

考 試 科 目	微積分	系 所 別	科技管理與智慧財產研究所	考 試 時 間	2 月 12 日 (三) 第 二 節
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以下各題，每大題 10 分，共 100 分。請以藍（黑）筆依序作答，並寫明計算程序，無過程 0 分計算

1.(1)試求 $\int \sin 4x \cos 3x dx = ?$

(2)試求 $\lim_{x \rightarrow 0} \frac{\sin(a+4x) - 2 \sin(a+2x) + \sin a}{x^2} = ?$

2.試求 $\int_0^1 \cos^{-1} x dx = ?$

3.設 $f(x) = \int_0^x \frac{e^{t^2} - 1}{t^2} dt$ ，試求 $f^{(3)}(0) = ?$

4.試解微分方程式 $y^4 e^{2x} + \frac{dy}{dx} = 0$ 。

5.設 $f(x) = 4 + \int_1^{x^2} \sec(t-1) dt$ ，試求 $f(x)$ 在 $x = -1$ 的切線方程式為何？

6.某一汽車經銷商計算其每賣一部新車的獲利（以十萬元為單位）為 $Y = X^2$ ，若 X 的機率密度函數如下：

$$f(X) = \begin{cases} c(1-x), & 0 < x < 1, c \text{ 為常數} \\ 0, & \text{其他} \end{cases}$$

(1)試求常數 c 的值為何？(2 分)

(2)試推導 Y 的機率密度函數 $f(Y)$ 為何？(5 分)

(3)試計算該經銷商在賣下一部新車時，其獲利小於四千元之機率為何？(3 分)

7.試求雙紐線 $(x^2 + y^2)^2 = a^2(x^2 - y^2)$ ， $a > 0$ 在平面上所圍的面積為何？

8.試求 $\int \frac{2}{e^{2x} + e^{-2x}} dx = ?$

9. 設某產品的產量函數為 $f(x, y) = 200x^{\frac{3}{4}}y^{\frac{1}{4}}$ 。[x 為勞力財的單位數， y 為資本財的單位數]

若每一單位的勞力財為 100 元，且每一單位的資本財為 200 元，試求在總花費不超過 80000 元的最大產量為何？

10.試求 $\int_0^{\sqrt{\frac{\pi}{2}}} \int_x^{\sqrt{\frac{\pi}{2}}} \int_1^5 \sin y^2 dz dy dx = ?$

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。
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I. Multiple Choice (1 points each)

Identify the letter of the choice that best completes the statement or answers the question.

- Holding all other forces constant, when the price of gasoline rises, the number of gallons of gasoline demanded would fall substantially over a ten-year period because
 - buyers tend to be much less sensitive to a change in price when given more time to react.
 - buyers tend to be much more sensitive to a change in price when given more time to react.
 - buyers will have substantially more real income over a ten-year period.
 - the quantity supplied of gasoline increases very little in response to an increase in the price of gasoline.
- Suppose that Mike is filling out a survey. The survey asks him what he would do if the price of his favorite shaving gel increased. Mike reports that he would switch to a different brand. The survey then asks what he would do if the price of all shaving gels increased. Mike reports that he must use shaving gel, so he would have to adjust his spending elsewhere. These examples illustrate the importance of
 - changes in total revenue in determining the price elasticity of demand.
 - the definition of a market in determining the price elasticity of demand.
 - a necessity versus a luxury in determining the price elasticity of demand.
 - the time horizon in determining the price elasticity of demand.
- Suppose that Elmer's hourly wage increases, and he decides to work fewer hours. For Elmer, the substitution effect of the wage change is
 - greater than the income effect.
 - equal to the income effect.
 - smaller than the income effect.
 - We do not have enough information with which to answer the question.
- In the work-leisure trade-off analysis, if tax reductions induce a consumer to work more, an explanation for this occurrence is that
 - leisure is an inferior good.
 - leisure is a Giffen good.
 - the substitution effect dominates the income effect.
 - the income effect dominates the substitution effect.

備註

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- 試題請隨卷繳交。

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5. Which of the following production functions exhibits constant returns to scale?

- A. $Q = K \cdot L$.
- B. $Q = K^{0.4} \cdot L^{0.4}$.
- C. $Q = K + L - 2$.
- D. $Q = 2K + 3L$.

