

考試科目	基礎數學	所別	統計44	考試時間	3月7日(四)第一節
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1. (21pts). For each of the following statements, determine whether it is true or false. (True or false questions. Do not give explanation)

- (a) If A and B are $n \times n$ matrices, then $(A + B)(A + B) = A^2 + 2AB + B^2$.
- (b) Let A be an $m \times n$ matrix, $O_{n \times n}$ be $n \times n$ zero matrix, and $O_{m \times n}$ be $m \times n$ zero matrix. If $A^T A = O_{n \times n}$, then $A = O_{m \times n}$.
- (c) Let A be a square matrix, then $\det(-A) = -\det(A)$.
- (d) Every orthonormal set of five vectors in R^5 is a basis for R^5 .
- (e) Let λ_1 and λ_2 be two distinct eigenvalues of an $n \times n$ matrix A . Then the eigenspaces associated with λ_1 and λ_2 are orthogonal.
- (f) Any 4×3 matrix has linear dependent rows.
- (g) Suppose A is a nonsingular matrix. If A is symmetric and positive definite, then A^{-1} is also symmetric and positive definite.

2. (5pts). An automobile rental company has three locations, which we designated as P, Q, and R. When an automobile is rented at one of the locations, it may be returned to any of the three locations. Experience has shown the weekly rental distribution history as summarized below:

Location P:	80% stay at P,	10% go to Q,	10% go to R.
Location Q:	10% go to P,	60% stay at Q,	30% go to R.
Location R:	10% go to P,	20% go to Q,	70% stay at R.

Suppose the automobile rental company has a fleet of 300 cars. Initially an equal number of cars is based at each location. Please compute the steady-state vector.

考試科目	基礎數學	所別	統計441	考試時間	3月7日(五)第一節
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3. Let $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation for which

$$L\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \text{ and } L\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}.$$

(a) (4pts). Please compute $L\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$ and $L\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$. Show your work.

(b) (2pts). Find the standard matrix representing L .

4. (4pts). Is the set of all 3×3 matrices whose determinant is 1 a subspace of the vector space M_{33} . Justify your answer.

5. (5pts). Use the Gram-Schmidt process to generate an orthonormal basis for

$$W = \text{Span}\{w_1, w_2, w_3\}, \text{ where } w_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, w_2 = \begin{bmatrix} 3 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \text{ and } w_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}.$$

6. Let $A = \begin{bmatrix} 2 & 1 & 2 & 3 \\ 1 & 2 & 2 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

(a) (2pts). Find the characteristic polynomial of A .

(b) (2pts). Find eigenvalues of A .

(c) (3pts). Please give bases for the eigenspaces associated with each eigenvalue of A .

(d) (2pts). Is A diagonalizable? Explain your answer.

考試科目	基礎數學	所別	統計4141	考試時間	3月7日(日)第一節
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7. (10分) Sketch the curve $y = \frac{2x^2}{x^2 - 1}$.

8. (5分) (a) Evaluate $\lim_{x \rightarrow 3} \left(\frac{x}{x-3} \int_3^x \frac{\sin t}{t} dt \right)$ (5分) (b) Evaluate $\int_{-\infty}^0 x e^x dx$

9. (5分) (a) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$$

(5分) (b) Find $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2 y}{x^2 + y^2}$ if it exists.

10. (10分) Evaluate $\iint_R (3x + 4y^2) dA$, where R is the region in the upper half-plane

bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$

11. (10分) Find the maximum and minimum values of the function

$f(x, y, z) = x + y + z$ subject to the given constraints $x^2 - y^2 = z$, $x^2 + z^2 = 4$

考試科目	數理統計	所別	統計 4141	考試時間	3月7日(日)第三節
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1. Two basketball teams, Tiger and Lion, play a series of 5 games that ends whenever one of them has won 3 games. Suppose that each game is independently played, and is won by Team Tiger with probability p . Let Y be the number of games that are played for the series.

