國立政治大學九十 ／＼學年度研究所(博/碩)士班入學考試命題紙

考試科目	基礎數學	所別	統計研究所	考試時間	3 月 15 日 星期 日 第 1 節

1. (10 points)Find the area of the region bounded by the graphs of $y^2=1-x$ and $2y=x+2$.

2. (10 points)Evaluate $\int \frac{6x-1}{x^3(2x-1)}dx$

3. (10 points)Evaluate $\iint_R xe^{y^2}\,dA$ over the region R in the first quadrant bounded by the graphs of $y=x^2$, $x=0$, $y=4$

4. (10 points)Find the volume of the solid in the first octant bounded by the graphs of $z=1-y^2$, $y=2x$, and $x=3$.

5. (10 points)Solve $\frac{dy}{dx}+2xy=x$ subject to $y(0)=-3$.

6. (10 points)Prove that the series $\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}(x-1)^k$ represents $f(x)=\ln x$ on the interval $(0, 2]$

7. (10 points)Suppose that **A** is a nonsingular matrix whose entries are integers. Prove that the entries in $\mathbf{A}^{-1}$ are integers if and only if $\det(\mathbf{A})=\pm 1$.

8. (10 points)A, B are matrices. Show that $\text{rank}(\mathbf{A}+\mathbf{B})\leq\text{rank}(\mathbf{A})+\text{rank}(\mathbf{B})$.

9. (10 points)Prove that: For symmetric matrices **A** and **B** of the same order, with **A** being positive definite, there exists a nonsingular matrix **P**(not necessarily orthogonal), such that $\mathbf{P}'\mathbf{A}\mathbf{P}=\mathbf{I}$ and $\mathbf{P}'\mathbf{B}\mathbf{P}$ is a diagonal matrix of the solutions for λ to $|\mathbf{B}-\lambda\mathbf{A}|=0$.

10. (10 points)Prove that: A linear operator **T** on a vector space **V** is an isometry if and only if **T** maps an orthonormal basis of **V** onto an orthonormal basis of **V**

備考	試題隨卷繳交

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考試科目	數理統計	所別	統計學系	考試時間	3月15日 星期日 第3節

1. (25pts). For each of the following statements, determine whether it is true or false. (True or false questions. Do not give explanation)

   (a) Suppose two continuous random variables X and Y have a bivariate normal distribution. If X and Y have the same marginal distributions, then the conditional distribution of X, given $Y = a$, and the conditional distribution of Y, given $X = a$, are also the same.

   (b) The assertion that random variables X and Y are uncorrelated is equivalent to that random variables X and Y are independent.

   (c) Suppose $\{X_n\}$ and $\{Y_n\}$ are two sequences of random variables and X and Y are two random variables. If the sequence X_n converges to X in distribution and the sequence Y_n converges to Y in distribution, then the sequence $X_n + Y_n$ converges to $X + Y$ in distribution as well.

   (d) Let $X_1, \ldots, X_n$ have joint pdf $f(x_1, \ldots, x_n; \theta)$. If S is a vector of jointly complete sufficient statistics for θ, then S is a uniformly minimum variance unbiased estimator (UMVUE) of $\mathrm{E}(S)$.

   (e) Consider a random sample of size n from a normal distribution, $X_i \sim \mathcal{N}(\mu, 1)$. A uniformly most powerful (UMP) test of size α of $H_0 : \mu = 0$ versus $H_a : \mu \neq 0$ is to reject H_0 iff $|\frac{\bar{X}-0}{\sqrt{\frac{1}{n}}}| > z_{1-\frac{\alpha}{2}}$, where $\bar{X} = \frac{X_1+\ldots+X_n}{n}$ and $z_{1-\frac{\alpha}{2}}$ is the $1-\frac{\alpha}{2}$ percentile of a standard normal distribution.

2. (5pts). Let $X_1, \ldots, X_n$ be a random sample of size n from the normal distribution $\mathcal{N}(\mu, \sigma^2)$. Consider two random variables $Y \equiv \Sigma_{i=1}^n a_i X_i$ and $Z \equiv \Sigma_{i=1}^n b_i X_i$. When, and only when, are Y and Z independent? Only give your answer to this question and do not show your work.

3. Suppose $(X, Y)'$ is a random vector uniformly distributed on the disk with center $(0, 0)'$ and radius 1.

   (a) (10pts). Please give the marginal probability density function (pdf) for X. Show your work.

   (b) (6pts). Are X and Y independent? Explain.

備考	試題隨卷繳交
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考試科目	數理統計	所別	統計學系	考試時間	3月15日 星期日 第3節

4. Let X be the number of independent trials of some component until it fails, where $1-p$ is the probability of failure on each trial. We record the exact number of trials $Y = X$, if $X \leq r$; otherwise we record $Y = r + 1$, where r is a fixed positive integer.

   (a) (6pts). Give the discrete pdf of Y, $f(y;p)$. Show your work.

