# 國立政治大學　104　學年度碩士班招生考試試題



| 考試科目 | 基礎數學 | 所別 | 統計學系 | 考試時間 | 2 月 28 日(六) 第 一 節 |
|---|---|---|---|---|---|

1. (15 points) Find the following integrals.

   (a) $\int_0^1 3.1^x dx$.

   (b) $\int_0^1 \log_{3.1}(x) dx$.

   (c) $\int_0^1 x^{3.1} dx$.

   (d) $\int_0^1 \sin(x) dx$.

   (e) $\int_0^1 \cos(x) dx$.

2. (20 points) Suppose that $f$ is a differentiable function such that $f(0) = 1$ and

$$f'(x) = \frac{x}{2 + \sin(x)}$$

   for $x \in (-\infty, \infty)$.

   (a) Find the minimum of $f$ on $(-\infty, \infty)$.

   (b) Show that $\lim_{x \to \infty} f(x) = \infty$.

3. (10 points) Suppose that $a_1 = b_1 = 1$ and for $n \geq 2$,

$$a_n = a_{n-1} + \frac{n}{2 + \sin(n)}$$

   and $b_n = 1/n$.

   (a) Determine whether $\lim_{n \to \infty} a_n b_n$ exists. Justify your answer.

   (b) Determine whether $\sum_{n=1}^{\infty} a_n b_n$ is finite. Justify your answer.

4. (5 points) Let $D_1 = \{(x, y) : x < 0 \text{ and } y < 0\}$ and $D_2 = \{(x, y) : x > 1 \text{ and } y > 1\}$. Define

$$f(x,y) = \begin{cases} 0 & \text{if } (x,y) \in D_1; \\ x/(1-y+x) & \text{if } y \leq x \text{ and } (x,y) \notin D_1 \cup D_2; \\ y/(1+y-x) & \text{if } y > x \text{ and } (x,y) \notin D_1 \cup D_2; \\ 1 & \text{if } (x,y) \in D_2. \end{cases}$$

   Determine whether $f$ is continuous at $(0,0)$. Justify your answer.

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
|---|---|

| 考試科目 | 基礎數學 | 所別 | 統計學系 | 考試時間 | 2 月 28 日(六) 第 一 節 |
|---|---|---|---|---|---|

5. (20 points) Suppose that $V$ is a vector space, and three vectors $e_1$, $e_2$ and $e_3$ form a basis for $V$. Suppose that $L$: $V \to V$ is a linear transform such that $L(e_1) = e_1 + e_2$, $L(e_2) = e_2$ and $L(e_3) = e_1$. Find the dimension for the space $\{v \in V : L(v) = 0\}$ and the dimension for the range of $L$. Justify your answers.

6. (20 points) Suppose that $A$ is a $3 \times 3$ real matrix with eigenvalues 1, 2, 3 and associated eigenvectors $v_1$, $v_2$ and $v_3$ respectively.

   (a) Can we conclude that $v_1$, $v_2$ and $v_3$ are linearly independent? Justify your answer.

   (b) Suppose that $v_1$, $v_2$ and $v_3$ are orthogonal. Can we conclude that $A$ is symmetric? Justify your answer.

7. (10 points) Suppose that $-1 < a < 1$ and

$$A = \begin{pmatrix} 1 & a & 0 \\ a & 1 & 0 \\ 0 & a & 1 \end{pmatrix}$$

Find the eigenvalues of $A$. Show your work.

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
|---|---|

| 考試科目 | 數理統計學 4141 | 所別 | 統計 | 考試時間 | 2月 28日(六) 第3節 |
|---|---|---|---|---|---|

1. (25pts) Let Y~ Uniform(0, 1). Let $X = \theta Y^{1/3}$. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. with distribution same as X.
   (a) (5pts) Find the probability density function of X and the cumulative distribution function of $X_{(n)}$.
   (b) (5pts) Obtain a complete and sufficient statistic for $\theta$.
   (c) (5pts) Obtain the MLE $\hat{\theta}$ of $\theta$.
   (d) (5pts) Find $E(\hat{\theta})$ and derive an unbiased estimator for $\theta$.
   (e) (5pts) Find UMVUE of $\theta$.