- (a) Write down the probability function of Y . (6pts)
- (b) Sales in tickets depends on the number of games played. Suppose that the basic sales per game is \$1,000,000, while each additional game, compared to the minimum requirement to end the series, brings \$200,000 more out of basic sales. What is the expected sales for the series? (6pts)

2. Let $f(x, y) = 1/4$, if $|x| < y$, $0 < y < 2$ be the joint pdf of (X, Y) .

- (a) What is the marginal pdf of X ? (5pts)
- (b) Find $f(y|x)$ and $E(Y|x)$. (7pts)
- (c) Find $E(XY)$. (5pts)
- (d) Find the pdf of $W = X + Y$. (7pts)

3. Let $X_1, \dots, X_n$ be a random sample from a continuous uniform distribution, $\text{UNIF}(a, b)$, where $a < b$. Let $X_{(n)}$ be the largest order statistic.

- (a) What does $\exp[X_{(n)}]$ converge in probability to? Show your work. (6pts)
- (b) Find the limiting distribution of $\exp[-n(b - X_{(n)})/(b - a)]$. Show your work.. (8pts)

4. In each of the families of distributions listed below, checks whether they have a monotone likelihood ratio in the parameter specified and finds the relevant statistic:

- (a) Poisson distribution. (4pts)
- (b) Exponential distribution. (4pts)
- (c) Gamma distribution, shape parameter. (4pts)

考試科目	數理統計	所別	統計 4141	考試時間	3 月 7 日 (B) 第三節
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5. Let $X_1, \dots, X_n$ be a random sample from the distribution $f(x; \theta) = \left(\frac{1}{\theta}\right) \left[\frac{\theta}{\theta+1}\right]^x$,

where $x = 1, 2, \dots$. Determine the UMP test of the hypothesis $H_0: \theta = \theta_0$ against the alternative $H_1: \theta > \theta_0$. (15pts)

6. Let $X_1, \dots, X_n$ be a random samples from uniform distribution on $(0, \theta)$.

(a) Find the sufficient statistic for θ , called T_1 . (3pts)

(b) Show the three estimators, T_2, T_3 and T_4 , are unbiased estimators of θ ,

where $T_2 = T_1 + \min(X_1, \dots, X_n)$, $T_3 = 2\bar{X}$ and $T_4 = (n+1)\min(X_1, \dots, X_n)$.

(6pts)

(c) For T_2, T_3 and T_4 , based on T_1 find the improved estimators T_2^*, T_3^* and T_4^*

those have less variance than T_2, T_3 and T_4 , i.e. $\text{Var}(T_i) > \text{Var}(T_i^*)$, $i = 2, 3, 4$.

(8pts)

(d) What do you find from part (c)? Which theorem can explain the result from part (c)? (6pts)

考試科目	統計方法	所別	統計4141	考試時間	3月7日(日)第四節
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Choose ONE best answer for Questions 1-5.

- 1) (4%) Which of the following is not the result of the Central Limit Theorem?
 - (a) the binomial distribution can be approximated by the normal distribution;
 - (b) the distribution of the sample means is approximately normal when the sample size is large;
 - (c) the z value can be used to compute the probabilities related to the sample mean;
 - (d) the t distribution gets closer to the z distribution when the degrees of freedom become larger.
- 2) (4%) Which of the following is best for interpreting the 95% confidence interval for the mean?
 - (a) 95% of the population means will fall within the confidence interval;
 - (b) for each confidence interval the probability of including the true population mean is 0.95;
 - (c) the probability that the midpoint of the confidence interval is equivalent to the true population mean is 0.95;
 - (d) 95% of the constructed confidence intervals will include the true population mean.
- 3) (4%) Consider testing the hypotheses $H_0: \mu \leq 10$ and $H_1: \mu > 10$ based on a random sample, where μ is the population mean. Which of the following statement may not be correct?
 - (a) A good decision rule will reject H_0 if the observed sample mean is large.
 - (b) A good decision rule will reject H_0 if the observed p -value is small.
 - (c) Once the probability of Type I Error is fixed, the probability of Type II Error is also fixed.
 - (d) This is a right-tailed test.
- 4) (4%) Which of the following may not increase the power of a test?
 - (a) obtain a new sample; (b) increase the significance level;
 - (c) reduce the probability of Type II Error; (d) increase the sample size.
- 5) (4%) When using the t distribution to develop the confidence interval for the population mean, which of the following assumptions is not required?
 - (a) the population is normal or approximately normal; (b) the sample size needs to be large;
 - (c) the population standard deviation is unknown; (d) the sampled observations are independent.
- 6) According to a report last year, the proportion of families with total income less than a million in Taiwan is 70%. This year, the governor is concerned about that the proportion has changed. A survey is then conducted by randomly selecting 200 families in Taiwan, and 128 of them reveal to have total income less than a million.
 - a) (6%) Suppose the proportion is unknown for this year, construct a 99% confidence interval for the proportion.
 - b) (6%) Continue with part a), if we want the confidence interval to have the margin of error to be less than 0.05, what is the required sample size?
 - c) (6%) Please perform a hypothesis testing procedure (using significance level 0.01) to validate the concern of the governor.

