

考 試 科 目	基礎數學	系 所 別	統計學系	考 試 時 間	2 月 3 日(五) 第 一 節
Part I: Calculus 1. (15%) State and prove the mean value theorem. 2. (10%) Use ϵ-δ argument to show that $\lim_{x \rightarrow \infty} \frac{1}{x^2} = 0$. 3. (20%) Find the following integrals: (a) $\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$ (b) $\int_{-2}^3 \frac{1}{x^4} dx$ (c) $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx$ (d) $\int_0^{\infty} e^{-x^2} dx$ (e) $\int \sin^5 x \cos^2 x dx$. 4. (5%) Find $\frac{dy}{dx}$ if $\tan\left(\frac{x}{y}\right) = x + y$. 5. (10%) Find the extreme value of the function $f(x, y, z) = x + 2y + 3z$ subject to constraints $x - y + z = 1$ and $x^2 + y^2 = 1$. Part II: Linear Algebra 6. (10%) Suppose that A is a matrix satisfying $A^2 = A$ and $A^T = A$. (a) Find possible eigenvalues of A. (b) Determine the rank of A. 7. (10%) Prove that if Q is an orthogonal matrix, then $\det(Q) = 1$. 8. (15%) Let V and W be two vector spaces and let $T: V \rightarrow W$ denote a linear transformation. Suppose that $\mathcal{N}(T)$ and $\mathcal{R}(T)$ are null space and range of T, respectively. Show that $\mathcal{N}(T)$ and $\mathcal{R}(T)$ are subspaces of V and W, respectively. 9. (5%) Let $A = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$. Compute A^n for any positive integer n.					
備 註	一、作答於試題上者, 不予計分。 二、試題請隨卷繳交。				

考試科目	數理統計學	系所別	統計學系	考試時間	2 月 3 日(星期五) 第 2 節
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• Note. All parts in the same problem are worth the same points.

1. (40 pts) Suppose that X is a random variable whose distribution is $N(\theta, 1)$, the normal distribution with mean θ and variance 1. Suppose that we have IID (independent and identically distributed) observations $Y_1, \dots, Y_n$, and the distribution of Y_1 is the same as the distribution of X^3 . Suppose that the parameter $\theta \in (-\infty, \infty)$ is unknown.

- Find a PDF of X^3 .
- Find the maximum likelihood estimator for θ based on the observations $Y_1, \dots, Y_n$.
- For $\alpha \in (0, 1)$, find the likelihood ratio test of size α for testing

$$H_0 : \theta = 0 \text{ versus } H_1 : \theta \neq 0.$$

Express the rejection region in terms of a test statistic and give the distribution of the test statistic under H_0 .

- Find the (uniformly) minimum variance unbiased estimator for θ^2 based on the observations $Y_1, \dots, Y_n$.
2. (60 pts) Suppose that $(X_1, \dots, X_n)$ is a random sample from the uniform distribution on $[-\theta, \theta]$, where $\theta > 0$ is an unknown parameter. Let

$$X_{(n)} = \max_{1 \leq i \leq n} X_i$$

and

$$X_{(1)} = \min_{1 \leq i \leq n} X_i.$$

- Show that $X_{(n)}$ is a consistent estimator for θ .
- Determine whether $X_{(n)}$ and $X_{(1)}/X_{(n)}$ are independent. Justify your answer.
- Find $\hat{\theta}$: an estimator for θ based on the data such that

$$E((\hat{\theta} - \theta)^2) < E((X_{(n)} - \theta)^2)$$

for every $\theta > 0$. Justify your answer.

- Construct a 95% confidence interval for θ based on the statistic $(X_{(1)}, X_{(n)})$.

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註

- 作答於試題上者，不予計分。
- 試題請隨卷繳交。

考 試 科 目	統計方法	系 所 別	統計學系	考 試 時 間	2 月 3 日 (五) 第四節
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1. To understand whether the average monthly salary of recent college graduates reaches \$32,000, a survey of 1,000 sampled graduates was conducted. Since salary is a sensitive issue, each respondent answers only one of the 2 questions according to the approach described below:

---If the month of birth is January to August, answer:

Question A: Is your monthly salary at least \$32,000?

---If the month of birth is from September to December, answer:

Question B: Is your monthly salary less than \$32,000?

---Respondents only need to answer "yes" or "no", no need to provide month of birth or the question asked.

Of the 1,000 respondents, 550 answered "yes".