2. (10pts) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables from Gamma $(\alpha, \beta)$ distribution, where $\beta > 0$ is the unknown parameter and $\alpha > 0$ is a known constant. Show that $\sqrt{n}(\hat{\beta} - \beta)$ converges to a non-degenerate asymptotic distribution as $n \to \infty$ and identify the distribution.

3. (10pts) Suppose that X has pdf $f(x|\theta) = 2\theta(1-2x) + 2x$ on [0,1] for $\theta \in \Theta = [0,1]$. A Bayesian wants to test $H_0: \theta \leq 0.4$ *vs* $H_a: \theta > 0.4$. If the Bayesian's prior distribution is uniform on [0,1], what is the pearson's (0-1 loss optimal) test?

4. (10pts) Let the random variable X has p.d.f $f(x;\theta) = \frac{1}{\theta^2} x e^{-\frac{x}{\theta}}, x > 0$, (and 0 otherwise), $\theta \in \Omega = (0, \infty)$.
   What are the $E_\theta \tilde{\theta}_n$ *and* $\sigma_\theta(\tilde{\theta}_n)$. $\tilde{\theta}_n$ is the moment estimator of $\theta$, $\tilde{\theta}_n = \tilde{\theta}_n(\mathbf{X}_n)$, $\mathbf{X}_n = (X_1, X_2, \ldots, X_n)$.

5. (45pts) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables from the Uniform(0, 1),
   $Y_n = (\prod_{i=1}^{n} X_i)^{-\frac{1}{n}}$, and $Z = X_{(n)} - X_{(1)}$.
   (a)(20pts) Show that $\sqrt{n}(Y_n - e) \Rightarrow N(0, e^2)$.
   (b)(25pts) Derive the probability density function of Z. (20pts). Is Z independent of $X_{(n)}$? (5pts).

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
|---|---|

| 考試科目 | 統計方法 | 所別 | 統計學系 | 考試時間 | 2 月 28 日( 六 ) 第 四 節 |
|---|---|---|---|---|---|

(每小題5分，請依序作答)

1) The following ANOVA table is the output of a simple linear regression analysis for a dependent variable $Y$ and an independent variable $X$.

| Source | SS | *df* | MS | *F* |
|---|---|---|---|---|
| Regression | 300 | 1 | 300 | 4.5 |
| Error | 600 | 9 | 66.67 | |
| Total | 900 | 10 | | |

**Q1)** The proportion of variation in $Y$ explained by the variation in $X$ = _____.

**Q2)** If the relationship between $X$ and $Y$ is negative, the coefficient of correlation = _____.

**Q3)** Given $\alpha = 0.05$, is the slope of the regression line significant (yes/no)? _____.

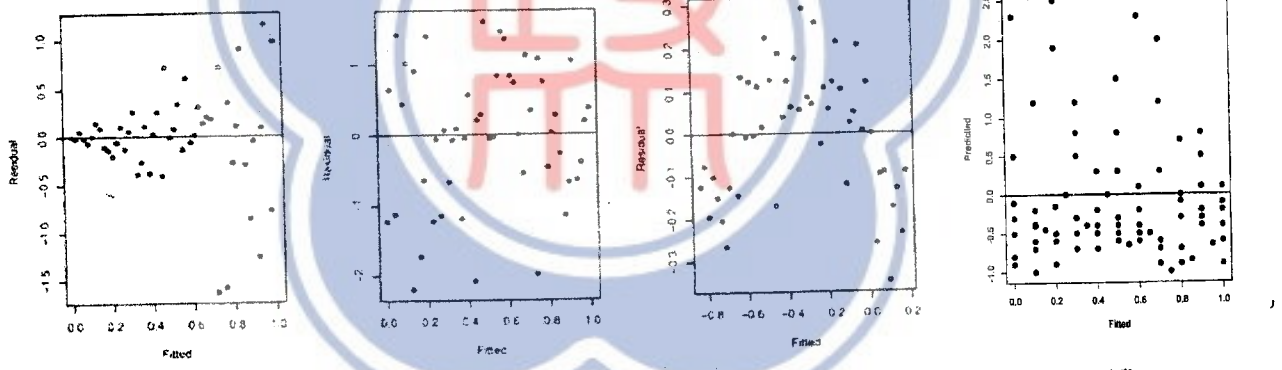
(**Hint:** The critical values $F_{0.95}(1, 9) = 5.12$, $F_{0.95}(1, 10) = 4.96$, $F_{0.95}(9, 10) = 3.02$)