考試科目	統計方法	所別	統計4141	考試時間	3月7日(日)第四節
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d) (6%) Compute the p -value for the test in part c). Is your decision based on the p -value agrees with that in part c)? Please explain.

e) (6%) Suppose the true proportion for this year is 74%, what is the power of the test in part c)?

- 7) (35%) 某英語教師針對學生閱讀能力進行相關研究，將 30 位同學分成兩組，進行閱讀測驗，第一組同學的測驗卷內容較容易，而第二組同學的測驗卷則較為困難。以下為同學測驗成績。

第一組	35	75	25	80	45	45	40	45	25	60	70	90	45	65	20
第二組	50	30	35	48	30	63	35	40	35	50	30	48	50	43	20

	平均數	標準差	個數
第一組	51.0	21.4	15
第二組	40.5	11.1	15

甲、(10%) 假設測驗成績服從常配分佈，請檢定第二組同學的測驗成績是否顯著較低? ($\alpha=5\%$ ，臨界點請見附錄一)

乙、(25%) 除了閱讀測驗成績(R)，另蒐集每位同學字彙能力(V)與文法能力(G)的相關測驗資料。某同學考慮針對各組資料進行迴歸分析。以字彙(V)與文法(G)為解釋變數，對閱讀測驗分數(R)配適一迴歸模型： $\mu_R = \alpha + \beta_1 V + \beta_2 G$ 。相關報表見附錄二。請根據結果回答下列問題。($\alpha=5\%$)

1. (5%) 請寫出各組的預測模型。
2. (5%) 請針對第二組預測模型，說明各迴歸係數的意義。
3. (5%) 請檢定第二組模型的顯著性，包括整體與個別解釋變數的效果。
4. (5%) 針對第二組資料之殘差圖，請診斷資料之 homoscedasticity 與變數轉換的需要性。
5. (5%) 請根據分析結果，判斷測驗卷難易度與閱讀測驗分數是否相關。若欲將測驗卷難易度也同時列入迴歸模型中，請寫出適當的迴歸模型，並解釋相關的迴歸係數。

註：回答時請寫出依據的報表以及理由，否則一律不予給分。

備註	試題隨卷繳交
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考試科目	統計方法	所別	統計414	考試時間	3月7日(日)第4節
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- 8) (8%) TVBS 民意調查中心在 2009 年 4 月針對電玩爭議廣告作調查，主要詢問民眾是否認為該電玩廣告為內容不雅，或同意其為廣告創意展現。以下為此 400 位受訪民眾的回應與其【性別】的交叉表格，請檢定民眾對該廣告的看法與其性別是否顯著相關。 $(\alpha=5\%$ ，臨界點請見附錄一)

	性別	
	Male	Female
內容不雅	86	134
廣告創意	114	66
總計	200	200

- 9) (7%) A school supervisor is reviewing initial wages of former students (in \$000). Independent random samples of size 3 were taken over 2 years (2003, 2005) for 4 different majors (accounting, administration, finance, and marketing). The results are given in the following. Test the significance of major and year and their interaction on the mean salary and give a brief report. ($\alpha=5\%$)

Major	Accounting		Finance		Administration		Marketing	
Year	2003	2005	2003	2005	2003	2005	2003	2005
Number	3	3	3	3	3	3	3	3
Mean	69.2	71.1	68.7	73.0	61.2	66.6	69.0	61.3
Variance	43.2	75.1	20.3	35.2	1.3	65.4	6.0	1.6

ANOVA

Source of Variation	SS	Df	MS	F	P-value
Major	220.3	3	73.4	2.37	0.109
Year	5.9	1	5.9	0.19	0.668
Interaction	160.9	3	53.6	1.73	0.201
Error	496.1	16	31.0		
Total	883.2	23			

考試科目	統計方法	所別	統計4141	考試時間	3月7日(日)第四節
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附錄一：Critical values

