國立政治大學　111　學年度 轉學生 招生考試試題

考試科目	統計學	系所別	統計學系 二年級	考試時間	7 月 6 日(三) 第 二 節

1. (15%) (a) A study surveyed 64 randomly selected people living in Taipei to determine the mean amount spent on lunch. The sample mean was 90. The population distribution is a normal distribution with the mean μ and standard deviation 10. What is the 95% confidence interval estimate of μ? (8%)
   (b) Suppose that the 95% interval estimate in (a) is $[c_1, c_2]$, where c_1 and c_2 are real numbers and $c_1 < c_2$. There is a statement about the conclusion: "$[c_1, c_2]$ contains μ with probability 95%." Is the statement correct or incorrect? Please give your reason. (7%)

2. (20%) The manager of a department store wished to study shopping habits of their customers. Based on earlier studies, customers used to spend, on average, 1 hour at the department store, with a standard deviation of .8 hour. Recently, the manager added some specialty restaurants. To evaluate the effects of the restaurants, a sample of 100 shoppers resulted in a sample mean of 1.2 hour. Suppose that the population standard deviation remains the same. Develop a hypothesis test to determine if the population mean time spent in the department store changed. Use the .05 significance level.
   (a) State the null and alternative hypotheses. (2%)
   (b) What is the p-value? What is your conclusion about the test? (9%)
   (c) What is the power of the statistical test when the actual new population mean is .78? (9%)

3. (15%) To test for any significant difference in the mean number of hours between breakdowns for three machines, an employee recorded the following sample data. Assume that each of the three population follows a normal distribution.

Machine 1	Machine 2	Machine 3
5.3	9.1	9.9
7.1	10	10.5
8.2	9	11.7
6.4	8.7	11.5
7.8	7.6	10.6
7.2	9.6	11.8

   (a) To determine any significant difference in the mean number of hours between breakdowns for three machines, what are the null and alternative hypotheses? (2%)
   (b) Using a 0.05 significance level, complete the hypothesis testing and what is the conclusion? (13%)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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4. (20%) The ANOVA table for a multiple regression is given in the following. Please answer questions (a) to (d).

Source of Variation	Sum of Squares	Degree of Freedom	Mean Square	F	p-value
Regression	77.907	2	(3)	(5)	(6)
Error	(1)	(2)	(4)		
Total	661.600	64			

(a) What is the sample size in the regression analysis? (3%)

(b) How many independent variables involved in the multiple regression? (3%)

(c) Conduct a global test of hypothesis to determine whether any of the regression coefficients for independent variables are not equal to 0. Using a .05 significance level. What are the null and alternative hypotheses? What is the value of the test statistic? What is your conclusion? (9%)

(d) Compute the coefficient of determination. What is your conclusion about the regression analysis based on the coefficient of determination? (5%)

5. (10%) Given below is a bivariate distribution, $p(x,y)$, for random variables X and Y. In specific, $p(x,y)$ stands for $P(X=x, Y=y)$, where x and y are real numbers.

$p(x,y)$	x	y
.2	50	80
.3	30	50
.3	40	60
.1	20	60
.1	30	100

(a) Develop the probability distribution for $X+Y$. (5%)

(b) Compute $E(X+Y)$. (5%)

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6. (5%) An internal study by a high-tech company revealed that their employees receive an average of 1.5 non-work-related emails per hours. Assume that the arrival time between these emails follows an exponential distribution. What is the probability that one employee receives 4 or more non-work-related emails between 9 a.m. to 11 a.m. tomorrow?

7. (5%) The probability density function of X is given by

$$f(x) = \begin{cases} c \cdot x & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases},$$

where c is a constant. Compute $E(X)$.

8. (10%) Suppose that X is a discrete random variable with its probability mass function $p(x)$. Let $E[X] = \mu$. Prove that $E[(X-\mu)^2] = E[X^2] - \mu^2$.
Note: The following property might be useful in your proof. For any real-valued function $g(\cdot)$, $E[g(X)] = \sum_i g(x_i) \cdot p(x_i)$, where the summation is taken over all x_i's whose $p(x_i)$ larger than 0.

Statistical Table Values:

$F_{.025,2,10} = 5.456$, where .025 is the upper tail probability

$F_{.025,2,15} = 4.765$	$F_{.025,2,17} = 4.619$	$F_{.025,2,18} = 4.560$	
$F_{.025,2,62} = 3.917$	$F_{.025,2,63} = 3.914$	$F_{.025,2,64} = 3.910$	$F_{.025,2,65} = 3.906$
$F_{.025,3,10} = 4.826$	$F_{.025,3,15} = 4.153$	$F_{.025,3,17} = 4.011$	$F_{.025,3,18} = 3.954$
$F_{.025,3,62} = 3.335$	$F_{.025,3,63} = 3.331$	$F_{.025,3,64} = 3,328$	$F_{.025,3,65} = 3,324$
$F_{.05,2,10} = 4.103$	$F_{.05,2,15} = 3.682$	$F_{.05,2,17} = 3.592$	$F_{.05,2,18} = 3.555$
$F_{.05,2,62} = 3.145$	$F_{.05,2,63} = 3.143$	$F_{.05,2,64} = 3.140$	$F_{.05,2,65} = 3.138$
$F_{.05,3,10} = 3.708$	$F_{.05,3,15} = 3.287$	$F_{.05,3,17} = 3.197$	$F_{.05,3,18} = 3.160$
$F_{.05,3,62} = 2.753$	$F_{.05,3,63} = 2.751$	$F_{.05,3,64} = 2.748$	$F_{.05,3,65} = 2.746$

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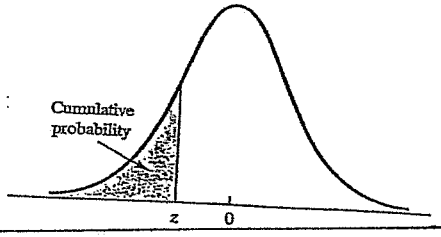
Table of the standard Normal Distribution (its cumulative probabilities)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

國立政治大學 /// 學年度轉學生招生考試試題

考試科目	微積分	系所別	國貿、金融、統計 企管、資管二年級	考試時間	7 月 6 日(三)第四節

選擇題請在答案卡上作答，否則不予計分。

第一部份: 選擇題 (單選題, 每題 5 分, 共 50 分)

1. Suppose that

$$f(x) = \begin{cases} 0 & \text{if } x \le -1; \\ x^2 - 1 & \text{if } -1 < x \le 1; \\ \ln(x) & \text{if } 1 < x \le e; \\ g(x) & \text{if } x > e, \end{cases}$$

where g is a continuous function on (e, ∞) and $\lim_{x \to e+} g(x) = 1$. Suppose that $\int_e^\infty g(x)dx = 1$. Which of the following statements is false?

(a) $\int_{-1}^1 f(x)dx > -2$

(b) $\int_1^e f(x)dx = 1$.

(c) $\int_1^\infty f(x)dx < 3$

(d) $\int_0^\infty f(x)dx = 5/3$

(e) $\int_{-\infty}^\infty f(x)dx = 2/3$.

2. For every positive integer n, let $a_{k,n} = k/n$ for $k \in \{1, \ldots, n\}$. Which of the following statements is false?

(a) $\lim_{n\to\infty} \sum_{k=1}^n \sqrt{a_{k,n}}/n = 2/3$.

(b) $\lim_{n\to\infty} \sum_{k=1}^n a_{k,n}^6/n = 1/7$.

(c) $\lim_{n\to\infty} \sum_{k=1}^n \tan(\pi/4 - \pi a_{k,n}/4)/n = 2\ln(2)/\pi$.

(d) $\lim_{n\to\infty} \sum_{k=1}^n \sin(\pi a_{k,n}/2)/n = 1/\pi$.

(e) $\lim_{n\to\infty} \sum_{k=1}^n \cos(\pi a_{k,n})/n = 0$.

3. Suppose that f and g are two functions such that $g'(x) = f'(x) = xe^x$ for $x \in (-\infty, \infty)$ and $f(0) = -1$. Which of the following statements is false?

(a) $f(x) = -1 + \int_0^x te^t dt$ for $x \in (-\infty, \infty)$.

(b) If $g(3) = e^3$, then $g(x) = f(x)$ for $x \in (-\infty, \infty)$.

(c) If $g(2) = e^2$, then $g(x) = f(x)$ for $x \in (-\infty, \infty)$.

(d) If $g(1) = 0$, then $g(x) = f(x)$ for $x \in (-\infty, \infty)$.

(e) $f(100) > e^{100}$.

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4. Let $f(x) = \int_0^x t\sin(t)dt$ and $g(x) = \int_0^x t^2\cos(t)dt$ for $x \in (-\infty, \infty)$. Which of the following statements is false?

   (a) $g(x) = x^2\sin(x) - 2f(x)$ for $x \in (-\infty, \infty)$.

   (b) There exists some constant c such that $f(x) + x\cos(x) = c\sin(x)$ for $x \in (-\infty, \infty)$.

   (c) $f(x) = -f(-x)$ for $x \in (-\infty, \infty)$.

   (d) $f(\pi) = \pi$.

   (e) $g(\pi/2) = (\pi/2)^2 + 2$.

5. Let $f(x, y) = x^2 + y^2$ for $x, y \in (-\infty, \infty)$, $D_1 = \{(x, y) : x \in [0, 1] \text{ and } y \in [0, 1]\}$ and $D_2 = \{(x, y) : x^2 + y^2 \leq 1\}$. Which of the following statements is false?