**Q4)** If the estimated regression equation is $\hat{Y} = 0.5 - 4X$, and we know $\overline{X} = 16$, $\sum(X - \overline{X})^2 = 256$.

Given $X = 10$, what is the 95% confidence interval for the mean of $Y$? _____.

(**Hint:** The critical values $t_{0.975}(9) = 2.262$, $t_{0.95}(9) = 1.833$, $t_{0.975}(10) = 2.228$, $t_{0.95}(10) = 1.812$)

**Q5)** The following are four residual plots (vs the fitted values), which one(s) violate the assumptions of usual regression analysis? Answer: _____.



(a)　(b)　(c)　(d)

2) The production manager of Apple wants to compare the mean number of defective iPhone 6s produced on the factories in New York and Seattle. The manager obtained the following sample information regarding the number of defects:

| | Mean | Sample Variance | Sample Size |
|---|---|---|---|
| New York | 8 | 2.25 | 10 |
| Seattle | 10 | 5.11 | 10 |

**Q6)** Given $\alpha = 0.10$, would you conclude that the variances of the two factories are the same? (yes/no)_____

(**Hint**: $F_{0.95}(10, 10) = 2.98$, $F_{0.90}(10, 10) = 2.32$, $F_{0.95}(9, 9) = 3.18$, $F_{0.90}(9, 9) = 2.44$)

**Q7)** What is the 90% confidence interval for the mean of New York? _____.

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
|---|---|

| 考試科目 | 統計方法 4144 | 所別 | 統計學系 | 考試時間 | 2 月 28 日( 六 ) 第 四 節 |
|---|---|---|---|---|---|

**Q8)** If one believes that the variances for the two factories are the same, and the manager claims that the true mean number of defects in Seattle is larger. Compute the *t* statistic for testing such a claim. The *t* statistic = _____.

**Q9)** If one believes that the variances for the two factories are different, will the computed test statistic be different from the one obtained in Q8? (yes/no) _____.

**Q10)** Are the critical values of the *t* statistics used to perform the tests in Q8 and Q9 the same? (yes/no)_____

3) The following observed scores were randomly selected from three independent populations A, B, C.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Population A | 12 | 15 | 19 | 23 | 29 | 40 | 51 |
| Population B | 13 | 21 | 16 | 17 | 33 | 43 | |
| Population C | 11 | 18 | 30 | 29 | 24 | 45 | |

**Q11)** What is the computed test statistic if the ANOVA technique is used to compare the three population means? The computed *F* statistic = _____.

**Q12)** The Kruskal-Wallis test can also be considered to determine whether there is a difference among populations A, B, C by using ranked data. The computed test statistic *H* = _____.

**Q13)** Given the same level of significance $\alpha = 0.05$, are the decisions based on ANOVA and Kruskal-Wallis test the same? (yes/no) _____ (**Hint**: $F_{0.95}(2, 16) = 3.63$, $F_{0.95}(2, 18) = 3.55$, $F_{0.95}(3, 19) = 3.13$)

**Q14)** Consider the scores of populations B and C as paired data, the Pearson's coefficient of correlation between the two scores = _____.

**Q15)** Consider the scores of populations B and C as paired data, the Spearman's coefficient of correlation between the two scores = _____.

