

考 試 科 目	基礎數學	系 所 別	統計學系	考 試 時 間	2 月 18 日 (一) 第一節
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1. (20 pts) Let W be a subspace of R^n .
 - (a) (5 pts) It's known that W is a subset of R^n and W itself is a vector space. What is a vector space?
 - (b) (5 pts) Define the orthogonal complement of W , $W^\perp$.
 - (c) (10 pts) Show that $R^n = W \oplus W^\perp$.
2. (10 pts) Let W be the subspace of the plane $3x - 2y + z = 0$ in R^3 . Find the shortest distance of the vector $u = (2, 3, -1)$ to W .
3. (20 pts) Consider an experimental design. We have 3 factors of interest. Each treatment has two possible levels, H(HIGH) and L(LOW). Thus, there are 8 possible combinations in total. Define the following parameters for the response Y ,

$$\begin{aligned}
 \mu &= \text{overall mean,} \\
 \delta_1 &= \text{mean increment due to } X_1, \\
 \delta_2 &= \text{mean increment due to } X_2, \\
 \delta_3 &= \text{mean increment due to } X_3.
 \end{aligned}$$

The responses $Y_1, \dots, Y_8$ under the 8 levels are expressed in the following way, where $\epsilon_i, i = 1, \dots, 8$, are random errors.

Trial	Treatment Level			Response model
	X_1	X_2	X_3	Y
1	H	H	H	$Y_1 = \mu + \delta_1 + \delta_2 + \delta_3 + \epsilon_1$
2	H	H	L	$Y_2 = \mu + \delta_1 + \delta_2 - \delta_3 + \epsilon_2$
3	H	L	H	$Y_3 = \mu + \delta_1 - \delta_2 + \delta_3 + \epsilon_3$
4	H	L	L	$Y_4 = \mu + \delta_1 - \delta_2 - \delta_3 + \epsilon_4$
5	L	H	H	$Y_5 = \mu - \delta_1 + \delta_2 + \delta_3 + \epsilon_5$
6	L	H	L	$Y_6 = \mu - \delta_1 + \delta_2 - \delta_3 + \epsilon_6$
7	L	L	H	$Y_7 = \mu - \delta_1 - \delta_2 + \delta_3 + \epsilon_7$
8	L	L	L	$Y_8 = \mu - \delta_1 - \delta_2 - \delta_3 + \epsilon_8$

- (a) (10 pts) Express the model of the response variable in matrix form.
- (b) (10 pts) Find the least squared estimates of $\mu, \delta_1, \delta_2, \delta_3$.

備

註

- 一、作答於試題上者, 不予計分。
- 二、試題請隨卷繳交。

考 試 科 目	基礎數學	系 所 別	統計學系	考 試 時 間	2 月 18 日 (一) 第 一 節
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For the following problems, show your calculation to receive full credit.

4. (10 pts) Evaluate the following integral:

$$\int_0^{\infty} \frac{\sin(x)}{x} dx.$$

5. (10 pts) Let

$$f(x, y) = \begin{cases} \frac{xy}{x^2+y^2} & (x, y) \neq (0, 0), \\ 0 & (x, y) = (0, 0). \end{cases}$$

Is f continuous at $x = y = 0$? (Give an explanation.)

6. (30 pts) Let $f(\mathbf{c}) = \mathbf{c}^T A \mathbf{c}$, where $\mathbf{c}$ is a 2×1 column vector and A is a 2×2 matrix:

$$\mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \text{ and } A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}.$$

- (a) (20 pts) Find $\mathbf{c}^*$ that maximizes $f(\mathbf{c})$ subject to $c_1^2 + c_2^2 = 1$.
 (b) (10 pts) Find $f(\mathbf{c}^*)$.