   (b) (10pts). Let $Y_1, \ldots, Y_n$ be a random sample from $f(y;p)$. Find the maximum likelihood estimator (MLE) of p. Show your work.

5. Consider a distribution with pdf $f(x;\theta) = \theta x^{\theta-1}$ if $0 < x < 1$ and zero otherwise.

   (a) (12pts). Based on a random sample of size $n = 1$, find the most powerful test of $H_0 : \theta = 1$ against $H_a : \theta = 2$ with significant level $\alpha = 0.05$. Show your work.

   (b) (6pts). Compute the power of the test in (a) for the alternative $\theta = 2$. Show your work.

6. Let Y_n be the nth order statistic of a random sample of size n from a distribution with pdf $f(x|\theta) = \frac{1}{\theta}$, $0 < x < \theta$, zero elsewhere.

   (a) (12pts). Assume the prior density of θ is $\pi(\theta) = \beta\frac{\alpha^\beta}{\theta^{\beta+1}}$, $\alpha < \theta < \infty$. What is the posterior distribution of θ given $Y_n = y_n$? Show your work.

   (b) (8pts). Using the squared error loss, find the Bayes estimator of θ. Show your work.

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考試科目	統計方法	所別	統計	考試時間	3 月 15 日 星期日 第四節

1. (10%) An institute wants to conduct a survey to investigate the health status of Taiwanese people of the four different age groups: child, teenager, adult and senior. Which sampling scheme will you recommend practically? Please give a brief illustration on the sampling process and explain why you prefer the method.

2. (35%) Falls are the leading cause of accidental deaths for senior citizens. It's known that 18.7% of senior people self-reported to have falls in 1999.
   i. (10%) Falls may lead to hip fractures. Assume 0.85% of senior people had hip fractures in 1999 and 65% of the hip fractures were caused by falls. According to the record of 1999, given that a person did not have falls, what is the chance that he did not have a hip fracture?
   ii. (10%) In the health survey, one is interested in finding the proportion(π) that the senior citizens have in-home falls last year. How large a sample of senior citizens should be taken for an estimate to have sampling error at most 3% with 95% confidence?
   iii. (15%) Consider a small-scale study in a county of 2,000 senior citizens. 250 citizens are drawn without replacement and 65 of them report to have falls last year. Please give a point estimation for π and its standard error. Can we conclude a significant increment in the proportion of falls from 1999?

3. (15%) Consider the four local musical radio stations, UFO, ICRT, WAVE, HIT-FM. One is interested in studying the number of minutes of music played per hour. From a random sample from each station, the following statistics were offered.

	UFO	ICRT	WAVE	HIT-FM
mean	52	43	47	48
n	7	9	11	13
$SS_{Total} = 795.9$				

Is there a difference in the mean amount of music time between the 4 stations at the 0.05 significance level? Please write down the statistical hypotheses, distributional assumptions, critical region and the observed value of the test statistic and the conclusion.

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考試科目	統計方法	所別	統計	考試時間	3 月 15 日 星期日 第四節

4. (40%) One teacher aims to investigate whether a new teaching skill can raise students' study motive. Two classes of students are selected. Students of class A are taught by the new skill and students of class B are treated by the traditional method. A student's study motive is scored by a questionnaire. The following data are collected in the beginning and the end of the semester, where Pre_score is the score obtained in the beginning of the semester and Post_score is the score obtained in the end of the semester.

Class A (New method)			Class B (Traditional method)		
ID	Pre_score (X)	Post_score (Y)	ID	Pre_score (X)	Post_score (Y)
1	3.7	4.3	1	5.2	5.1
2	2.7	2.2	2	3.8	4.0
3	4.2	4.9	3	4.3	4.2
4	5.3	6.0	4	3.8	4.4
5	5.2	6.1	5	5.6	5.7
6	5.7	6.9	6	4.5	4.3
7	1.8	1.5	7	6.6	5.9
8	3.8	4.2	8	4.1	4.6
9	5.1	5.8	9	3.1	3.6
10	2.9	3.1	10	3.0	4.3
mean	4.0	4.5	mean	4.4	4.6
S^2	1.68	3.20	S^2	1.27	0.54
S	1.30	1.79	S	1.13	0.74
Σxy	202.5		Σxy	209.6	

