考 試 科 目	計算機數學	系 所 別	資訊安全碩士學位學程	考 試 時 間	2月 5日(四) 第 二 節

本次考試共 25 題單選題，每題 4 分。選擇題請在答案卡上作答，否則不予計分。

1. If the following matrix equation holds:

$$\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} 1 & d & e \\ 0 & 1 & f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 2 \\ -2 & 5 & 3 \end{bmatrix}$$

Please determine the value of $a+b+c+d+e+f$.

(A) 10 (B) 11 (C) 12 (D) 13

2. How many of the following statements are correct?

Let A be an $n \times m$ matrix whose null space has dimension k. Consider the following statements:

1. The dimension of $NULL(A^T)$ is $n-m+k$.
2. The dimension of $CS(A)$ is $m-k$.
3. The dimension of $RS(A)$ is $m-k$.
4. The dimension of $RS(A)$ is $n-k$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

3. Let $E=\{v_1, v_2\}=\left\{\begin{bmatrix} 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 14 \end{bmatrix}\right\}$ and $F=\{u_1, u_2\}=\left\{\begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}\right\}$ be two bases for $\mathbb{R}^2$.

If $[I]_E^F=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the transition matrix from E to F, where $a, b, c, d \in \mathbb{R}$. Please determine the value of $a+b+c+d$.

(A) 3 (B) 5 (C) 7 (D) 9

考 試 科 目	計算機數學	系 所 別	資訊安全碩士學位學程	考 試 時 間	2月 5日(四) 第 二 節

4. How many of the following statements are True?

(1) Suppose that $T:\mathbb{R}^2 \to \mathbb{R}^3$ is linear and $T(1,2)=(1,0,2)$ and $T(2,3)=(1,-1,4)$. Is $\mathrm{Ker}(T)=\{0\}$? (2) The linear transformation $L:P_2 \to P_2$ defined by $L(at^2+bt+c)=2a+b$ is one to one. (3) Suppose that $T:\mathbb{R}^2 \to \mathbb{R}^3$ is defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)=\begin{bmatrix} 2x_1-x_2 \\ -x_1 \\ 6x_1 \end{bmatrix}$. Is T onto?

(A) 0 (B) 1 (C) 2 (D) 3

5. How many of the following statements are False?

(1) If A is invertible and 1 is an eigenvalue of A, then 1 is also an eigenvalue of A^{-1}. (2) If A contains a row or column of zeros, then 0 is a eigenvalue of A. (3) Each eigenvector of A is also an eigenvector of A^2. (4) Each eigenvalue of A is also an eigenvalue of A^2. (5) Eigenvectors must be nonzero vectors.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

6. Find an orthonormal basis for the column space of D:

$$D=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 0 \\ 1 & 4 & 6 \\ 1 & 4 & 6 \end{bmatrix}$$

Let the orthonormal basis obtained by the Gram-Schmidt process be $\{q_1,q_2,q_3\}$, where:

$$q_1=\frac{1}{2}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix},\quad q_2=\frac{1}{\sqrt{12}}\begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix},\quad q_3=\frac{1}{\sqrt{6}}\begin{bmatrix} i \\ j \\ k \\ l \end{bmatrix}$$

Please determine the value of the sum of all numerators: $a+b+c+d+e+f+g+h+i+j+k+l$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

備 註	作答於試題上者，不予計分。 試題請隨卷繳交。

考試科目	計算機數學	系所別	資訊安全碩士學位學程	考試時間	2月 5日(四) 第二節

7. Given the data points $(x, y) = (0, 1), (3, 4), (6, 5)$, find the best squares fit by a linear function $y = ax + b$.

Please determine the value of $a + b$.

(A) 1 (B) 2 (C) 3 (D) 4

For problems 8-10, please find a singular value decomposition for the following matrix:

$$B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 1 & -3 \end{bmatrix} = U\Sigma V^T$$

8. $U = ?$

$(A) \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix}$ $(B) \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ $(C) \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$ $(D) \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \end{bmatrix}$

9. $\Sigma = ?$

$(A) \begin{bmatrix} 16 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $(B) \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $(C) \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $(D) \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$

10. $V = ?$

$(A) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $(B) \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix}$ $(C) \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$ $(D) \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix}$

備註	作答於試題上者，不予計分。 試題請隨卷繳交。

考 試 科 目	計算機數學	系 所 別	資訊安全碩士學位學程	考 試 時 間	2月 5日(四) 第 二 節

11. Let set $A = \{1,2,3,4\}$. Define a relation R on A where $R = \{(1,1),(1,2),(2,2),(2,3),(1,3),(3,3),(4,4)\}$, which of the following properties does this relation have?

(1) Symmetric (2) Asymmetric (3) Antisymmetric (4) Reflexive (5) Irreflexive (6) Transitive

(A) (3), (4), (6) (B) (1), (4), (6) (C) (3), (5) (D) (2), (5), (6)

12.

$$47^{245} \equiv a \pmod{19}, \text{find } a.$$

(A) 4 (B) 5 (C) 9 (D) 11

13. Let $A = \{\emptyset, 1, \{1\}, \{1, \emptyset\}\}$, and $P(A)$ denote the power set of A. How many of the following statements are TRUE?