備

註

- 一、作答於試題上者，不予計分。
 二、試題請隨卷繳交。

考 試 科 目	數理統計學	系 所 別	統計學系	考 試 時 間	2 月 18 日(一) 第三節
1. A system has 4 components. It works if both component 1 and at least 2 of the other components operate, otherwise it fails. Let Y be the lifetime of the system, X_i be the lifetime of the i^{th} component, and X_i's are <i>i.i.d.</i> with a continuous distribution function $F(x)$. Derive the distribution function and the pdf of Y. (10%) 2. Let $X_1, \dots, X_n$ be a random sample from $f(x) = \theta \eta^\theta x^{-(\theta+1)}$, $x \geq \eta$; $\theta, \eta > 0$. (1) Find the MLEs (maximum likelihood estimators) of θ and η. (8%) (2) Suppose $\eta=1$, then the pdf becomes $f(x) = \theta x^{-(\theta+1)}$, $x \geq 1$; $\theta > 0$. (a) Find MVUE (minimum variance unbiased estimator) of $\theta + \theta^2$. (12%) (b) Find an approximate $100(1-\alpha)\%$ confidence interval for θ. (8%) (c) Let $X_{(1)}$ be the smallest order statistic. (12%) (c1) Is it true that $\ln(X_{(1)}) \xrightarrow{P} 0$? Show your work. (c2) Find the limiting distribution for $n(X_{(1)} - 1)$. 3. The joint distribution of X and Y is $f(x, y) = \begin{cases} 8xy, & 0 \leq x \leq y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$. (1) Suppose $Y=0.6$ is observed. In order to have the smallest MSE (mean square error), what is the predicted value of X? (8%) (2) Let $Z = \frac{Y}{X}$. (12%) (a) Find the joint pdf of (X, Z). (b) Find the marginal pdf of Z, $E(Z)$, and $V(Z)$. 4. Consider two independent random samples: $X_i \sim N(\mu_1, \sigma_1^2)$, $i = 1, \dots, n_1$, and $Y_j \sim N(\mu_2, \sigma_2^2)$, $j = 1, \dots, n_2$. Assume μ_1 and μ_2 are known. (1) Find the MLE of $\frac{\sigma_1^2}{\sigma_2^2}$. (6%) (2) Find the exact $100(1-\alpha)\%$ confidence interval for $\frac{\sigma_1}{\sigma_2}$. (8%) (3) Derive the size α LRT (likelihood ratio test) of $H_0: \sigma_1^2 = \sigma_2^2$ against $H_a: \sigma_1^2 \neq \sigma_2^2$. (16%)					
備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。				

考 試 科 目	統計方法	系 所 別	統計系	考 試 時 間	2 月 18 日 (一) 第四節
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1. (25%) The following table shows the contingency table of sales for paper towels.

Location	Package type	
	Two rolls	Dozen rolls
Rural	89	111
Suburban	75	125
Urban	82	118

Let P_{rural} , $P_{suburban}$, and P_{urban} denote the population proportions of sales of two-roll packages in rural, suburban, and urban locations, respectively. Use the 0.05 level of significance.

- (1) Test $H_0: P_{rural} = P_{suburban}$ versus $H_1: P_{rural} \neq P_{suburban}$. (8%)
 - (2) Test $H_0: P_{rural} = P_{suburban} = P_{urban}$ versus $H_1: P_{rural}, P_{suburban}, \text{ and } P_{urban} \text{ are not all equal.}$ (10%)
 - (3) Construct the 95% confidence interval for $P_{suburban} - P_{urban}$. (7%)
2. (25%) A traffic engineering study on traffic delay was conducted at intersections with signal on urban streets. Three types of traffic signals were utilized and five intersections were used for each type of signal. The measure of traffic delay was the average stopped time per vehicle at each of the interactions (seconds/vehicle) as follows:

	Type of signal		
	Pretimed	Semi-actuated	Fully actuated
	36.6	17.5	15.0
	39.2	20.6	10.4
	30.4	18.7	18.9
	37.1	25.7	10.5
	34.1	22.0	15.2
mean	35.48	20.90	14.00
variance	11.357	10.185	12.915

- (1) Write the linear statistical model for this study. Explain the model components and state the necessary assumptions for your model. (8%)
- (2) Compute the analysis of variance table for the data. (10%)
- (3) Compute the 95% confidence interval of the mean traffic delays for Pretimed. (7%)

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3. (50%) The following table shows a sample of 30 purchase transactions for houses in an area in three-month period.

Price	SqFeet	Bathrooms	Fireplace	Price	SqFeet	Bathrooms	Fireplace
200	3.7	1	yes	172	3.8	2	no
121	2.3	1	yes	301	2.4	2	yes
192	2.8	1	no	243	2.9	2	no
111	2.2	1	yes	345	4.0	3	no
209	3.7	1	yes	298	5.7	3	yes
265	2.9	1	no	429	4.1	3	no
484	7.9	2	yes	285	4.6	3	yes
207	3.1	2	yes	459	5.9	3	yes
256	3.4	2	yes	380	4.9	3	no
203	2.1	2	no	452	5.0	3	yes
260	2.3	2	no	523	9.6	4	yes
345	5.3	2	no	531	6.4	4	yes
275	4.5	2	yes	408	6.5	4	yes
284	5.4	2	no	317	6.1	4	yes
202	2.3	2	no	371	6.1	4	yes

Price: price of the home (in thousand US dollars)

SqFeet: area of the home (in thousand square feet)

Bathrooms: number of bathrooms

Fireplace: with or without fireplaces in the home

Consider the regression model using *Price* as the response variable and the remaining variables are explanatory variables. The following table shows the estimation output.

