國立政治大學 106 學年度 碩士班 招生考試試題

考 試 科 目	基礎數學 41412	系 所 別	統計學系	考 試 時 間	2 月 18 日(六) 第一節

Note. You need to show your work in your solutions for the following problems instead of giving final answers only.

1. (20 points) Suppose that

$$f(x) = \begin{cases} 0 & \text{if } x = 0; \\ x^{-3} & \text{if } x < 1 \text{ and } x \neq 0; \\ -1 + x & \text{if } 1 \leq x < 2; \\ xe^{-x^2} & \text{if } x \geq 2. \end{cases}$$

   Find $\int_1^\infty f(x)dx$ and $\int_{-\infty}^\infty f(x)dx$.

2. (16 points) Find the following limits.

   (a) $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x$.

   (b) $\lim_{x\to\infty} \frac{\int_x^\infty e^{-t^2}dt}{xe^{-x^2}}$.

   (c) $\lim_{x\to\infty} \frac{x + \cos(x)}{x - \sin(x)}$.

   (d) $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{k}{n} \sin\left(\frac{2\pi k}{n}\right)$.

3. (14 points) Suppose that u is a differentiable function of x such that

$$u + x\cos(u) = 1$$

   and $u = 0$ when $x = 1$. Let

$$f(x, y) = \int_0^x \int_y^{y+s} s(t-y)dtds + (u+y)^2 - \frac{x}{2}$$

   for $x, y \in (-\infty, \infty)$. Determine whether f has a local minimum or a local maximum at the point $(1, 0)$. Justify your answer.

4. (12 points) Suppose that a is a real number and $a \neq 0$. Let

$$A = \begin{pmatrix} 1 & a & 0 \\ a & 1 & a \\ 0 & a & 1 \end{pmatrix}.$$

   (a) Show that 1 is an eigenvalue of A.

   (b) Find an eigenvector of A associated with the eigenvalue 1.

   (c) Find all eigenvalues of A that are not equal to 1.

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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5. (24 points) Suppose that n is a positive integer and let

$V_n = \{p:\ p$ is a polynomial of real coefficients on $[0,1]$ of degree at most $n.\ \}$.

Then it is clear that V_n is a linear space, where the vector addition is the usual addition for polynomials and a vector multiplied by a scalar means a polynomial multiplied by a real constant. Let

$$W = \left\{ p \in V_n : \int_0^1 p(x)dx = 0 \right\}$$

and

$$W^* = \left\{ q \in V_n : \int_0^1 q(x)p(x)dx = 0 \text{ for every } p \in W \right\}.$$

(a) Show that W and W^* are linear spaces.

(b) Find the dimension of W.

(c) Find the dimension of W^* and give a set of basis for W^*.

6. (14 points) Suppose that A is a 3×3 matrix of the following form

$$\begin{pmatrix} 1 & B \\ O & D^{-1} \end{pmatrix},$$

where $B = \begin{pmatrix} 2 & 3 \end{pmatrix}$ is a 1×2 matrix, O is a 2×1 matrix of zeros, and D^{-1} is the inverse matrix of a 2×2 matrix D. Express the inverse matrix of A in terms of D.

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考 試 科 目	數理統計學 41413	系 所 別	統計學系	考 試 時 間	2 月 18 日(六) 第三節

1. Let Y follow a truncated normal distribution with pdf

$$f(y)=\frac{\phi(y)}{1-\Phi(a)}, \quad y>a,$$

where $\phi(y)$ and $\Phi(a)$ are respectively the pdf and cdf of N(0,1).

Find $\mu=E(Y)$ and $\sigma^2=V(Y)$. (10%)

2. Let $(X,Y)\sim f(x,y)=\exp(-x)$, $x>y>0$.

1) Find $Cov(X,Y)$. (6%)

2) Find the joint pdf of $U=X$ and $V=X+Y$. (6%)

3) Find the marginal pdf of U, and the marginal pdf of V. (8%)

4) Find $P[U\le 1.5\,|\,V=2]$. (6%)

3. Let $X_1,\ldots,X_n$ be a random sample from $f(x;p)=p(1-p)^{x-1}$, $x=1,2,\ldots$,

with $\mu=1/p$ and $\sigma^2=(1-p)/p^2$.

1) Find the MLE for $P[X>m]=(1-p)^m$, for some $m=1,2,\ldots$ (6%)

2) Find the limiting distribution for $\sqrt{n}(1/\bar{X}_n-p)$. (6%)

3) Find the UMVUE for $P[X=1]=p$. (12%)

4. For a simple linear regression model with no intercept:

$Y_i=\beta x_i+\varepsilon_i$, $i=1,\cdots,n$, and $\varepsilon_i\sim iid\ \ N(0,\sigma^2)$.

1) Assume that σ^2 is known.

1a) Find MLE of β, *i.e.*, $\hat{\beta}$, and specify the distribution of $\hat{\beta}$. (10%)

1b) Derive the UMP size α test of $H_o:\beta=0$ vs. $H_a:\beta>0$. (10%)

1c) A test rejects H_o if $\dfrac{\hat{\beta}}{\sigma/\sqrt{\sum_{i=1}^n x_i^2}}>z_\alpha$. Find the power for it at $\beta=1$. (6%)

2) Assume that β and σ^2 are both unknown.