4) Four manufacturers of light bulbs are being considered for the declaration of Xmas trees on campus. The director of purchasing asked for 100 samples from each manufacturer. The numbers of acceptable and unacceptable bulbs from each manufacturer are show below.

| | Manufacturer | | | |
|---|---|---|---|---|
| | A | B | C | D |
| Unacceptable | 12 | 8 | 5 | 11 |
| Acceptable | 88 | 92 | 95 | 89 |
| Total | 100 | 100 | 100 | 100 |

**Q16)** Based on all the samples, what is the 95% confidence interval for the "proportion of acceptable light bulbs"? _____ (**Hint**: $z_{0.95} = 1.645$, $z_{0.975} = 1.96$)

**Q17)** Continuing with Q16, assume that no estimate is available for the population proportion and one wants the estimation margin of error to be within 0.01 of the true proportion, then what is the required sample size? _____

**Q18)** What is the name of the hypothesis test one can use to validate if the quality of bulbs "related to" manufacturer? _____

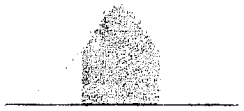
**Q19)** Continuing with Q18, what is the computed test statistic? _____

**Q20)** The director claims that 85% of light bulbs are acceptable and 15% of light bulbs not acceptable. Consider a hypothesis test to validate the director's claim at the level of significance 0.10, do you agree with him? (yes/no) _____

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
|---|---|

| 考試科目 | 統計方法 41414 | 所別 | 統計學系 | 考試時間 | 2 月 28 日( 六 ) 第 四 節 |
|---|---|---|---|---|---|

## 1. Student's t Distribution

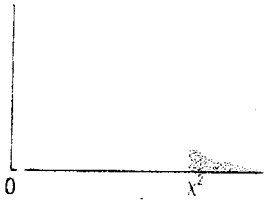



Confidence interval


Left-tailed test

| | Confidence Intervals, c | | | | | |
|---|---|---|---|---|---|---|
| | 80% | 90% | 95% | 98% | 99% | 99.9% |
| | Level of Significance for One-Tailed Test, α | | | | | |
| df | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0005 |
| | Level of Significance for Two-Tailed Test, α | | | | | |
| | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.001 |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 636.619 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 31.599 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 12.924 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 8.610 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 6.869 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.959 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 5.408 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 5.041 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.781 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.587 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.437 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 4.318 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 4.221 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 4.140 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 4.073 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 4.015 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.965 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.922 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.883 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.850 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.819 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.792 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.768 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.745 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.725 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.707 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.690 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.674 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.659 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.646 |
| 31 | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 | 3.633 |
| 32 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 3.622 |
| 33 | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 | 3.611 |
| 34 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 3.601 |
| 35 | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 | 3.591 |

## 2. Critical Values of Chi-Square




Example: With 17 *df* and a .02 area in the upper tail, $\chi^2 = 30.995$

| Degrees of Freedom, df | Right-Tail Area | | | |
|---|---|---|---|---|
| | 0.10 | 0.05 | 0.02 | 0.01 |
| 1 | 2.706 | 3.841 | 5.412 | 6.635 |
| 2 | 4.605 | 5.991 | 7.824 | 9.210 |
| 3 | 6.251 | 7.815 | 9.837 | 11.345 |
| 4 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 14.684 | 16.919 | 19.679 | 21.666 |
| 10 | 15.987 | 18.307 | 21.161 | 23.209 |
| 11 | 17.275 | 19.675 | 22.618 | 24.725 |
| 12 | 18.549 | 21.026 | 24.054 | 26.217 |
| 13 | 19.812 | 22.362 | 25.472 | 27.688 |
| 14 | 21.064 | 23.685 | 26.873 | 29.141 |
| 15 | 22.307 | 24.996 | 28.259 | 30.578 |
| 16 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 33.196 | 36.415 | 40.270 | 42.980 |
| 25 | 34.382 | 37.652 | 41.566 | 44.314 |
| 26 | 35.563 | 38.885 | 42.856 | 45.642 |
| 27 | 36.741 | 40.113 | 44.140 | 46.963 |
| 28 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 40.256 | 43.773 | 47.962 | 50.892 |

| 備註 | 一、作答於試題上者，不予計分。<br>二、試題請隨卷繳交。 |
|---|---|