國立政治大學 102 學年度轉學生招生考試試題

考試科目	微積分	系別	商學院共同科 4141, 4151, 4161, 4181	考試時間	7月 12日(五) 第四節

Part I. (50 pts)

1. (10pts)Show that the function f(x)=|x|+1 is differentiable everywhere except at 0.

2. (10pts)Evaluate the following limits.

   (a) $\lim_{x\to 0} x^3 \sin\frac{1}{x}$ (b) $\lim_{x\to\infty}\frac{3x+3}{\sqrt{x^2-1}}$

3. (10pts) Find dy/dx and d^2y/dx^2 in terms of x and y , if $2xy+x^3=4$.

4. (10pts) Find an equation of the tangent line at the point of the graph of $y=x^2\sin 2x$, where $x=\pi/2$.

5. (10pts) Find f(x) given that $f'(x)=x^3(x^2+1)^{1/2}$ and f(0)=1/3

備註	試題隨卷繳交

國立政治大學 102 學年度轉學生招生考試試題

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Part II. (50 pts)

1. (15 pts) Find the following integrals.

   (a) (5 pts) $\int_0^1 (\ln x)^2\, dx$.

   (b) (5 pts) $\int_0^{\pi} \sin^2 x\, dx$.

   (c) (5 pts) $\int \frac{1}{x^2-3}\, dx$.

2. (10 pts) Show that the following series converges and find its sum,

$$\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}+\frac{1}{3^n}\right).$$

3. (15 pts) Find the radius and interval of convergence of the power series, $\sum_{n=1}^{\infty}\frac{3^n x^n}{\sqrt{n+1}}$.

   (Note: The convergence of the boundary points should be discussed as well.)

4. (10 pts) Evaluate the integral, $\int_0^1 \int_x^1 \sin(y^2)\, dy dx$.

備註	試題隨卷繳交

國立政治大學 102學年度轉學生招生考試試題

考試科目	統計學	系別	統計學系 4141	考試時間	7月12日(五)第二節

1. 某一玩具工廠宣稱他所製造的產品的不良率小於 3%。消基會從這家工廠隨機抽取 500 件產品，發現不良率為 5%。
   (a) 請寫出案例中之母體(Population)為何？(5 分)
   (b) 請寫出案例中之母體參數(Parameter)為何？(5 分)
2. 請寫出下列分配函數(Distribution)的平均數(Mean)、中位數(Median)與眾數(Mode)的大小關係。
   (a) 對稱分配(Symmetrical)？(5 分)
   (b) 左偏分配(Skewed to the Left)？(5 分)
3. 請寫出二項分配(Binomial Probability Distribution)與超幾何分配(Hypergeometric Probability Distribution)的不同點？(10 分)
4. 請解釋下列名詞：
   (a) 貝氏定理(Bayes' Theorem)。(5 分)
   (b) 統計量(Statistics)。(5 分)
   (c) 抽樣分配(Sampling Distribution)。(5 分)
   (d) 中央極限定理(Central Limit Theorem)。(5 分)
   (e) 不偏性(Unbiased)。(5 分)
   (f) 一致性(Consistency)。(5 分)
5. (a) 若 H_0：此人是不該愛的 v.s. H_1：此人是該愛的。試寫出此例之型 I 錯誤與型 II 錯誤的事件。(5 分)
   (b) 若 H_0：此人是該愛的 v.s. H_1：此人是不該愛的。試寫出此例之型 I 錯誤與型 II 錯誤的事件。(5 分)
   (c) 就(a)與(b)依照正常的角度，統計假設會傾向採用何者？為什麼？(5 分)
6. 容易生氣的人似乎較可能得心臟病，根據某醫院從台北市隨機抽取的 200 位上班族做調查，得到下表之資料。

	易怒指標			
類別	低	中	高	合計
有心臟病	15	15	50	80
沒有心臟病	50	50	20	120
合計	65	65	70	200

   (a) 請寫出統計假設(Null Hypothesis & Alternative Hypothesis)。(5 分)
   (b) 請寫出檢定統計量(Test Statistics)。(5 分)
   (c) 若檢定結果 p 值<0.01，我們是否可以下結論說容易發怒會導致心臟病？(5 分)
7. 考慮下列兩種線性迴規模型：

$$Y_i=\beta_0+\beta_1 X_i+U_i,\quad i=1,2,\cdots,n$$

$$X_i=\beta_0+\beta_1 Y_i+W_i,\quad i=1,2,\cdots,n$$

   (a) 這兩條估計的迴歸線都會通過(X,Y)平面上的那一點？(5 分)
   (b) 這兩種模型的判定係數(R^2)是否相同？(5 分)

考試科目	高等微積分	系別	統計學系/三年級 4146	考試時間	7月12日(五) 第 一 節

Part I. (5 × 10 = 50 pts) For each of the following problems, state that it is True or False.

Q1) Let $f(x)=\sin(\frac{\pi}{x})$ defined on (0, 1), then $f(x)$ is uniformly continuous on (0, 1).

Q2) The sequence $f_n(x)=x^n$ converges uniformly on $[0, \frac{1}{2}]$.

Q3) Let $f_n(x)=nxe^{-nx^2}$, $n=1,2,3,\ldots$, and $0\le x\le 1$. It follows that $\lim_{n\to\infty}\int_0^1 f_n(x)dx=\int_0^1 \lim_{n\to\infty} f_n(x)dx$.