Derive the likelihood ratio test for $H_o:\beta=0$ vs. $H_a:\beta\ne 0$. (14%)

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考試科目	統計方法 41414	系所別	統計學系	考試時間	2月 18日(六) 第四節

1. (22pts) Please answer the following short questions. Answer should be brief and clear.
   (a) When we perform two-sample test of means, whenever there is insufficient evidence that the variance are unequal, it is preferable to perform the equal variance t-test. Why? (4pts)
   (b) 100 households were surveyed as part of a study on water consumption. The mean water usage per day per household in this survey was 370 gallons, and a 95% confidence interval for the mean water usage per household is (364.3,376.2). If we perform an identical study of households that have dogs and found that they use an average of 388.6 gallons per day, with a 95% confidence interval of (381.2,393.5). Can we conclude that having a dog causes households to use more water? Why? (4pts)
   (c) In the regression analysis, what is the major difference between confidence interval and prediction interval? Which one is narrower? Why? (6pts)
   (d) In the regression analysis, adjusted R^2 is the coefficient of determination adjusted for degree of freedom. When should we use adjusted R^2 instead of R^2? Why? (4pts)
   (e) In the regression analysis, what is multicollinearity? Why is multicollinearity a problem? (4pts)

2. (10pts) In each of the following, indicate what would be the best statistical method to use.
   (寫出方法即可，例如: paired data test on means, paired data test on proportions, independent samples test on means, independent samples test on proportions, etc)
   (a) Cathy notes that the most recent Student Government Association election at the NCCU was somewhat controversial. She asks a random sample of 200 students about their choice in the election (candidate A, candidate B, candidate C, or didn't vote), and their major (Sciences, Business, or Education). What test should she do to check whether there's a relation between electoral choice and major? (2pts)
   (b) Helen is conducting a study to determine whether students are more likely to fall asleep in statistics or accounting class. She asks the Institutional Research Office for the names of twenty students who are taking both classes this semester, and contacts them to ask whether they have fallen asleep in stats and/or accounting class this term. What test should she conduct? (2pts)
   (c) A statistics professor believes the final grade of a student depends on how many hours that student has studied statistics and how many hours that student has spent on Facebook. A random sample of 38 students was selected to collect data on their final grade, study hours, and hours spent on Facebook. Which method will he perform to draw conclusion on the belief? (2pts)
   (d) An agronomist wants to compare the crop yield of 3 varieties of chickpea seeds. She plants all 3 varieties of the seeds on each of 5 different patches of fields. She then measures the crop yield in bushels per acre. She has found out that the different varieties do have an impact on crop yield. Which test will be the most appropriate to find out which variety will produce the highest yield? (2pts)

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(e) Two new different models of compact SUVs have just arrived at the market. You are interested in comparing the gas mileage performance of both models to see if they are the same. You are told that the gas mileage population distributions for both models are not normally distributed. Which test will be the most appropriate to perform? (2pts)

3. (6pts) Write down the distribution model for the following random variables.(寫出分配即可，例如: normal distribution)
   (a) the number of times you have to shoot a basketball before you make a basket(投籃得分). (2pts)
   (b) the number of car accidents per day in Taipei. (2pts)
   (c) the number of people you have to interview before you find the fifth left-handed person. (2pts)

4. (6pts) In statistics we are able to make inference because statistics have a predictable distribution called a sampling distribution. One method of inference we have discussed is the idea of hypothesis testing. Explain briefly how the sampling distribution is used in the process of hypothesis testing.

5. (10pts) Player A makes a series of independent bets, winning each with probability θ. When A wins a single bet, his fortune increases by c, when he loses, it decreases by c. His initial fortune is 3. He bets repeatedly until either he increases his fortune to 9 or he is ruined and his fortune is zero. If A reaches 9 before being ruined, we say A wins.
   (a) If the size of each bet is $c = 1$ and if $\theta = 2/3$, find the probability that A wins. (3pts)
   (b) Suppose that $\theta = (2 + X)/(4 + X)$ where $X \sim \text{Binomial}(2, 1/3)$ and the size of each bet is $c = 1$. Find the probability that A wins if θ is chosen at random. (Hint: Use the Law of Total Probability.) (7pts)

6. (10pts) Assume IQs have mean 100 and standard deviation 16. You have created a pill that increases IQ by 10 points. You want to prove that your pill works. Suppose type I error $\alpha = 0.05$ and type II error $\beta = 0.1$. What sample size do you need?

7. (10pts) The number of alcoholic beverages a NCCU freshman consumes in a week has the Poisson distribution with parameter $\lambda = 3.5$. If you sample 20 students, what is the approximate probability that $\overline{X}$ is greater than 3.6?

8. (10pts) Assume the number of phone calls you receive in an hour has a Poisson distribution with parameter λ. Let X be the number of calls you receive between noon and 1 p.m., and let Y be the number of calls you receive between noon and 3:00 p.m. What is the correlation between X and Y ?

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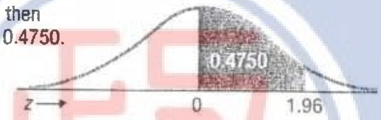
9. (16pts) A filling machine at a local soft drinks company is calibrated to fill the cans at a mean amount of 12 fluid ounces and a standard deviation of 0.5 ounces. The company wants to test whether the standard deviation of the amount filled by the machine is 0.5 ounces. A random sample of 15 cans filled by the machine reveals a standard deviation of 0.67 ounces.
   (a) Which is the appropriate test to use? (2pt)
   (b) In order to perform the test you perform in (a), what assumptions you need to assume? (6pts)
   (c) Please perform the test at 0.05 level of significance. You have to write down the null and alternative hypothesis, test statistic, and rejection region. (8pts)

Areas under the Normal Curve

Example:
If $z = 1.96$, then
$P(0 \text{ to } z) = 0.4750$.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。