   (a) $\int_{D_1} f(x, y)d(x, y) = 2/3$.

   (b) $\int_{D_2} f(x, y)d(x, y) > 1$

   (c) $\int_{D_1 \cap D_2} f(x, y)d(x, y) < 1/2$

   (d) $\int_{D_1 \cup D_2} f(x, y)d(x, y) > 2.5$

   (e) $\int_{D_1 \cup D_2} 1d(x, y) = 1 + 3\pi/4$.

6. Let $D_1 = \{(x, y) : |x + y| \leq 1\}$ and $D_2 = \{(x, y) : |y - x| \leq 1\}$. Which of the following statements is false?

   (a) $\int_{D_1 \cap D_2} (x + y)d(x, y) = 0$.

   (b) $\int_{D_1 \cap D_2} (x - y)d(x, y) = 0$.

   (c) $\int_{D_1 \cap D_2} (x^2 - y^2)d(x, y) = 0$.

   (d) $\int_{D_1 \cap D_2} x^2 d(x, y) = 1/3$.

   (e) $\int_{D_1 \cap D_2} y^2 d(x, y) = 2/3$.

7. Let $f(u) = \int_0^u e^{-x^2}dx$ for $u \in (-\infty, \infty)$ and let $I = \lim_{u \to \infty} f(u)$. Which of the following statements is false?

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(a) f is strictly increasing on $(-\infty, \infty)$.

(b) $\int_{-\infty}^{\infty} e^{-x^2/2} dx = 2\sqrt{2}I$.

(c) $\int_{R^2} e^{-x^2-y^2} d(x,y) = 4I^2$.

(d) There exists some positive integer k such that $k\pi \int_0^{\infty} re^{-r^2} dr = I^2$.

(e) There exists some positive integer k such that $I = k\sqrt{\pi}/2$.

8. For $a \in (-\infty, \infty)$ and $b > 0$, define $G(a,b) = \int_0^{\infty} x^{a-1} e^{-x/b} dx$. Which of the following statements is false?

(a) $\lim_{a\to\infty} G(a,1) < \infty$.
(b) $G(a,b) < \infty$ for $a > 0$, $b > 0$.
(c) $G(a,b) = b^a G(a,1)$ for $a > 1$, $b > 0$.
(d) $G(a+1,1) = aG(a,1)$ for $a > 1$.
(e) $G(a,1) = \infty$ for $a < 0$.

9. For $m \geq 0$, $n \geq 0$, define $G(m,n) = \int_0^1 x^m (1-x)^n dx$. Which of the following statements is false?

(a) $0 \leq G(m,n) \leq 1$ for $m \geq 0$, $n \geq 0$.
(b) $(m+1)G(m,n+1) = (n+1)G(m+1,n)$ for $m \geq 0$, $n \geq 0$.
(c) $1/G(m,2) = (m+1)(m+2)(m+3)$ for $m \geq 0$.
(d) $G(m,n) = G(n,m)$ for $m \geq 0$, $n \geq 0$.
(e) $G(m,0) = 1/(m+1)$ for $m \geq 0$.

10. Which of the following integrals is ∞?

(a) $\int_0^2 x^{-0.5} dx$.

(b) $\int_0^1 \frac{\sin(x)}{x} dx$.

(c) $\int_2^{\infty} \frac{1}{x \ln(x)} dx$.

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(d) $\int_0^{\infty} 2^{-x}dx.$

(e) $\int_1^{\infty} \frac{\ln(x)}{x^2}dx.$

第二部份: 計算題 (共 50 分)

11. (20pt) **Short** questions, you just need to give the final answer **in each part.**

(此大題請直接填寫答案,不需要提供過程說明)

a. (5pt) $f\left(\frac{x^2-1}{x^2+1}\right) = x, x > 0$. Find $f'(0)$

b. (5pt) $f(x) = (\ln x)^{\frac{1}{x}}, x > 0$. Find $f'(e)$.

c. (5pt) $f(x) = (\ln x)^{\frac{1}{x}}, x > 0$. Find $\lim_{x\to\infty} f(x)$

d. (5pt) $f(x) = \frac{1-cos(x)}{1+cos(x)}$. Find $f'(x)$

12. (10pt) Show your work to get the points. **(請提供計算過程說明)**

Let $f(x) = 2^{-x} + 4^{-x} - 1, g(x) = (f(x))^{-1/x}$ for $x \geq -1, x \neq 0$. Find $g'(x)$.

13. (10pt) Show your work to get the points. **(請提供計算過程說明)**

Find the minimum distance from the origin to the surface $z^2 = xy + 1$.

14. (10pt) Show your work to get the points. **(請提供計算過程說明)**

Sketch graph of $f(x)$ and determine its maximum, minimum, inflection points, asymptotes if they exist

$$f(x) = \exp(-\frac{(x+1)^2}{4}), x \in R.$$

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