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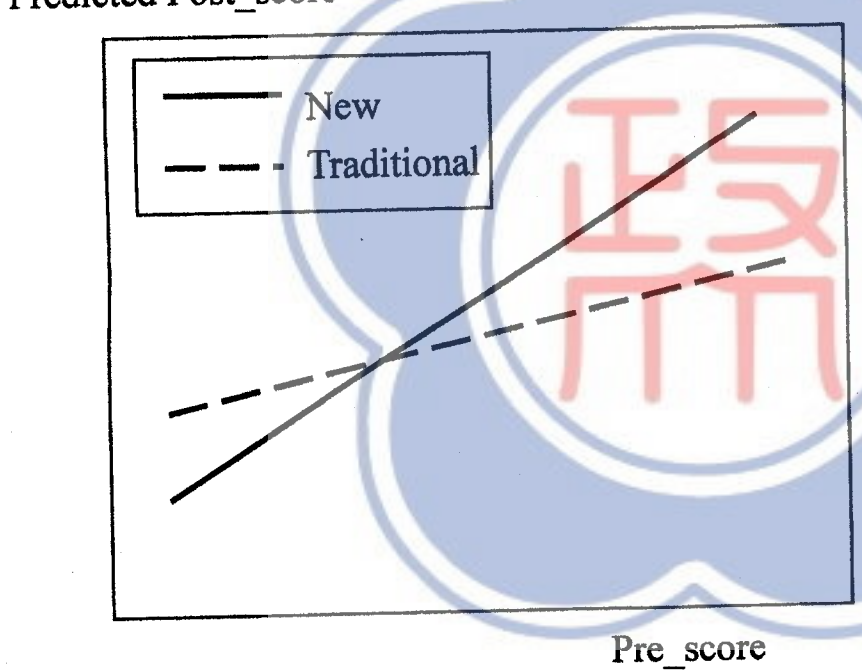
考試科目	統計方法	所別	統計	考試時間	3 月 15 日 星期日 第四節

i. (10%) Please determine whether the new teaching skill is effective at the 0.05 significance level.

ii. (10%) Please determine whether the new teaching skill is better than the traditional method at the 0.05 significance level.

iii. (10%) Use the data of Class A to develop a least square regression equation for Post_score based on Pre_score. Comment on the regression coefficients.

iv. (10%) If one add the variable of teaching method in the regression equation for Post_score and obtain the following prediction lines, please write down a possible prediction equation and comment on the regression coefficients.



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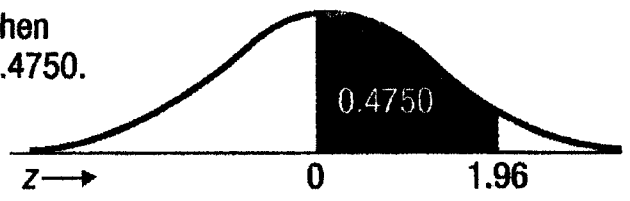
Appendix: Distribution and quantile tables.

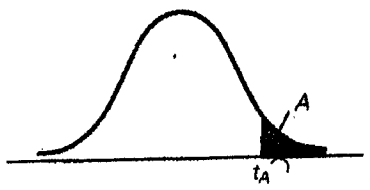
1. Normal

If $z = 1.96$, then $P(0 \text{ to } z) = 0.4750$.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952

2. Student's t distribution



DEGREES OF FREEDOM	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845

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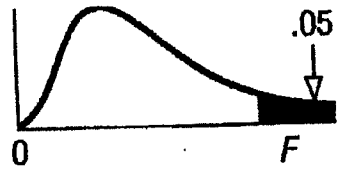
考試科目	統計方法	所別	統計	考試時間	3 月 15 日 星期日 第四節

3. F distribution:



Degrees of Freedom for the Denominator	Degrees of Freedom for the Numerator: 1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39

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3. 試題由郵寄遞者請以掛號寄出，以免遺失而示慎重。

考試科目	統計方法	所別	統計	考試時間	3 月 15 日 星期日 第四節

4. Distribution function of the Wilcoxon signed-rank test statistic :

	Sample Size					
t	7	8	9	10	11	12
1	0.0157	0.0079	0.0040	0.0020	0.0010	0.0005
2	0.0235	0.0118	0.0059	0.0030	0.0015	0.0008
3	0.0391	0.0196	0.0098	0.0049	0.0025	0.0013
4	0.0547	0.0274	0.0137	0.0069	0.0035	0.0018
5	0.0782	0.0391	0.0196	0.0098	0.0049	0.0025
6	0.1094	0.0547	0.0274	0.0137	0.0069	0.0035
7	0.1485	0.0743	0.0372	0.0186	0.0093	0.0047
8	0.1875	0.0977	0.0489	0.0245	0.0123	0.0062
9	0.2344	0.125	0.0645	0.0323	0.0162	0.0081
10	0.2891	0.1563	0.0821	0.0420	0.0210	0.0105
11	0.3438	0.1915	0.1016	0.0528	0.0269	0.0135
12	0.4063	0.2305	0.125	0.0655	0.0337	0.0171
13	0.4688	0.2735	0.1504	0.0801	0.0416	0.0213
14	0.5313	0.3204	0.1797	0.0967	0.0508	0.0262
15		0.3711	0.2129	0.1163	0.0616	0.0320
16		0.4219	0.2481	0.1377	0.0738	0.0386
17		0.4727	0.2852	0.1612	0.0875	0.0462
18		0.5274	0.3262	0.1875	0.1031	0.0550
19			0.3672	0.2159	0.1202	0.0647
20			0.4102	0.2461	0.1392	0.0757

備考	試題隨卷繳交
命題委員：	

命題紙使用說明：1. 試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。
2. 書寫時請勿超出格外，以免印製不清。
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