考試科目	統計學	所別	統計研究所	考試時間	3月20日 星期日 上(下)午第三節

1. A student has a class which is supposed to end at 10:00 am and another which is supposed to begin at 10:10 am. Suppose that the actual ending time of the 10 am class is a normally distributed random variable X_1 with mean 10:02 and standard deviation 1.5 min, and that the starting time of the next class is also a normally distributed random variable X_2 with mean 10:10 and standard deviation 1 min. Suppose also that the time necessary to get from one classroom to the other is a normally distributed random variable X_3 with mean 6 min and standard deviation 1 min. Assume independence of X_1, X_2, and X_3, what is the probability that the student makes it to the second class before the lecture starts? (15 pts)

2. Using a long rod which has length μ, you are going to lay out a square plot in which the length of each side is μ. Thus the area of the plot will be μ^2. However, you don't know the value of μ, so you decide to make n independent measurements $X_1, X_2, \ldots, X_n$ of the length. Assume that each X_i has mean μ (unbiased measurements) and variance σ^2.
   a. Show that $\overline{X}^2$ is not an unbiased estimator for μ^2. (10 pts)
   b. For what value of k is the estimator $\overline{X}^2 - ks^2$ unbiased for μ^2? (10 pts)

3. A paper reported that under conditions of the desegregation plan to be implemented, any students who travel more than 45 min one way on a bus would be required to have only three years of integrated schooling, while those traveling at most 45 min would face a five-year requirement. Suppose that travel time between two particular locations is normal, and that a random sample of 20 travel times yields observations

47.3	44.1	44.5	43.7	46.2	45.3	47.0	42.5	41.9	47.6
47.1	45.8	45.7	47.8	46.6	44.2	47.2	43.6	46.5	45.9

   Does this data indicate that only three years of integrated schooling is required? Use $\alpha = 0.1$. (15 pts)

4. In an experiment to investigate the performance of four different brands of sparkplugs intended for use on a 125-cc two-stroke motorcycle, five plugs of each brand were tested and the number of miles (at a constant speed) until failure was observed. The partial ANOVA table for the data appears below. Fill in the missing entries, state the relevant hypotheses, and carry out a test. Use $\alpha = 0.05$. (15 pts)

Source	d.f.	Sum of squares	Mean square	F-value
Brand				
Error		235,419.04		
Total		310,500.76		

5. In an experiment to study the linear relationship between age x and vocabulary size Y for young children (i.e., $Y = \beta_0 + \beta_1 x + \varepsilon$), 10 children of particular ages were selected and vocabulary size (measured by testing) was determined for each child, resulting in the following data.

x:	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
y:	100	250	460	890	1210	1530	1840	2060	2300	2500

   The summary statistics are

   $$\sum x_i = 37.5, \sum y_i = 13{,}140, \sum x_i^2 = 161.25, \sum y_i^2 = 24{,}050{,}400, \sum x_i y_i = 61{,}055.$$

   a. Compute $\hat{\beta}_0$ and $\hat{\beta}_1$. (15 pts)

備考	試題隨卷繳交

命題委員： 106 (簽章) 94年3月3日

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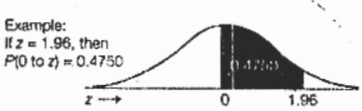
b. What would your estimate be for true average vocabulary size of all four-year-old children in the population sampled? (5 pts)

c. Compute a 95% confidence interval for it. (10 pts)

d. If you were to select at random a four-year-old child from the population, what would be your prediction for vocabulary size of the selected child? (5 pts)

Areas under the Normal Curve



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

備考	試題隨卷繳交

命題委員： 107 (簽章) 94年 3月3日

考試科目	統計學	所別	統計研究所	考試時間	3月20日 星期日 上(下)午第三節

Student's *t* Distribution

Example: With df = 9 and .10 area in the upper tail, $t = 1.383$

	Confidence Intervals					
	80%	90%	95%	98%	99%	99.9%
	Level of Significance for One-Tailed Test					
df	0.100	0.050	0.025	0.010	0.005	0.0005
	Level of Significance for Two-Tailed Test					
	0.20	0.10	0.05	0.02	0.01	0.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

Critical Values of the *F* Distribution at a 5 Percent Level of Significance

	Degrees of Freedom for the Numerator															
Degrees of Freedom for the Denominator	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39

備考	試題隨卷繳交

命題委員：　　108　　(簽章) 94 年 3 月 3 日

考試科目	數理統計	所別	統計系 4141 4146	考試時間	3月20日 星期日 (下)午第4節

1. Suppose $X \sim F(x)$, which is given by

$$F(x)=\begin{cases} 0, & x<0 \\ \dfrac{x+1}{4}, & 0\le x<1 \\ 1, & x\ge 1 \end{cases}.$$

Find E(X) and Var(X). (4%+4%)

2. Suppose $X_1, X_2, \cdots, X_n$ is a random sample from Uniform$[\theta, \theta+1]$. Let $Y_1 = \min\{X_1, X_2, \cdots, X_n\}$, $Y_n = \max\{X_1, X_2, \cdots, X_n\}$, and $R = Y_n - Y_1$.
   (a) Find the method of moments estimate of θ. (4%)
   (b) Find the maximum likelihood estimate of θ. (5%)
   (c) Find the density function of R. (15%)

3. Suppose $Y_1, Y_2, \cdots, Y_5$ are the order statistics of a random sample of size 5 from a continuous population with median θ. How much confidence can be associated with the random interval $[Y_2, Y_4]$ as a confidence interval for θ. (10%)

4. Suppose $X \sim f(x)$, which is given $f(x) = 3(x+1)^2 \cdot I_{(-2,-1)}(x)$. Find a transformation (function), ϕ, such that $Y = \phi(X)$ has a pdf given by $g(y) = e^{-y} \cdot I_{[0,\infty)}(y)$. (10%)

5. Suppose X_1, X_2 is a random sample of size 2 from a continuous distribution having p.d.f. $f(x|\theta) = (1/\theta)e^{-x/\theta}; x \ge 0$.
   (a) Show that X_2 is an unbiased estimator of θ. (4%)
   (b) Show that $T = X_1 + X_2$ is a sufficient statistic for θ. (5%)
   (c) Does T have a complete family of distributions? Explain. (9%)
   (d) Derive the minimum variance unbiased estimator of θ by finding the joint distribution of T and X_2 and computing a conditional expectation. (10%)

6. Suppose $X_1, X_2, \cdots, X_n$ is a random sample from a normal distribution whose mean is known to be 1 and whose variance σ^2, is unknown. The following simple hypothesis are to be tested:

$$H_0 : \sigma^2 = 2$$
$$H_1 : \sigma^2 = 3$$

   (a) Argue that among all test procedures whose type I error probability is no more than .05 the type II error probability is minimized by a procedure which rejects H_0 when $\sum_{i=1}^{n}(X_i - 1)^2 \ge c$. (10%)
   (b) Describe how you would find the value of c in part (a). (10%)

備考	試題隨卷繳交

命題委員： 109 (簽章) 94 年 3 月 7 日