| 考試科目 | 微積分 | 系所別 | 統計系、企管系、資管系、經濟系(二年級) | 考試時間 | 7 月 10 日(三) 第四節 |
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Multiple choice problems (5 points each; 100 points in total; single answer for each problem)

選擇題請在答案卡上作答，否則不予計分。

1. Let $f(x) = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$. Then
   (a) $f(x)$ does not exist
   (b) $f(x) = \sum_{n=0}^{\infty} x^n$
   (c) $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
   (d) $f(x) = e^{-x}$
   (e) None of the above.

2. Let $f(x) = \frac{\sin x}{x^2}$. Find $\lim_{x\to 0-} f(x)$.
   (a) 0
   (b) $\infty$
   (c) $-\infty$
   (d) Does not exist.
   (e) None of the above.

3. Find the integral $\int_0^\infty x^n e^{-x^2}dx$.
   (a) $\frac{1}{2}\Gamma(\frac{n+1}{2})$
   (b) $\frac{1}{2}\Gamma(\frac{n}{2})$
   (c) $\frac{1}{2}\Gamma(\frac{n-1}{2})$
   (d) $\Gamma(\frac{n}{2})$
   (e) None of the above.

4. Find the critical points of the function $f(x,y) = x^3 - y^2 - xy + 1$.
   (a) (0,0)
   (b) (0,0) and $(-\frac{1}{6}, \frac{1}{12})$
   (c) (0,0) and $(\frac{1}{6}, -\frac{1}{12})$
   (d) Does not exist.
   (e) None of the above.

5. Find the saddle point of the function $f(x,y) = x^3 - y^2 - xy + 1$.
   (a) (0,0)
   (b) $(-\frac{1}{6}, \frac{1}{12})$
   (c) $(\frac{1}{6}, -\frac{1}{12})$
   (d) Does not exist.
   (e) None of the above.

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
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| 考試科目 | 微積分 | 系所別 | 統計系、企管系、資管系、經濟系（二年級） | 考試時間 | 7月10日(三) 第四節 |
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6. Find the point that corresponds to the relative minimum of $f(x,y) = x^3 - y^2 - xy + 1$.
   (a) (0,0)
   (b) $(-\frac{1}{6}, \frac{1}{12})$
   (c) $(\frac{1}{6}, -\frac{1}{12})$
   (d) Does not exist.
   (e) None of the above.

7. Find the absolute maximum value of the function $f(x,y) = x^2 + xy + y^2$ on the circular region $R$, $R = \{x^2 + y^2 \leq 1\}$.
   (a) 3
   (b) 2
   (c) 3/2
   (d) 1
   (e) None of the above.

8. Let $f(\beta) = \sum_{i=1}^{n}(y_i - 1 - \beta x_i + 2x_i)^2$. Suppose that $\sum_{i=1}^{n} x_i = -5$, $\sum_{i=1}^{n} y_i = 5$, $\sum_{i=1}^{n} x_i^2 = 15$, $\sum_{i=1}^{n} x_i y_i = 10$. Find $\beta$ that corresponds to the minimum of $f$.
   (a) 2/3
   (b) 4/3
   (c) 8/3
   (d) 7/3
   (e) None of the above.

9. Find the Maclaurin series for $f(x) = 1/(1-x)$.
   (a) $\sum_{n=0}^{\infty} x^n$
   (b) $\sum_{n=0}^{\infty}(-x)^n$
   (c) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$
   (d) $\sum_{n=0}^{\infty} \frac{(-x)^n}{n!}$
   (e) None of the above.

10. Find the power series centered at $-2$ that is equal to $f(x) = 1/(1-x)$.
   (a) $\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^n}$
   (b) $\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^{n+1}}$
   (c) $\sum_{n=0}^{\infty} \frac{(x+2)^n}{3^n}$
   (d) $\sum_{n=0}^{\infty} \frac{(x+2)^n}{3^{n+1}}$
   (e) None of the above.

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
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| 考 試 科 目 | 微積分 | 系 所 別 | 統計系、企管系、資管系 經濟系（二年級） | 考 試 時 間 | 7 月 10 日(三) 第 四 節 |
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11. Suppose that $f(x) = x^4 - 2x^3 + 1$ for $x \in (-\infty, \infty)$ and $g$ is a function defined on $(-\infty, \infty)$ such that $g'(0) = 1$. Let $h(x) = f(x)g(x)$ for $x \in (-\infty, \infty)$. Which of the following statements is true?
    (a) $-\infty < h'(0) \le 0$.
    (b) $0 < h'(0) \le 1$.
    (c) $1 < h'(0) < \infty$.
    (d) $h'(0)$ exists but the value of $h'(0)$ cannot be determined based on the given information.
    (e) The existence of $h'(0)$ cannot be guaranteed based on the given information.