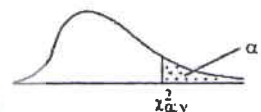
	Estimate	Std Error
<i>Constant</i>	58.65	32.18
<i>SqFeet</i>	37.15	9.27
<i>Bathrooms</i>	41.06	16.32
<i>Fireplace</i>	-24.77	25.06

Assume *Price* follow a normal distribution. Use the 0.05 level of significance if required.

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- (1) Construct the 95% confidence interval for the proportion of homes with fireplaces. (8%)
- (2) Assume the variances of home prices are equal no matter whether a house has a fireplace or not. Test whether the mean home prices are equal or not for houses with or without a fireplace. (12%)
- (3) Please use the result from part (2) and the estimation output to define the dummy variable for *Fireplace* in the regression model. What is the interpretation of the estimated coefficient for *Fireplace* in the regression model? (6%)
- (4) Examine which variables could be omitted from the regression model. (9%)
- (5) If $R^2 = 0.7342$, then construct the ANOVA table for the regression analysis. (10%)
- (6) Compute the adjusted R^2 . (5%)

Table of the Chi-square Distribution

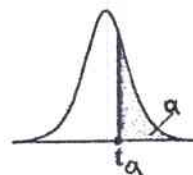


$\alpha =$	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001	$= \alpha$			
$v = 1$	0.000043	0.000157	0.000288	0.000482	0.000939	0.00158	0.00264	0.00429	0.00676	0.01064	0.01642	0.02537	0.03943	0.06057	0.09348	0.13781	0.19422	0.27488	0.38479	0.54154	0.82054	$v = 1$		
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.708	1.064	1.642	2.338	3.001	3.579	4.605	5.991	7.378	7.824	9.210	10.597	13.815	21.004	2		
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.492	2.366	3.219	4.108	5.013	5.841	7.142	8.538	9.348	9.837	11.345	12.838	16.268	23.685	3		
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	2.366	3.357	4.299	5.191	6.094	6.958	8.485	9.488	10.297	11.143	11.668	13.277	14.860	18.465	4		
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	3.219	4.299	5.224	6.251	7.289	8.235	9.236	11.070	12.832	13.388	15.086	16.750	20.517	26.151	5		
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	4.058	5.204	6.191	7.233	8.558	9.803	10.645	12.592	14.449	15.033	16.812	18.548	22.457	28.433	6		
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.903	6.094	7.142	8.291	9.547	10.802	12.017	14.067	16.013	16.622	18.475	20.278	24.322	30.820	7		
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.781	6.958	8.114	9.263	10.395	11.602	12.362	15.507	17.535	18.168	20.090	21.955	26.125	32.909	8		
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	6.634	7.879	9.047	10.264	11.451	12.601	14.684	16.919	19.023	19.679	21.666	23.589	27.877	34.201	9		
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	7.424	8.670	9.847	11.064	12.251	13.401	15.987	18.307	20.483	21.161	23.209	25.188	29.588	36.190	10		
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	8.291	9.547	10.764	11.981	13.171	14.321	17.275	19.675	21.920	22.618	24.725	26.757	31.264	38.155	11		
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	9.109	10.365	11.582	12.799	13.989	15.139	18.549	21.026	23.337	24.054	26.217	28.300	32.909	40.289	12		
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.936	11.192	12.409	13.626	14.816	15.966	19.812	22.362	24.736	25.472	27.688	29.819	34.528	42.782	13		
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.769	12.025	13.242	14.459	15.649	16.799	21.064	23.685	26.119	26.873	29.141	31.319	36.123	44.913	14		
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.609	12.865	14.082	15.299	16.489	17.639	21.311	23.907	26.488	27.248	28.259	30.578	32.801	37.697	46.797	15	
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	12.454	13.710	14.927	16.144	17.334	18.484	22.465	25.065	27.646	28.405	29.633	32.000	34.267	39.252	48.268	16	
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	13.304	14.560	15.777	16.994	18.184	19.334	23.469	26.069	28.650	30.191	30.995	33.409	35.718	40.790	50.000	17	