$F(14,14,0.05)=2.48$, $t(21, 0.05)=2.08$, $t(21, 0.025)=2.41$, $t(28, 0.05)=2.05$, $t(28, 0.025)=2.37$,
 $\chi^2(1, 0.05)=3.84$, $\chi^2(2, 0.05)=5.99$, $z(0.05)=1.645$, $z(0.025)=1.96$

附錄二：閱讀能力研究之統計分析：

I. Descriptive Statistics

Table I-1

變數 組別	字彙成績(V)			文法成績(G)		
	Group 1	Group 2	Pooled	Group 1	Group 2	Pooled
平均數	44.7	42.7	43.7	41.2	45.5	43.3
標準差	18.2	20.5	19.1	16.3	13.4	14.8
個數	15	15	30	15	15	30

Table I-1 (Cont.)

變數 組別	閱讀測驗成績(R)		
	Group 1	Group 2	Pooled
平均數	51	40.5	45.7
標準差	21.4	11.1	17.6
個數	15	15	30

考試科目

統計方法

所別

統計 4141

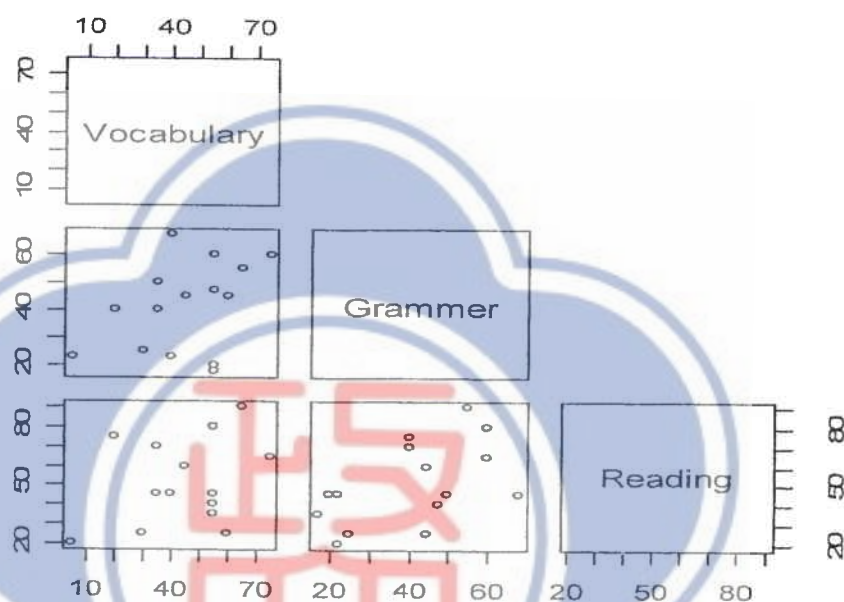
考試時間

3月7日(日)第四節

II. Regression Analysis by Group

GROUP 1:

Figure 1. The scatter plots of the three variables in group 1.



迴歸統計

R 的倍數	0.54
R 平方	0.29
標準誤	19.44

ANOVA

	自由度	SS	MS	F	顯著值
迴歸	2	1877.1	938.5	2.48	0.13
殘差	12	4532.9	377.7		
總和	14	6410			

	係數	標準誤	t 統計	P-值
截距	18.3	16.3	1.12	0.29
字彙成績	0.2	0.3	0.51	0.62
文法成績	0.6	0.3	1.80	0.10