12. Suppose that $f(x) = 2 - x + x^2 + \sin(x^2)$ for $x \in (-\infty, \infty)$ and $g$ is a function defined on $(-\infty, \infty)$ such that $g(0) = 4$ and $g'(0) = 2$. Let $h(x) = g(x)/f(x)$ for $x \in (-\infty, \infty)$. Which of the following statements is true?
    (a) $-\infty < h'(0) \le 0$.
    (b) $0 < h'(0) \le 1$.
    (c) $1 < h'(0) < \infty$.
    (d) $h'(0)$ exists but the value of $h'(0)$ cannot be determined based on the given information.
    (e) The existence of $h'(0)$ cannot be guaranteed based on the given information.

13. Suppose that $f(x) = \ln(1 + x^2) + 2x$ for $x \in (-\infty, \infty)$ and $h$ is a differentiable function defined on $(-\infty, \infty)$ such that $f(h(x)) = x^2 + x$. Which of the following statements is true?
    (a) $-\infty < h'(0) \le 0$.
    (b) $0 < h'(0) \le 1$.
    (c) $1 < h'(0) \le 2$.
    (d) $2 < h'(0) < \infty$.
    (e) $h'(0)$ cannot be determined based on the given information.

14. Let $f(x) = e^x + \sin(2x)$ and $g(x) = e^x + \cos(2x)$ for $x \in (-\infty, \infty)$. Which of the following statements is true?
    (a) $-\infty < \lim_{x\to\infty} g(x)/f(x) \le 1$.
    (b) $1 < \lim_{x\to\infty} g(x)/f(x) \le 2$.
    (c) $2 < \lim_{x\to\infty} g(x)/f(x) \le 3$.
    (d) $3 < \lim_{x\to\infty} g(x)/f(x) < \infty$.
    (e) $\lim_{x\to\infty} g(x)/f(x)$ does not exist.

15. Let $f(x) = \cos(4x) - 1 - x$ and $g(x) = e^{2x} - 1$ for $x \in (-\infty, \infty)$. Which of the following statements is true?
    (a) $\lim_{x\to 0^+} f(x)/g(x)$ does not exist.
    (b) $-\infty < \lim_{x\to 0^+} f(x)/g(x) \le -2$.
    (c) $-2 < \lim_{x\to 0^+} f(x)g/(x) \le -1$.
    (d) $-1 < \lim_{x\to 0^+} f(x)/g(x) \le 0$.
    (e) $0 < \lim_{x\to 0^+} f(x)/g(x) < \infty$.

| 備 註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
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| 考試科目 | 微積分 | 系所別 | 統計系、企管系、資管系、經濟系、(二年級) | 考試時間 | 7 月 10 日(三) 第四節 |
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16. Let $f(x) = x - \int_0^x e^{-t^2} dt$ for $x \in (-\infty, \infty)$. Which of the following statements is true?
   (a) $f$ is strictly decreasing on the interval $(0, \infty)$.
   (b) $f$ has a local minimum at 0.
   (c) $\lim_{x\to 0^-} f(x) = \infty$.
   (d) $f(x) < 0$ for $x < 0$.
   (e) None of the above statements is true.

17. Suppose that $c$ is a positive constant. What is $\int_0^{2\pi/c} x\sin(cx)dx$?
   (a) $2\pi/c$.
   (b) $\pi c$.
   (c) $-2\pi/c$.
   (d) $-\pi c$.
   (e) None of the above.

18. Suppose that $\{a_n\}_{n=1}^{\infty}$ is a sequence such that $a_1 = 5$ and $a_{n+1} = \sqrt{12 + a_n}$ for $n \geq 1$. Which of the following statements is true?
   (a) $-\infty < \lim_{n\to\infty} a_n \leq 1$.
   (b) $1 < \lim_{n\to\infty} a_n \leq 2$.
   (c) $2 < \lim_{n\to\infty} a_n \leq 3$.
   (d) $\lim_{n\to\infty} a_n$ does not exist.
   (e) None of the above statements is true.

19. Let $f_n(x) = \sum_{k=2}^{n} k(k-1)2^{-k}x^{k-2} + \sum_{k=0}^{n} x^{k+1}/k!$ for $x \in (-\infty, \infty)$. Which of the following statements is true?
   (a) $\lim_{n\to\infty} f_n(x)$ exists for every $x \in (-\infty, \infty)$.
   (b) $\lim_{n\to\infty} f_n(x)$ does not exist whenever $|x| > 0$.
   (c) $\lim_{n\to\infty} f_n(x)$ does not exist whenever $|x| > 0.5$.
   (d) $\lim_{n\to\infty} f_n(x)$ does not exist whenever $|x| > 1$.
   (e) None of the above statements is true.