Estimate the proportion of monthly salary being at least \$32,000 based on this survey. (10%)

2. The total score of the master's entrance examination is normally distributed with a mean of 200 and a standard deviation of 30. There are 500 examinees recently.

(1) Mary's total score is 185. How many examinees are worse than Mary? (6%)

(2) Examinee with top 30% of the total score will be eligible for oral. What is the minimum total score in order to be eligible for oral? (8%)

3. At the reception center in a hotel, the time between calls follow an exponential distribution with a mean of 2 minutes.

(1) Find the probability that after receiving a call, the next call comes in less than 1 minute. (4%)

(2) Today George is the only person on duty, and he is going to take a break for 10 minutes. What is the probability that 4 calls will he miss to answer? (5%)

(3) George will work 2 hours on next Monday. Find the probability that he will answer 50 to 70 calls approximately. (7%)

4. The marketing manager of a department store claimed that customers using store credit cards spent more on average than customers using bank credit cards. Recently, customer service department conducted a survey. Independent samples of credit cards' spending (unit: \$100), are summarized below. It is reasonable to assume spending to be normally distributed.

	Bank Card	Store Card
sample size	9	16
sample mean	25	34
sample st. dev.	10	5

(1) At $\alpha=0.10$, test for equality of the two population variances. (6%)

(2) At $\alpha=0.10$, does the sample results support the manager's claim? (14%)

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註

作答於試題上者，不予計分。
試題請隨卷繳交。

考 試 科 目	統計方法	系 所 別	統計學系	考 試 時 間	2 月 3 日 (五) 第四節
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5. The costs (unit:US\$1,000) and numbers of passengers for Boeing 737s of Northland Airline are listed below.

X (passengers)	61	63	67	69	70	74	76	81	86	91	95	97
Y (cost)	4.3	4.1	4.4	4.2	4.5	4.3	4.8	4.7	5.1	5.2	5.8	5.6

Notice that $\sum X^2=73764$, $\sum Y^2=274.220$, $\sum XY=4489.900$, $\sum X=930$, $\sum Y=57$; and $SSE=0.3665$.

Answer the questions by rounding to 4 decimal places! (四捨五入至小數點後第 4 位數!)

- (1) Derive the estimated regression function. (5%)
- (2) An industry comparison suggests the slope to be 0.027. At $\alpha=0.05$, test whether Northland has a slope larger than suggested. (7%)
- (3) A Boeing 737 flight has 75 passengers on board. Develop a 95% interval for its cost. (8%)

6. Three different hardwood concentrations are studied to determine their effect on the strength of the paper produced. The factory could only sustain 3 trials per day, so an experiment was carried out over 4 days.

Hardwood Concentration	Days				Subtotal
	1 st	2 nd	3 rd	4 th	
I	4	10	7	8	29
II	9	12	10	16	47
III	13	17	14	15	59
Subtotal	26	39	31	39	135

- (1) Based on the following partial ANOVA table, can we conclude there is a difference in the mean strength by centration at $\alpha=0.05$. Write down H_0 、 H_1 、test statistic、critical value, and conclusion. (9%)

Source	DF	SS
Concentration		
Day		
Error		15.3333
Total		170.2500

- (2) Develop the 95% confidence interval for difference between mean strength of concentration II and mean strength of concentration III. (6%)
- (3) The factory decides to rerun the experiment on next month. Do you suggest to ignore the effect of Day? Why or why not? (5%)

考 試 科 目	統計方法	系 所 別	統計學系	考 試 時 間	2 月 3 日 (五) 第四節
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P(0 < Z ≤ z) under Standard Normal distribution

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

Upper tailed values of t distribution

Probabilities in upper tail

df	.25	.20	.15	.10	.05	.025	.02	.01
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364

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作答於試題上者，不予計分。
試題請隨卷繳交。

考試科目	統計方法	系所別	統計學系	考試時間	2 月 3 日 (五) 第四節
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Table F distribution; F_{α, v_1, v_2} for $P(F \geq F_{\alpha, v_1, v_2}) = \alpha$

$F_{0.05;v_1,v_2}$												
$v_2 \backslash v_1$	1	2	3	4	5	6	7	8	9	10	12	15
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03

$F_{0.025; v_1, v_2}$													
$v_2 \backslash v_1$	1	2	3	4	5	6	7	8	9	10	12	15	
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	

備

註

作答於試題上者，不予計分。
試題請隨卷繳交。