國立政治大學 九十六學年度 碩士班暨碩士在職專班招生考試 命題紙

考試科目	基礎數學	所別	統計系 4141	考試時間	3月 17 日 星期六	第 3 節

1. (10 pts) Write down final answers only.

   (a) Find an anti-derivative for the function $f(x) = x\cos(x)$.

   (b) Suppose that $f(x) = (e^x - 1)/(x^2 + 1)$. Find $df(x)/dx$.

2. (10 pts) Find $\lim_{h\to 0} h^{-1}(2^{x+h} - 2^x)$.

3. (15 pts) Let $A = \{(x, y) : x \le y \le x + 1 \text{ and } 1 \le y \le 3\}$. Find $\iint_A (x + y^2)dxdy$.

4. (15 pts) Let $f(x) = x + x^{-2}$ for $x > 0$. Find the minimum of f on the interval $(0, \infty)$.

5. (10%) (a) 舉反例說明 AB＝AC，不見得 B＝C（A 不是零矩陣）；

   (b) 證明 $(I-A)^{-1} = I + A + A^2 + \ldots + A^n$，其中 $A^{n+1} = 0$。

6. (10%) 試求 A^n，若矩陣

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}。$$

7. (10%) 試求下述矩陣 A 的反矩陣 A^{-1}，並以 $AA^{-1} = I$ 驗證。

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 2 \\ -2 & -3 & 2 \end{bmatrix}$$

8. (10%) 試求 $\lambda(\neq -0.5)$，使下列三個向量為線性相關(Linear dependent)：

$$v_1 = \begin{bmatrix} \lambda \\ -0.5 \\ -0.5 \end{bmatrix}, v_2 = \begin{bmatrix} -0.5 \\ \lambda \\ -0.5 \end{bmatrix}, v_3 = \begin{bmatrix} -0.5 \\ -0.5 \\ \lambda \end{bmatrix}$$

9. (10%)試述何謂矩陣的 QR 分解(QR Decomposition)，與特徵值分解(Eigenvalue Decomposition)有何不同？將下列矩陣以 QR 分解展開：

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

考試科目	數理統計學	所別	統計	考試時間	3月17日 星期六	第四節

1. Suppose $Y_1,\ldots,Y_n$ forms a random sample from the uniform distribution having parameters 0 and $\theta>0$. Let $Y_{(1)},\ldots,Y_{(n)}$ represent the order statistics of the sample.

   (1) Find the cumulative distribution function (c.d.f) of $Y_{(n)}$. (5%)

   (2) Find the method of moments estimator (MOME) of θ. (2%)

   (3) Find the maximum likelihood estimator (MLE) of θ. When finding the likelihood, use $f(y)=(1/\theta)I_{(y\le\theta)}$ for $y>0$, where $I_{(y\le\theta)}=1$ if $y\le\theta$ and 0 if $y>\theta$. (5%)

   (4) Find the uniformly minimum variance unbiased estimator (UMVUE) of θ. (8%)

   (5) Are $\hat{\theta}_{MOME}$ and $\hat{\theta}_{MLE}$ consistent for estimating θ? (8%)

   (6) We want use the rejection region $RR=\{Y_{(n)}\le k \quad or \quad Y_{(n)}>1\}$, where $0\le k<1$, to test the null hypothesis $H_0:\theta=1$ versus $H_1:\theta\neq 1$. If α is the significance level, find k in terms of α, θ and n. (10%)

2. Let $Y_1,\ldots,Y_n \overset{i.i.d}{\sim} N(\theta,\theta^2),\theta>0$.

   (1) Find a sufficient statistic for θ. (5%)

   (2) Find a pivotal quantity of θ. (8%)

   (3) Use the pivotal quantity of (2) to find a 90% confidence Interval of θ. (7%)

3. Let $Y_i\sim LOGN(\mu_i,\sigma_i^2)$, $i=1,,\ldots,n$ are independent. Find the distribution of :

   (1) $\prod_{i=1}^{n}Y_i$. (5%) (2) Y_1/Y_2. (5%) (3) $E[\prod_{i=1}^{n}Y_i]$.(5%)

4. (1) Suppose X is a random variable with $P(X\ge 0)=1$. Assuming both expected values exist, show $E\sqrt{X}\le\sqrt{EX}$. (8%)

   (2) Let X be a random variable with pmf $f(x)=1/3$, $x=0,1,2$. Check that $EX^3>(EX)^3$. Give an example of a random variable for which $EX^3<(EX)^3$. Justify your answer. (7%)