20. Let $D = \{(x, y) : x \geq 0, y \geq 0, 0 \leq x^2 + 4y^2 \leq 4\}$ and let $f(x, y) = (3/\pi)(x^2 + 4y^2)^{1/2}$ for $(x, y) \in D$. Which of the following statements is true?
   (a) $-\infty < \int_D f(x, y)d(x, y) \leq 2$.
   (b) $2 < \int_D f(x, y)d(x, y) \leq 4$.
   (c) $4 < \int_D f(x, y)d(x, y) \leq 6$.
   (d) $6 < \int_D f(x, y)d(x, y) \leq 8$.
   (e) None of the above statements is true.

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
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| 考試科目 | 統計學 | 系所別 | 統計學系(二年級) | 考試時間 | 7月10日(三)第2節 |
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1. [33 pts] 下列敘述正確（T）或錯誤（F）？錯誤的請簡述原因或改成正確的敘述。(回答中英文皆可) (3 pts each)

A. Nonparametric tests do not require data of interval or ratio scale.

B. When some explanatory variables of a regression model are strongly correlated, this phenomenon is called serial correlation.

C. The correlation coefficient is unit-free.

D. The coefficients of each explanatory variable in a multiple linear regression model have the same interpretation as the coefficient of the explanatory variable in a simple linear regression model.

E. In general, a blocking variable is used to eliminate the variability in the response due to the levels of the blocking variable.

F. Statistical inference for $\sigma^2$ is based on the F distribution.

G. Unstructured data conforms to a predefined row-column format.

H. Both discrete and continuous variables may assume an uncountable number of values.

I. The probability density function for a continuous distribution is positive for all values between $-\infty$ and $+\infty$.

J. A parameter is a random variable, whereas a sample statistic is a constant.

K. Big data is a catchphrase that implies a complete set of population data.

2. [9 pts] 下列敘述採用何種抽樣方法 (i. Simple Random Sampling, ii. Systematic Random Sampling, iii. Stratified Random Sampling, iv. Cluster Sampling) (3 pts each)

A. A population can be divided into 50 city blocks. The sample will include all residents from two randomly chosen city blocks.

B. A population contains 10 members under the age of 25 and 20 members over the age of 25. The sample will include two people chosen at random under the age of 25 and four people chosen at random over 25.

C. A population contains 10 members under the age of 25 and 20 members over the age of 25. The sample will include six people chosen at random, without regard to age.

3. [9 pts] 寫出適當的分析方法(例如：回答 "2 sample t -test" 即可) (3 pts each)

A. Packaged candies have three different types of colors. A data scientist wants to determine if the population proportion of each color is the same.

B. A data scientist wants to compare prices of the same textbooks sold by two different vendors.

C. A professor has a hunch that people who do the lecture reviews tend to do better on midterm and final exam. He decides to investigate this hunch. He treats the students in his statistics course this semester as a sample of all possible statistics course students. He has data on their lecture review grade and on their average exams score. What test should he use?

| 考試科目 | 統計學 | 系所別 | 統計學系（二年級） | 考試時間 | 7月 10日(三) 第 2 節 |
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4. [9 pts] 寫出計算下列機率所需的分配 （例如：回答“exponential distribution”即可） (3 pts each)

A. It is known that 10% of the calculators shipped from a particular factory are defective. What is the probability that no more than one in a random sample of four calculators is defective?

B. Studies have shown that bats can consume an average of 10 mosquitoes per minute. What is the probability that a bat consumes four mosquitoes in a 30-second interval?

C. The quality assurance engineer of a television manufacturer inspects TVs in lots of 100. He selects 5 of the 100 TVs at random and inspects them thoroughly. Assuming that 6 of the 100 TVs in the current lot are defective, find the probability that exactly 2 of the 5 TVs selected by the engineer are defective.

5. [20 pts] If the mean of 10 nonnegative numbers is 100, can

A. three of these numbers be >350? Why? (10 pts)

B. all these numbers be <90? Why? (10 pts)

6. [9 pts] According to a research, 65% of preschool children living in poverty have been exposed to cigarette smoke at home. If 200 preschool children living in poverty are selected at random, what is the probability that at least 125 have been exposed to cigarette smoke at home? (Note: $P(z > 0.82) = 0.206$.)

7. [6 pts; 3 pts each] 選擇 fewer 或 more 填空

The U.S. Food and Drug Administration (FDA) requires that all potential new drugs be tested to see whether they actually are effective, before they can be released to the public. (They are testing a null of ineffective treatment, vs. an alternative that the drug is effective.) Suppose the FDA decides to reduce the significance level on this test from .05 to .01. What result will this have upon the number of drugs released to the public?