6. Consider a market demand function $Q = e^{3600 + \ln \frac{1}{(p+8)^3}}$. There is a monopolist in the market with no fixed cost, and its marginal cost is \$8 at all levels of output. What price will it charge to maximize its profits?

- A. 20
- B. 12
- C. 10
- D. 16

7. Josh sells mojos as a monopolist in a market with two consumers, Lyle and Yu-Min. Lyle's demand function for mojos is $Q_L = 20 - P_L$. Yu-Min's demand function is $Q_Y = 10 - P_Y$. Josh only knows there are two consumers but does not know who has a higher demand. Therefore, he decides to offer two options $O_1 = (q_1, m_1)$ and $O_2 = (q_2, m_2)$. If a consumer chooses O_i , he pays m_i dollars to buy q_i mojos. Consumers can only choose one offer, and resale is not allowed. If Josh wants to practice 2nd-degree price discrimination, which of the following offers should he use?

- A. $O_1 = (20, 200)$ and $O_2 = (10, 50)$
- B. $O_1 = (20, 125)$ and $O_2 = (8, 48)$
- C. $O_1 = (20, 140)$ and $O_2 = (6, 42)$
- D. $O_1 = (20, 165)$ and $O_2 = (4, 32)$

8. In the game matrix below, the first payoff in each pair goes to player A who chooses the row, and the second payoff goes to player B, who chooses the column. Let a, b, c and d be positive constants.

		Player B	
		Left	Right
Player A	Top	$a, 1$	$b, 0.5$
	Bottom	$1, c$	$2, d$

If the game only has a mixed-strategy Nash equilibrium, then we can conclude that

- A. $b > a$ and $d > c$.
- B. $a > 1$ and $d < 0.5$.
- C. $a < 1$ and $b > 2$.
- D. None of the above is correct.

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9. Concerts in Taipei Dome cause noise and earthquake-like vibrations for surrounding residents. According to an estimation, there are 1,000 residents affected, and the average mental damage per concert for each resident is $D_R = 5 + Q$, where Q stands for the number of concerts per month. On the other hand, the monthly concert revenue of Taipei Dome is $R_T = 10,000Q - 1,000Q^2$. The marginal cost of holding each concert is 1,000. What is the socially optimal number of concerts?

- A. 1
- B. 3
- C. 5
- D. 7

10. Which of the following is not a characteristic of a public good?

- A. It is not excludable.
- B. It is not diminished or depreciated as additional people consume the good.
- C. Its benefits cannot be withheld from anyone.
- D. Because it is a free good, there is no opportunity cost.

11. Which of the following events would decrease the unemployment rate?

- A. A decrease in the labor force combined with an increase in the total population.
- B. A decrease in the labor force, with no change in the number of employed workers.
- C. A decrease in the number of employed workers, with no change in the labor force.
- D. An increase in the labor force, with no change in the number of employed workers.

12. Which of the following policies is considered quantitative easing?

- A. The central bank prints money and directly transfers it to financial institutions.
- B. The central bank purchases short-term government bonds.
- C. The central bank sells short-term government bonds.
- D. The central bank purchases long-term government bonds.

13. In the long run, what is the primary cause of inflation?

- A. Increase in the minimum wage.
- B. Government's currency manipulation.
- C. Growth in the money supply.
- D. Protective trade policies and high tariff rates.

14. All else being equal, which of the following outcomes would we expect from a decrease in Taiwan's interest rates?

- A. Citizens decide to hold more foreign bonds.

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- B. Citizens choose to hold more currency.
- C. Domestic business owners decide to invest in new equipment for their factory.
- D. All of the above are correct.

15. If inflation expectations rise, the short-run Phillips curve shifts:

- A. to the right, so that at any inflation rate, unemployment is higher in the short run than before.
- B. to the left, so that at any inflation rate, unemployment is higher in the short run than before.
- C. to the right, so that at any inflation rate, unemployment is lower in the short run than before.
- D. to the left, so that at any inflation rate, unemployment is lower in the short run than before.

16. In a world described by the classical dichotomy, assuming GDP and the price level are given, if President Trump raises U.S. import tariffs while all other factors remain unchanged, how will this policy affect Taiwan's loanable funds market?

