cm}}$ .
- (10%) Find the Maclaurin series for  $f(x) = e^{3x+1}$  and find its radius of convergence.
- (10%) Let  $R$  be the region under the graph of  $f(x) = 3 - 2x$  over the interval  $-1 \leq x \leq 2$ . Compute the area of  $R$  as the limit of a sum.
- (10%) (a) For the series  $\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{2n+1}}{2n+1}$ , find its radius and interval of convergence. Then identify the values of  $x$  for which the series converges (b) absolutely and (c) conditionally.
- (10%) Find the area of one leaf of the four-leaved rose  $r = a \sin 2\theta$ , where  $a > 0$ .

備

註

- 一、作答於試題上者，不予計分。
- 二、試題請隨卷繳交。