Ans: ________(Fewer/More) ineffective drugs and ________(fewer/more) effective drugs will be released.

8. [5 pts] Suppose the distribution of college professor salaries in Taiwan has median \$61,500, and that the 25$^{th}$ and 75$^{th}$ percentiles are given by \$83,000 and \$40,000. Will there be any small salaries labeled as outliers below the median in this distribution? Justify your answer.

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
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# 國立政治大學　108　學年度 轉學生 招生考試試題



| 考 試 科 目 | 高等微積分 | 系 所 別 | 統計學系（三年級） | 考 試 時 間 | 7 月 10 日(三) 第一節 |
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1. (10 pts) For each of the following statements, determine whether it is true or false. Do not give explanation.

   (a) Let $\mathcal{N}$ be the set of all positive integers, then the set

   $$\{k^{\ell} : k \in \mathcal{N} \text{ and } \ell \in \mathcal{N}\}$$

   and the set $\mathcal{N}$ have the same cardinality.

   (b) Suppose that $\{f_n\}_{n=1}^{\infty}$ is a sequence of real-valued functions defined on the interval $[0,1]$. Suppose that $f_n$ converges pointwise to some function $f$ on $[0,1]$ as $n \to \infty$, and there exist two functions $g_1$ and $g_2$ that are continuous on $[0,1]$ such that $g_1 \le f \le g_2$ on $[0,1]$. Then, $f_n$ converges uniformly to $f$ on $[0,1]$ as $n \to \infty$.

2. (20 pts) Suppose that $\{A_n\}_{n=1}^{\infty}$ is a sequence of compact sets in $(-\infty, \infty)$. Prove or disprove that $\cap_{n=1}^{\infty} A_n$ is a compact set in $(-\infty, \infty)$.

3. (25 pts) Suppose that $f$ is a real-valued function defined on an open interval $(a,b)$. Prove or disprove that if $f$ is uniformly continuous on $(a,b)$, then $\lim_{x\to a^+} f(x)$ exists.

4. (20 pts) Suppose that $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are two sequences of real numbers. Prove or disprove that

   $$\limsup_n (a_n + b_n) \le \limsup_n a_n + \limsup_n b_n.$$

5. (25 pts) Suppose that $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are two sequences of real numbers. Suppose that the sequence $\{\sum_{k=1}^{n} a_k\}_{n=1}^{\infty}$ is bounded and the sequence $\{b_n\}_{n=1}^{\infty}$ is decreasing with $\lim_{n\to\infty} b_n = 0$. Show that $\{\sum_{k=1}^{n} a_k b_k\}_{n=1}^{\infty}$ is convergent.

| 備 註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
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國立政治大學 108 學年度 轉學生 招生考試試題



| 考試科目 | 數理統計學 | 系所別 | 統計學系（三年級） | 考試時間 | 7 月 10 日(三) 第二節 |
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1. Let $X_1, X_2, \cdots, X_n$ be a random sample of size $n$ from a population with the following pdf and cdf,

$$f(x;\delta,\theta) = \frac{1}{\theta} e^{-(x-\delta)/\theta}, x > \delta > 0,\ \ F(x;\delta,\theta) = \begin{cases} 0, & x < \delta \\ 1 - e^{-(x-\delta)/\theta}, & x \geq \delta \end{cases},$$

where $0 < \delta, \theta < \infty$.

(a) Obtain the maximum likelihood estimators (mle) of $\delta$, $\theta$, and $\delta + \theta$, respectively. (10 pts)

(b) Obtain minimum variance unbiased estimators of $\delta$ and $\theta$, respectively. (20 pts)

(c) Determine a size $\alpha$ likelihood ratio test for $H_0: \theta = \theta_0$ against $H_1: \theta > \theta_0$. (15 pts)

(d) Suppose $\delta = 1$ is known. Determine a size $\alpha$ uniformly most powerful test for $H_0: \theta = \theta_0$ against $H_1: \theta > \theta_0$. (15 pts)

(e) Suppose $\delta = 1$, and $\hat{\theta}$ is the mle of $\theta$. Determine the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$. (15 pts)

2. The number of typing errors per page $X$ is known to have a Poisson distribution with parameter $\theta$. However, $\theta$ itself is a random variable having an exponential distribution, i.e.,

$$f(x|\theta) = \frac{\theta^x e^{-\theta}}{x!}, x = 0, 1, 2, \ldots, \theta > 0;\ \ f(\theta) = e^{-\theta}, \theta > 0.$$

(a) Compute the expected number of typing errors per page. (10 pts)

(b) Determine the unconditional pdf of $X$. (15 pts)

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
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