- A. The real interest rate will increase.
- B. The real interest rate will decrease.
- C. The real interest rate will stay the same.
- D. It will shift both the demand and supply of loanable funds simultaneously.

17. Consider a partial equilibrium model for good *A* in Taiwan. If President Trump raises U.S. import tariffs on that good, thereby reducing Taiwan's export prices, and assuming Taiwan is a small country and the U.S. is a large country in the market for good *A*, how will this affect Taiwan's consumer surplus?

- A. It will increase.
- B. It will decrease.
- C. It will stay the same.
- D. The change will be ambiguous.

18. If President Trump raises U.S. import tariffs, assuming all other factors remain unchanged, how will this policy affect Taiwan's economy according to the classical Keynesian model?

- A. The output will decrease but the price level will increase.
- B. Both the output and price level will decrease.
- C. The output will decrease but the price level will stay the same.
- D. The output will increase but the price level will stay the same.

19. If President Trump raises U.S. import tariffs and leads to workers in the associated industries in Taiwan losing their jobs and other things being equal, how will this affect the natural rate of unemployment in Taiwan?

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- A. It will increase.
- B. It will decrease.
- C. It will stay the same.
- D. It depends on how the cyclical unemployment rate changes.

20. In the long run, assuming GDP and price level are given, if investors in Taiwan fear that President Trump will raise the U.S. import tariff in the near future and other things equal, how will this affect Taiwan's economics?

- A. An increase in net export of Taiwan and a rise in the real interest rate
- B. A drop in net export of Taiwan and a rise in the real interest rate
- C. An increase in net export of Taiwan and a drop in the real interest rate
- D. A drop in net export of Taiwan and a drop in the real interest rate

II. Problems and Short-essay Questions

1. A competitive market currently has 120 firms, all of which has a total cost function of $TC(q) = 9 + 11q + q^2$, where q denotes the quantity produced by each firm.

a. (10 points) Currently, the market price $p = 19$. What is the total quantity supplied in the market? How much profit does each firm make in the short-run equilibrium?

b. (5 points) What is the long-run equilibrium price? Suppose the market demand is given by $p = 35 - \frac{1}{30}Q$, where Q denotes the market quantity demanded. How many firms are there in the long-run equilibrium?

c. (5 points) Suppose now the government levies a \$2 specific tax on each unit of the output produced. How many firms are there in the long-run equilibrium?

2. An industry has two firms, A and B , producing output at constant unit costs of 1 and $C > 1$ per unit, respectively. The demand function for the industry is

$$Q = \frac{V}{P}$$

with $V > C$.

a. (10 points) Suppose two firms are Cournot competitors. Find the equilibrium price, quantities produced, and profits for two firms.

b. (10 points) Suppose two firms are Bertrand competitors. Moreover, when both firms set identical prices, all consumers purchase from the more cost-efficient firm. Find the equilibrium price, quantities produced, and profits for two firms.

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3. Consider consumers in Country *A* and Country *B*, who each consumes 2 units of bananas and 5 units of pizza every year. The currency unit of Country *A* is AD, and the currency unit of Country *B* is BD. Please answer the following questions in detail:

a. (10 points) Assume that in year 2000, the prices of bananas and pizza in Country *A* were 3 AD and 5 AD, respectively, and the prices of these items in Country *B* were 2 BD and 2 BD. Suppose the purchasing power parity holds between the two countries. What was the nominal exchange rate of AD in terms of BD in year 2000?

b. (5 points) Does the law of one price hold according to the information in (a)? Why or why not?

c. (5 points) In 2001, the price of bananas and pizza in Country *A* became 2 AD and 6 AD, respectively. Please calculate the CPI index of Country *A* in 2001, using 2000 as the base year.

4. In this question, we will examine the monetary and fiscal policies of Country *Z*. The economy of Country *Z* is described by the following equations:

$$Y = C + I + G$$

$$C = 100 + 0.5(Y - T)$$

$$I = 300 - 50r$$

where *Y* is GDP, *C* is consumption, *I* is investment, *G* is government spending, *T* is taxes, and *r* is the real interest rate (for example, if the real interest rate is *q*%, then $r = q$).