- $\{1\} \in A$
- $\{1\} \subseteq A$
- $\{\emptyset\} \in A$
- $\{\emptyset\} \subseteq A$
- $\{1, \emptyset\} \in A$
- $\{1, \emptyset\} \subseteq A$
- $\{\{1\}\} \subseteq P(A)$
- $\{\emptyset, \{1\}\} \in P(A)$
- $\{\emptyset, \{1\}\} \subseteq P(A)$

(A) 6 (B) 7 (C) 8 (D) 9

備 註	作答於試題上者，不予計分。 試題請隨卷繳交。

考 試 科 目	計算機數學	系 所 別	資訊安全碩士學位學程	考 試 時 間	2 月 5 日(四) 第 二 節

The following questions constitute a set. Please answer accordingly.

Suppose A and B are events in a sample space, such that $P(A)=\frac{4}{9}, P(B)=\frac{5}{11}, P(A|B)=\frac{2}{5}$, and $P(B|A)=\frac{a}{b}$ (where $\frac{a}{b}$ is an irreducible fraction).

14. $a=?$

(A) 7 (B) 8 (C) 9 (D) 10

15. $b=?$

(A) 20 (B) 21 (C) 22 (D) 23

The following questions constitute a set. Please answer accordingly.

Let $X=\{1,2,3,4,5,6\}$ and $Y=\{a,b,c,d\}$. For a function f and a set S, define $f(S)=\{f(i)\mid i\in S\}$. Please find the answers for the following questions.

16. How many functions $f: X\to Y$ are onto (surjective)?

(A) 1024 (B) 1560 (C) 2048 (D) 4096

17. How many functions $f: X\to Y$ are there such that $|f(X)|=3$?

(A) 540 (B) 1080 (C) 1260 (D) 2160

18. How many functions $f: X\to X$ are there such that $f(\{1,2,3\})=\{1,2\}$ and for all $x\in X, f(x)\neq 6$?

(A) 192 (B) 384 (C) 512 (D) 768

備 註	作答於試題上者，不予計分。 試題請隨卷繳交。

考試科目	計算機數學	系所別	資訊安全碩士學位學程	考試時間	2月 5日(四) 第二節

For the following problems, please solve the linear recurrence relation $a_n - 4a_{n-1} + 4a_{n-2} = 2^n$ with $a_0 = 1$ and $a_1 = 2$, and let the solution be in the form $a_n = (i + jn + kn^2) \cdot 2^n$.

19. $i =$?

(A) 0 (B) 1 (C) 2 (D) -1

20. $j =$?

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) -1

21. $k =$?

(A) 1 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

22. Consider a graph $G = (V, E)$ with vertices $V = \{1, 2, 3, 4, 5, 6, 7\}$ and edges $E = \{(1,2), (2,3), (3,1), (3,4), (4,5), (5,6), (6,4), (4,7)\}$. How many of the following statements are TRUE?

- It is a bipartite graph.
- The length of the longest simple path is 6.
- It has an Euler path.
- It is a planar graph.

(A) 0 (B) 1 (C) 2 (D) 3

23. Let A and B be sets. Which of the following statements is TRUE?

(A) If $A \subseteq B$, then $A \cup B = A$. (B) If $A \subseteq B$, then $A \cap B = A$. (C) If $A \in B$, then $A \subseteq B$. (D) If $A \subseteq B$ and $B \in C$, then $A \in C$.

備註	作答於試題上者，不予計分。 試題請隨卷繳交。

考試科目	計算機數學	系所別	資訊安全碩士學位學程	考試時間	2月 5日(四) 第二節

24. Let $G = (V, E)$ be a simple undirected graph with $|V| = n$ and $|E| = m$. Let G be a bipartite graph with partition $V = V_1 \cup V_2$. How many of the following statements are ALWAYS TRUE?

- If G is connected and $m = n - 1$, then G is a tree.
- The number of vertices with odd degree in G is even.
- If G has a Hamiltonian Cycle, then $|V_1| = |V_2|$.
- If G has a Hamiltonian Path, then $|V_1| = |V_2|$.

(A) 1 (B) 2 (C) 3 (D) 4

25. How many of the following statements are TRUE? (Note: $\mathbb{Z}$ is the set of integers, $\mathbb{R}$ is the set of real numbers, and $\mathbb{N} = \{1, 2, 3, \ldots\}$)

- $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, defined by $f(m, n) = 2m + 3n$, is onto (surjective).
- $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = x^3 + x$, is one-to-one (injective).
- $f : \mathbb{Z} \to \mathbb{Z}$, defined by $f(n) = \lfloor \frac{n}{2} \rfloor$, is one-to-one.
- $f : \mathbb{R}^2 \to \mathbb{R}^2$, defined by $f(x, y) = (x + y, x + y)$, is bijective.
- $f : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$, defined by $f(n) = (n, n + 1)$, is onto.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

備註	作答於試題上者，不予計分。 試題請隨卷繳交。