

考 試 科 目	基礎數學	所 別	統計學系 444	考 試 時 間	2 月 22 日 第 1 節 (A)
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(15 pts) Q1. Given that $\lim_{x \rightarrow a} x = a$, prove $\lim_{x \rightarrow a} x^2 = a^2$, where a is any real number, by using the precise definition ($\varepsilon - \delta$ notation) of a limit.

(15 pts) Q2. If f is a continuous function on an interval, and if a is any number in that interval, then the function defined on the interval as follows is an antiderivative of f :

$$F(x) = \int_a^x f(t) dt$$

(10 pts) Q3. Decide whether $\int_1^{\infty} \frac{\sin x + 3}{\sqrt{x}} dx$ converges or diverges. Show your work.

(10 pts) Q4. Consider the function $f(x, y) = x^{\frac{1}{2}} y^{\frac{1}{2}}$. Show that the partial derivatives $f_x(0, 0)$ and $f_y(0, 0)$ exist, but that f is not differentiable at $(0, 0)$.

(15 pts) Q5. Let W be an m -dimensional subspace of $\mathbb{R}^n$ with orthonormal basis $\{X_1, X_2, \dots, X_m\}$. Then every vector X in $\mathbb{R}^n$ can be written as $X = Z + Y$ where Z is in W and Y is orthogonal to every vector in W .

(15 pts) Q6. If $L: V \rightarrow W$ is a linear transformation, then $\dim(\ker L) + \dim(\text{range } L) = \dim V$, assuming that $1 \leq \dim L \leq \dim V$.

Q7. (5 pts) (a) If A is a $n \times n$ matrix, show that if X and Y are vectors in $\mathbb{R}^n$, then

$$(AX) \cdot Y = X \cdot (A^T Y)$$

(15 pts) (b) If A is a symmetric matrix, then eigenvectors that belong to distinct eigenvalues of A are orthogonal.

考 試 科 目	數理統計學	所 別	統計學系 414	考 試 時 間	2 月 22 日(六) 第三節
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1. (15%) Consider an infinite sequence of independent tosses of a coin that comes up heads with probability 0.2.
 - (a) (5%) Compute the probability that the 5th head appears on the 10th toss.
 - (b) (10%) Let X be such that the first heads appears on the X th toss. In other words, X is the number of tosses required to obtain a heads. Compute the expectation $E[X]$.
2. (50%) Consider a random sample of size 2, X_1, X_2 , from the uniform distribution over the interval $(0, \theta)$ for $\theta > 0$.
 - (a) (10%) Find the p.d.f. of the sample range $R = |X_1 - X_2|$.
 - (b) (10%) Find an unbiased sufficient estimator of θ , denoted as U , and the p.d.f. of U .
 - (c) (10%) Find the maximum likelihood estimator (MLE) of θ , denoted as T .
 - (d) (10%) It can be seen in (c) and (d) that the MLE T is a function of the sufficient estimator U . Show the following general result: Let $X_1, \dots, X_n$ be a random sample from a distribution that has pdf $f(x; \theta)$, $\theta \in \Omega$. If a sufficient statistic U for θ exists and if a MLE T also exists uniquely, then T is a function of U .
 - (e) (10%) Which one is a better estimator for θ ? R , U or T ? Please consider at least two criterion for comparison and explain the results.
3. (10%) Suppose $X \sim \text{Bin}(n_X, \pi_X)$, $Y \sim \text{Bin}(n_Y, \pi_Y)$ and X, Y are independent. Derive the conditional distribution of X given $X + Y$.
4. (10%) Let $X_1, \dots, X_n$ be a random sample from a gamma distribution with known parameter α_0 and unknown $\beta > 0$. Construct a confidence interval for β .
5. (15%) If $X_1, \dots, X_n$ is a random sample from a distribution with the following p.d.f.

$$f(x; \theta) = \begin{cases} \frac{1}{2}\theta^3 x^2 \exp\{-\theta x\}, & 0 < x < \infty, \\ 0 & \text{elsewhere} \end{cases}$$

where $0 < \theta < \infty$. Find the unbiased minimum variance estimator of θ .