Country *Z* has recently experienced a significant negative economic shock. The president plans to implement policies aimed at stimulating the economy. At first, the president considers an expansionary fiscal policy and the corresponding multiplier effect.

a. (2 points) What is the marginal propensity to consume (MPC) in this economy?

b. (3 points) Calculate the fiscal multiplier.

c. (5 points) Suppose Country *Z* receives foreign aid, which enables an increase in government spending without raising taxes. Given a change in government spending ($\Delta G = 50$) and no change in taxes ($\Delta T = 0$), what is the resulting increase in aggregate demand?

In Country *Z*, the nominal interest rate is fixed at 8%. The current growth rate of the money supply is 7%, and the current inflation rate is 5%. To stimulate investment, the central bank increases the growth rate of the money supply to 9%.

d. (2 points) What is the inflation rate after the central bank's monetary policy?

e. (3 points) Referring to the Fisher effect (Fisher equation), calculate the new real interest rate after the monetary policy.

f. (5 points) Given the change in the real interest rate, calculate the change in investment (ΔI).

What is the resulting increase in aggregate demand?

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考 試 科 目	管理個案分析	系 所 別	科技管理與智慧財產研究所 科技管理組一般生	考 試 時 間	2 月 12 日 (三) 第二節
<p>一、隨著 ChatGPT 推出而掀起生成式 AI 風潮，有愈來愈多競爭者加入這個市場，包括 Google Gemini、Claude AI、Perplexity 等，雖然 ChatGPT 已主宰生成式 AI 市場，Open AI 仍積極開發 SearchGPT，欲從 AI 聊天功能擴展至 AI 搜尋引擎，將來勢必衝擊 Google 在搜尋引擎市場的龍頭地位。如果你是 Google 的執行長，你會如何維持自身搜尋引擎的競爭優勢，並提升 Gemini 的市場佔有率？(20%)</p> <p>二、我國環境部於 2024 年 12 月 30 日表示，為了配合 2030 年減碳新目標，將擴大應盤查登錄對象，將年排碳量達 1 萬噸的中小型製造廠、百貨公司、量販店、連鎖超商、電信業、運輸業、旅館業、大專校院、醫學中心等約 500 家企業或組織列入。雖初期僅要求盤查登錄，無須查驗，現階段也不納入碳費徵收對象。然而，若你是四大連鎖超商其中一家的總經理，你會如何規劃企業整體的減碳策略，來因應未來可能增收碳費的影響？(20%)</p> <p>三、近年在循環經濟的全球趨勢下，許多產業紛紛開創或導入循環模式，包含減少自然資源的開採和消耗，減少廢棄物的產生，增加重複使用，以及促進回收再利用的機制等。然而，包含筆記型電腦（筆電）在內的電子垃圾是近年成長最快速的廢棄物種類之一，為了減少電子廢棄物和挑戰主流筆電品牌計畫性淘汰的傳統做法，2020 年成立於美國加州矽谷的模組化筆電新創公司 Framework，推出了一款具可升級性和可修復性的筆電，該筆電所有的組件，包括處理器、鍵盤、螢幕、硬碟和擴充卡等，都被設計成獨立模組，可拆解和更換，讓使用者在需要維修或升級時能輕鬆更換組件，緩解通常需要昂貴維修或購買新筆電才能解決的問題，藉以倡導消費者的維修權，延長筆電使用的生命週期。該公司企盼在永續與獲利之間找到平衡，啟發更負責任的生產與消費方式。為此，Framework 與多家供應商合作製造其筆電模組，消費者可在 Framework 的平台或直接向供應商購買單獨的模組，並由供應商直接運送到消費者手中。此外，透過安裝和維修指引，消費者可獲取必要的操作資訊來維護自己的筆電。Framework 並利用線上論壇建立用戶社群，分享如何以創意客製化筆電的經驗和方法。請根據上述資訊，分析以下問題：(60%)</p> <ol style="list-style-type: none"> 1. Framework 模組化筆電的循環商業模式會遭遇哪些關鍵的阻礙？ 2. 在目前競爭激烈的筆電市場中，Framework 可採取什麼策略來擴大其市場佔有率？ 3. 在筆電產業中，除了模組化的商業模式之外，你認為筆電品牌商可以有什麼創新的做法來延長產品使用生命週期、減少電子廢棄物？而這樣的做法又可在永續與獲利之間取得平衡。 					
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