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	14.159	15.415	16.632	17.849	19.039	20.189	24.319	26.919	29.500	31.526	32.346	34.805	37.156	42.312	52.000	18	
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	15.018	16.274	17.491	18.708	19.898	21.048	25.178	27.778	30.359	32.385	33.687	36.191	38.582	43.820	54.000	19	
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.880	17.136	18.353	19.570	20.760	21.910	26.040	28.640	31.221	33.247	34.550	37.052	39.997	45.315	56.000	20	
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.747	17.964	19.181	20.398	21.588	22.738	26.868	29.468	32.049	34.075	35.378	36.343	38.932	41.401	46.797	58.000	21
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.616	18.833	20.050	21.267	22.457	23.607	27.737	30.337	32.918	34.944	36.781	37.659	40.289	42.796	48.268	60.000	22
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	18.489	19.706	20.923	22.140	23.330	24.480	28.610	31.210	33.791	35.817	38.076	38.968	41.638	44.181	49.728	62.000	23
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.364	20.581	21.798	23.015	24.205	25.355	29.485	32.085	34.666	36.692	39.364	40.270	42.980	45.558	51.179	64.000	24
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	20.242	21.459	22.676	23.893	25.083	26.233	30.363	32.963	35.544	37.570	40.242	41.566	44.314	46.928	52.620	66.000	25
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	21.122	22.339	23.556	24.773	25.963	27.113	31.243	33.843	36.424	38.450	41.223	42.856	45.642	48.290	54.052	68.000	26
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	22.015	23.232	24.449	25.666	26.856	28.006	32.136	34.736	37.317	39.343	42.116	44.140	46.963	49.645	55.476	70.000	27
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	23.027	24.244	25.461	26.678	27.868	29.018	33.148	35.748	38.329	40.355	43.128	45.154	47.977	50.993	56.893	72.000	28
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.691	24.908	26.125	27.342	28.532	29.682	33.812	36.412	38.993	41.019	43.792	45.693	49.588	52.336	58.302	74.000	29
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.579	25.796	27.013	28.230	29.420	30.570	34.700	37.300	39.881	41.907	44.679	46.762	50.892	53.672	59.703	76.000	30
40	20.706	22.164	23.838	24.433	26.509	29.051	32.345	34.769	37.193	39.617	42.041	44.465	46.889	51.019	55.759	59.342	60.436	63.691	66.766	73.402	80.000	90.000	40	
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	44.209	46.969	49.729	52.489	55.249	57.999	62.129	67.069	70.652	71.420	72.613	76.154	79.490	86.661	100.000	50	
60	35.535	37.485	39.699	40.482	43.188	46.459	50.641	53.902	57.163	60.424	63.685	66.946	69.707	73.837	79.082	83.298	84.580	88.379	91.952	99.607	110.000	120.000	60	
70	43.275	45.442	47.893	48.758	51.739	55.329	59.898	63.467	67.036	70.605	74.174	77.743	81.312	85.527	90.531	95.023	96.388	100.425	104.215	112.317	124.000	140.000	70	
80	51.171	53.539	56.213	57.153	60.391	64.278	69.207	73.136	77.065	81.004	84.933	88.862	92.791	96.578	101.880	106.629	108.069	112.329	116.321	124.839	140.000	160.000	80	
90	59.196	61.754	64.634	65.646	69.126	73.291	78.558	83.825	88.092	92.359	96.626	100.893	105.160	109.565	113.145	118.136	119.648	124.116	128.299	137.208	150.000	170.000	90	
100	67.327	70.065	73.142	74.222	77.929	82.358	87.945	93.532	98.119	102.706	107.293	111.667	116.667	121.498	124.342	129.561	131.142	135.807	140.170	149.449	170.000	200.000	100	

考試科目	統計方法	系所別	統計系	考試時間	2月18日(一) 第四節
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Percentage Points of the t Distribution; $t_{v, \alpha}$

$$P(T > t_{v, \alpha}) = \alpha$$



v	α													
	0.40	0.30	0.20	0.15	0.10	0.05	0.025	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960	2.054	2.170	2.326	2.432	2.576	2.807	3.291

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註

- 一、作答於試題上者，不予計分。
二、試題請隨卷繳交。