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試題隨卷繳交

考試科目

統計方法

所別

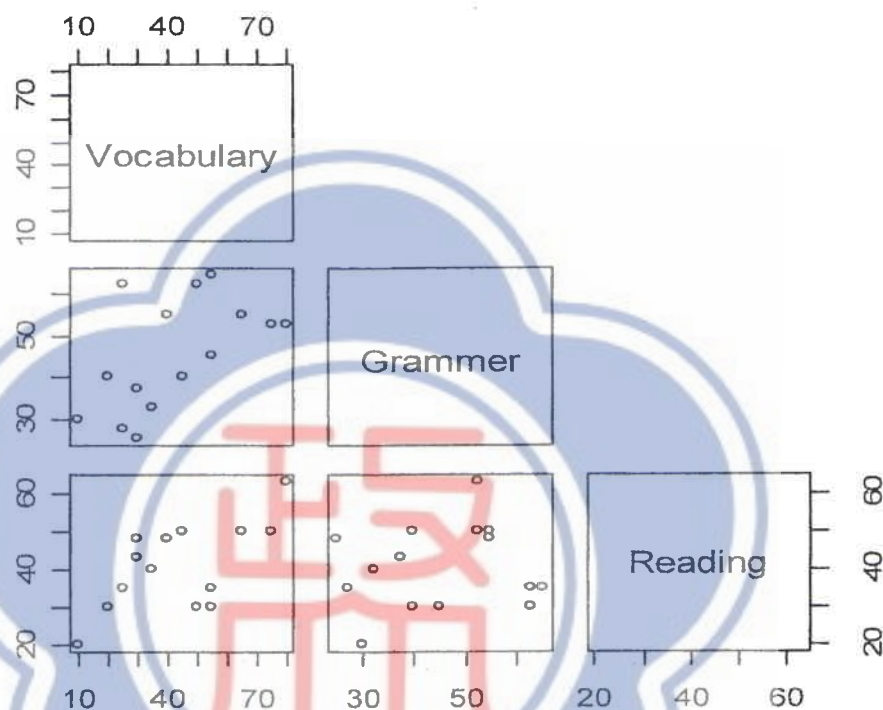
統計414

考試時間

3月7日(日)第四節

GROUP 2:

Figure 2. The scatter plot of the three variables in group 2.



迴歸統計

R 的倍數	0.70
R 平方	0.49
標準誤	8.57

ANOVA

	自由度	SS	MS	F	顯著值
迴歸	2	856.7	428.4	5.83	0.02
殘差	12	881.0	73.4		
總和	14	1737.7			

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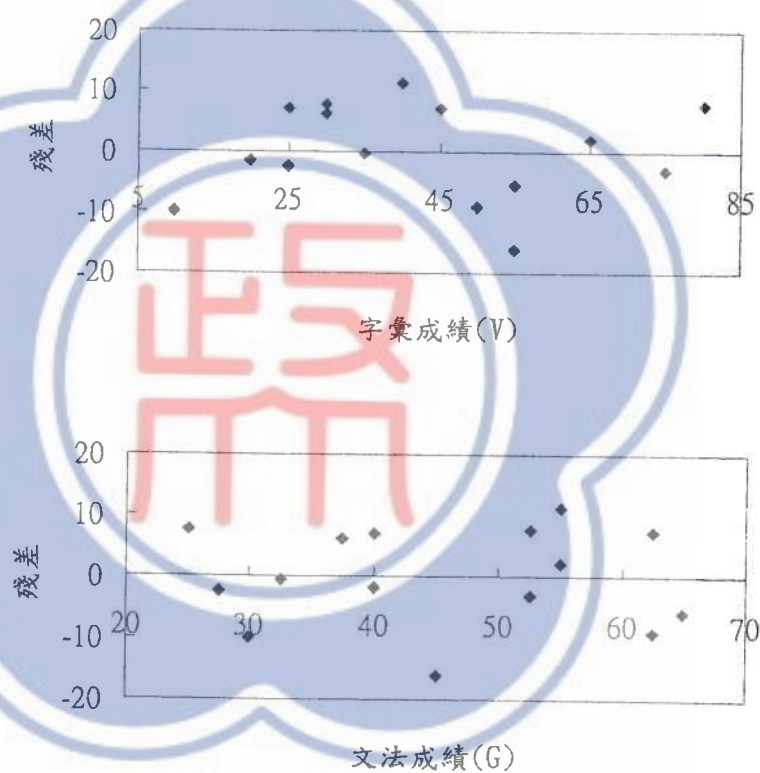
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試題隨卷繳交

考試科目	統計方法	所別	統計 4141	考試時間	3月7日(日)第四節
------	------	----	---------	------	------------

	係數	標準誤	t 統計	P-值
截距	33.8	8.1	4.17	0.00
字彙成績	0.5	0.1	3.36	0.01
文法成績	-0.3	0.2	-1.34	0.20

Figure 3. Residual plots for Group 2.



考試科目	統計方法	所別	統計 414	考試時間	3 月 7 日(日)第四節
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Areas under the Normal Curve

Example:
If $z = 1.96$, then
 $P(0 \leq z) = 0.4750$.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990