5. Suppose $Y_1\sim U(0,4)$, $Y_2\sim U(3,5)$, and Y_1,Y_2 are independent. Find the joint probability density of $U_1=Y_2-Y_1$ and $U_2=Y_2$. (12%)

國立政治大學 九十六學年度 碩士班暨碩士在職專班招生考試 命題紙

考試科目	統計方法	所別	統計學系 4141	考試時間	3 月 18 日 星期日 第 3 節

1. The number of hours of 8 NCCU students spent in studying the day before the Statistics final exam was recorded as follows:

4 3 0 5 2 9 4 5

a) Compute coefficient of variation and mean absolute deviation.

b) Draw the Box-Whisker plot. (You need to indicate the median, the first quartile (Q1), the third quartile (Q3), and the end points of the two whiskers.)

(5% + 5%)

2. What are the required assumptions when performing one-way ANOVA?

(6%)

3. A scatter diagram includes the following data points:

x	3	2	5
y	8	6	12

Two regression models are proposed:

Model 1: $\hat{y} = 1.2 + 2.5x$

Model 2: $\hat{y} = 5.5 + 4.0x$

Using the least squares method, which of these regression models provide the better fit to the data? Why?

(6 %)

4. An investor owns three common stocks. Each stock, independently of the others, has equally likely chances of (1) increasing in value, (2) decreasing in value, or (3) remaining the same value. What is the probability that at least two stocks increase in value?

(10%)

5. Will the power of a hypothesis test (for single population mean) increase or decrease when

a) α increases?

b) the population standard deviation increases?

c) sample size n increases? (2% + 2% + 2%)

備考	試題隨卷繳交
命題委員：	143 （簽章） 96 年 3 月 5 日

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6. The Taipei Labors Bureau wishes to estimate the proportion of the workforce that has two or more jobs. How large a random sample of the workforce must be examined to be 95% certain that the estimate does not differ from the true proportion by more than 0.02?
(10%)

7. You want to test whether or not a coin is fair, that is, $H_0 : p = .5$ vs. $H_1 : p \neq .5$. You flip the coin 5 times. If you get 1, 2, 3, or 4 heads, you will conclude that the coin is fair.
a) What is the probability of committing a Type I error?
b) Suppose you get 4 heads out of 5 coin tosses. What is the p-value?
(5% + 5%)

8. A random sample is drawn from a population with a mean of 20 and a standard deviation of 21. If only 33% of time the sample mean is less than 18.68, what is the sample size?
(10%)

9. A random sample of 12 observations for an independent variable and a dependent variable with $s_x = .11$ and $s_y = .33$ is fitted into a simple linear regression model. It is also known that the correlation coefficient between the independent variable X and the dependent variable Y is 0.9. Using above information to complete the following ANOVA summary table, what are the values of (A), (B), (C), and (D)?

Source	SS	df	MS	F
Regression			(C)	
Error	(A)		(D)	
Total	(B)			

(E) Using the t-test to test the null hypothesis that the slope =0 at α=0.05.
(F) If the fitted least squares regression equation passes through the point (2, 3), then what is the fitted least squares regression equation?
(2% + 2% + 2% + 2% + 4% + 5%)

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10. You are give the following information for 3 treatment groups:

	Treatment		
Statistic	1	2	3
n	5	5	5
$\bar{x}$	10	15	20
s^2	50	50	50

In the following AVONA summary table for testing the three treatment means are equal, what are the values for (A), (B), and (C)?

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F
Treatments	(A)			(C)
Error		(B)		
Total				

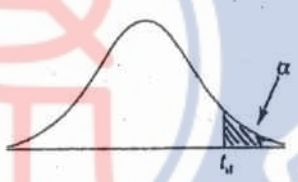
(D) What happens to the F statistic if we double the sample sizes each (i.e. changing the sample sizes to 10 each)?

(E) What happens to the F statistic if we subtract 10 to each of the sample means?

(F) What happens to the F statistic if we change all 3 sample means to 88, respectively?

(2% + 2% + 2% + 3% + 3% + 3%)

Critical values from the t Distribution (partial table)



Values of α for one-tailed test and $\alpha/2$ for two-tailed test						
df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.894
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686

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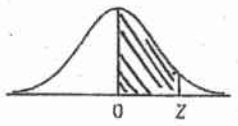
考試科目	統計方法	所別 4141	統計學系	考試時間	3月18日 星期日 第 3 節

Areas of the Standard Normal Distribution



The entries in this table are the probabilities that a standard normal random variable is between 0 and Z (the shaded area).

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998

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