考 試 科 目	統計方法	所 別	統計碩 4141	考 試 時 間	2 月 22 日 (天) 第 4 節
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1. 假設某公司想了解三種不同品質的材料(A, B, C)所生產的觸控面板之平均銷售額是否有異。隨機調查四個月獲得之觸控面板銷售額資料如表所示：
(以顯著水準 $=0.05$ 來檢定, 銷售額單位: 百萬元)

	A	B	C
銷	50	50	65
售	50	60	65
額	55	70	70
	55	55	70

- (1) 請建立上述資料之變異數分析表 (ANOVA) ? (10分)
(2) 請檢定材料品質是否對產品的銷售額有影響? 並說明數據之假設條件為何 (10分)

2. 一家公司之總裁認為產品的銷售額是跟產品之種類有關。隨機抽取產品 A 在 6 個月的銷售額，產品 B 在 8 個月的銷售額。它們之銷售額如下(銷售額單位: 百萬元):

產品 A: 15, 20, 25, 20, 30, 25

產品 B: 20, 25, 30, 25, 30, 35, 20, 35

假設銷售額服從常態分配，

- (1) 產品 A 和產品 B 之平均銷售額之點估計值分別為何? (6%)
(2) 產品 A 和產品 B 之銷售額標準差之點估計值分別為何? (6%)
(3) 檢定產品 A 和 B 之銷售額標準差是否相等，顯著水準為 0.05。 (8分)
(4) 依 (3) 的結果，說明銷售額標準差之點估計值。 (6分)
(5) 依 (3) 的結果，在顯著水準為 0.05 下，檢定產品 A 和 B 之平均銷售額是否相同? (8分)

3. 假設隨機樣本 $X_i \sim N(\mu, \sigma^2) \quad i=1, 2, \dots, n$, μ 未知但 σ 已知。在顯著水準為 0.1 下，

檢定 $H_0: \mu = \mu_0$ vs $H_1: \mu > \mu_0$ 。

- (1) 在 H_0 為真下，檢定統計量分配上接受區的臨界點為何? (5分)
(2) 已知檢定結果拒絕 H_0 ，若事實上 $\mu = \mu_0 + 0.5\sigma$ ，取樣本大小 $n=16$ ，則檢定力為何? (10分)
(3) 續題(2)，若(2)中之 n 未知，欲使型 II 誤差機率 $(\beta) < 0.2$ 則 n 至少應為多少? (10分)

考 試 科 目	統計方法	所 別	統計碩 4441	考 試 時 間	2 月 22 日 (天) 第 4 節
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4. 假設 X 為生產之晶圓厚度，已知 $X \sim N(\mu, \sigma^2)$ 。

$$\text{令 } L_0 = \mu, L_1 = \mu + \sigma, L_2 = \mu + 2\sigma, L_3 = \mu + 3\sigma,$$

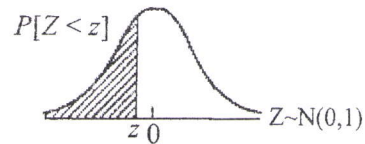
$$L_4 = \mu - \sigma, L_5 = \mu - 2\sigma, L_6 = \mu - 3\sigma.$$

- (1) 若 $(X \geq L_3 \text{ or } X \leq L_6)$ 則視為不合格品。計算不合格品之機率。(3 分)
- (2) 計算 $P(L_2 \leq X \leq L_3 \text{ or } L_6 \leq X \leq L_5)$ 之機率值。(3 分)
- (3) 自一批晶圓中隨機抽取 4 個晶圓，若至少有 3 個晶圓厚度落在 $(L_2 \text{ 和 } L_3 \text{ 區間})$ 或 $(L_5 \text{ 和 } L_4 \text{ 區間})$ 則此批晶圓需被退貨。計算退貨之機率。(8 分)
- (4) 自一批晶圓中隨機抽取 5 個晶圓，計算最多有 4 個晶圓厚度落在 $(L_2 \text{ 和 } L_3 \text{ 區間})$ 或 $(L_6 \text{ 和 } L_5 \text{ 區間})$ 之機率。(7 分)



考 試 科 目	統計方法	所 別	統計碩	考試時間	2 月 22 日 (天) 第 4 節
			4141		

【註】附統計表：標準常態分配機率分配表; r. v. $Z \sim N(0,1)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
-.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