Q4) Let $x_n=(-1)^n+\frac{1}{n}$, then $\limsup x_n=\liminf x_n$.

Q5) The sequence defined as

$$0,\ 1,\ \tfrac{3}{2},\ 2,\ \tfrac{7}{3},\ \tfrac{8}{3},\ 3,\ \tfrac{13}{4},\ \tfrac{14}{4},\ \tfrac{15}{4},\ 4,\ \ldots$$

is a Cauchy sequence.

Q6) Let $f,g,h:R\to R$ be three functions. If f and g are both differentiable at x, $f'(x)=g'(x)$, and $f\le h\le g$, then h is also differentiable at x and $h'(x)=f'(x)$.

Q7) Let J be an open interval in R. If $f:J\to R$ is differentiable and

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=0$$

for every $x\in J$, then f is constant on J.

Q8) Define $f:[-1, 1]\to R$ by

$$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right), & \text{if } x\neq 0\\ 0, & \text{if } x=0.\end{cases}$$

The set of all x such that $f(x)=0$ is a compact subset of $[-1, 1]$.

Q9) The equation

$$x^{180}+\frac{84}{1+x^2+\cos^2 x}=119$$

has at least two solutions in R.

Q10) If $f(x,\theta)$ is differentiable in θ for every x in R, then

$$\frac{d}{d\theta}\int_{-\infty}^{\infty} f(x,\theta)dx=\int_{-\infty}^{\infty}\frac{\partial}{\partial\theta}f(x,\theta)dx.$$

備註	試題隨卷繳交

考試科目	高等微積分	系別	統計學系/三年級 4146	考試時間	7月12日(五) 第 一 節

Part II. Comprehension Problems (10 × 5 = 50 pts)

Q11) Consider the function $f(x,y)=\frac{1}{384}x^2y^4e^{-y-(x/2)}$, $x>0$ and $y>0$. Compute $\int_0^\infty\int_0^\infty f(x,y)\,dx\,dy$.

Q12) Let $f:[0,\infty)\rightarrow R$ be given by $f(x)=\sqrt{x}$ and $\varepsilon=\frac{1}{2}$. Find a δ such that if $d(x,y)<\delta$, then $d(f(x),f(y))<\varepsilon$.

Q13) Show that $\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}$ is uniformly and absolutely convergent in $[0, 2\pi]$.

Q14) Estimate the value of the integral $\int_0^1 e^{x^2}\,dx$ by using Taylor's Theorem applied to the series for e^x.

Make sure that your estimation has the maximum error less than 0.01 (accurate to two decimal places).

Q15) If $f'(x)>0$ at all points of the interval (a, b), prove that $f(x)$ is strictly increasing in (a, b).

備註	試題隨卷繳交

國立政治大學 102 學年度轉學生招生考試試題

考試科目	數理統計學(含機率論)	系別	統計 4146	考試時間	7 月 12 日(五) 第二節

1. A high school requests its seniors to take a proficiency test before graduation. A student passing all 3 subjects (Literature, Science, and Ethics) would be awarded a diploma; otherwise, he (she) would receive only a certificate of attendance. A test given to N senior students this year resulted in the following table:

Subject Area	Number of students failing
Literature	n_1
Science	n_2
Ethics	n_3

Also, $n_1 + n_2 + n_3 < N$.

(a) Let F_1 be "Student fails Literature", F_2 be "Student fails Science" and F_3 "Student fails Ethics". Assume they are independent events, what proportion of next year's seniors can be expected to fail to qualify for a diploma? (6%)

(b) If F_1 and F_2 are dependent, while both of them are independent to F_3, what is the upper bound for the proportion of next year's seniors can be expected to fail to qualify for a diploma? (9%)

2. A boy pays \$1 a throw in order to win a \$3 Spiderman, and his probability of winning on each throw is 0.2. Let X denote the number of throws required to win the Spiderman.

(a) Derive the moment generating function for X. (7%)

(b) What is the probability that his net return is non- negative? (5%)

3. Let X and Y be continuous random variables with joint pdf: $f(x,y) = k(x+y)$, $0 < x < y < 2$; and 0 otherwise.

(a) Find k. (4%)

(b) Let $U=X$ and $V=XY$, find the joint pdf of U and V. (8%)

(c) Find the marginal pdf of V. (6%)

4. Let $X_1, \cdots, X_n$ be a random sample from $f(x) = \dfrac{3}{\theta(1+x/\theta)^4}$, $x > 0$, $\alpha > 0$.

Let $\overline{X}_n$ be the sample mean.

(a) Show $\overline{X}_n \xrightarrow{P} \theta/2$. (6%)

(b) Find the approximate distribution for $\exp(-\overline{X}_n)$. (9%)

5. Let $X_1, X_2, \cdots, X_n$ be a random sample from $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta > 0$.

(a) Find the MME (method of moment estimator) of θ. (6%)

(b) Find the MLE of θ. (6%)

(c) Find the UMVE(unbiased minimum variance estimator) of θ. (14%)

(d) Find the UMP(uniformly most powerful) size α test for $H_0: \theta \ge \theta_0$ vs. $H_1: \theta < \theta_0$. (14%)

備註	試題隨卷繳交