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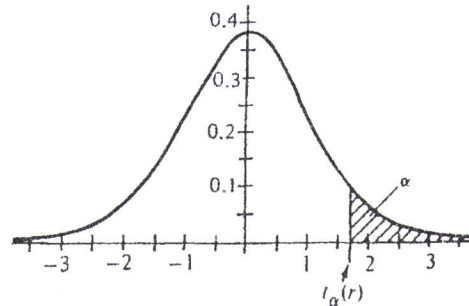
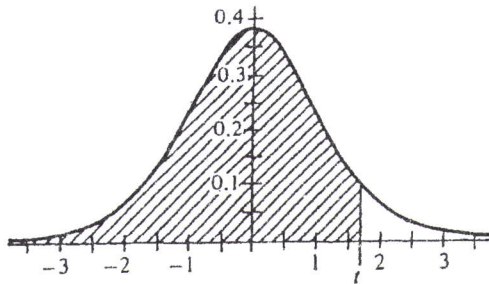
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考試科目 統計方法

所別 統計碩

考試時間 二月 22 日 (八) 第 4 節

4441

The t Distribution

$$P(T \leq t) = \int_{-\infty}^t \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r} \Gamma(r/2) (1 + w^2/r)^{(r+1)/2}} dw$$

$$[P(T \leq -t) = 1 - P(T \leq t)]$$

	$P(T \leq t)$						
	0.60	0.75	0.90	0.95	0.975	0.99	0.995
r	$t_{0.40}(r)$	$t_{0.25}(r)$	$t_{0.10}(r)$	$t_{0.05}(r)$	$t_{0.025}(r)$	$t_{0.01}(r)$	$t_{0.005}(r)$
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012
14	0.258	0.692	1.345	1.761	2.145	2.624	2.997
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576

This table is taken from Table III of Fisher and Yates: *Statistical Tables for Biological, Agricultural, and Medical Research*, published by Longman Group Ltd., London (previously published by Oliver and Boyd, Edinburgh), by permission of the authors and publishers.

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考 試 科 目 統計方法

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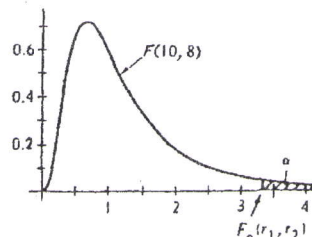
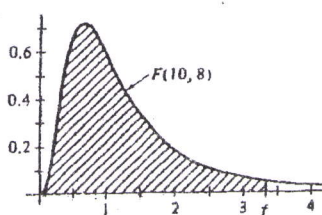
統計碩

考 試 時 間

2月22日(天) 第4節

4441

The F Distribution



$$P(F \leq f) = \int_0^f \frac{\Gamma[(r_1 + r_2)/2] (r_1/r_2)^{r_1/2} w^{r_1/2 - 1}}{\Gamma(r_1/2) \Gamma(r_2/2) (1 + r_1 w/r_2)^{(r_1 + r_2)/2}} dw$$

α	$P(F \leq f)$	Den. d.f. r_2	Numerator Degrees of Freedom, r_1									
			1	2	3	4	5	6	7	8	9	10
0.05	0.95	1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9
0.025	0.975	1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63
0.01	0.99	1	4052	4999.5	5403	5625	5764	5859	5928	5981	6022	6056
0.05	0.95	2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
0.025	0.975	2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40
0.01	0.99	2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
0.05	0.95	3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
0.025	0.975	3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42
0.01	0.99	3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
0.05	0.95	4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
0.025	0.975	4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84
0.01	0.99	4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
0.05	0.95	5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
0.025	0.975	5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62
0.01	0.99	5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
0.05	0.95	6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
0.025	0.975	6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46
0.01	0.99	6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
0.05	0.95	7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
0.025	0.975	7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76
0.01	0.99	7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
0.05	0.95	8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
0.025	0.975	8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30
0.01	0.99	8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
0.05	0.95	9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
0.025	0.975	9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96
0.01	0.99	9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
0.05	0.95	10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
0.025	0.975	10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72
0.01	0.99	10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
0.05	0.95	12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
0.025	0.975	12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37
0.01	0.99	12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
0.05	0.95	15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
0.025	0.975	15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06
0.01	0.99	15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
0.05	0.95	20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
0.025	0.975	20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77
0.01	0.99	20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
0.05	0.95	24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
0.025	0.975	24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64
0.01	0.99	24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
0.05	0.95	30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
0.025	0.975	30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51
0.01	0.99	30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
0.05	0.95	40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
0.025	0.975	40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39
0.01	0.99	40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
0.05	0.95	60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
0.025	0.975	60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27
0.01	0.99	60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
0.05	0.95	120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91
0.025	0.975	120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16
0.01	0.99	120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
0.05	0.95	∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83
0.025	0.975	∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05
0.01	0.99	∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32