

考試科目	計算機數學	系所別	資訊安全碩士學位學程 一般生	考試時間	2月12日(三)第2節
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I. 離散數學：60% (第 1~11、17~19 題)

II. 線性代數：40% (第 12~16 題)

請書寫必要的解題過程，僅提供答案而無必要過程，將無法獲得該題滿分。可使用中文或英文作答，力求書寫工整，如字跡潦草，無法閱讀，將影響評分。

第一部分、單選題 (第 1~14 題)，每題 3 分，無需提供解題過程。

1. Which of the following pairs of propositions is logically equivalent?

- (a) $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$
- (b) $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$
- (c) $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \wedge \neg r)$
- (d) $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (\neg q \wedge r)$

2. Let $A = \{a, b, c, d, e, f, g\}$ and $R = \{(a, a), (b, b), (c, c), (c, d), (c, g), (d, g), (e, e), (f, f), (g, g)\}$ be a relation on A. Then R is

- (a) reflexive and symmetric
- (b) antisymmetric and not reflexive
- (c) reflexive and antisymmetric
- (d) symmetric and not reflexive

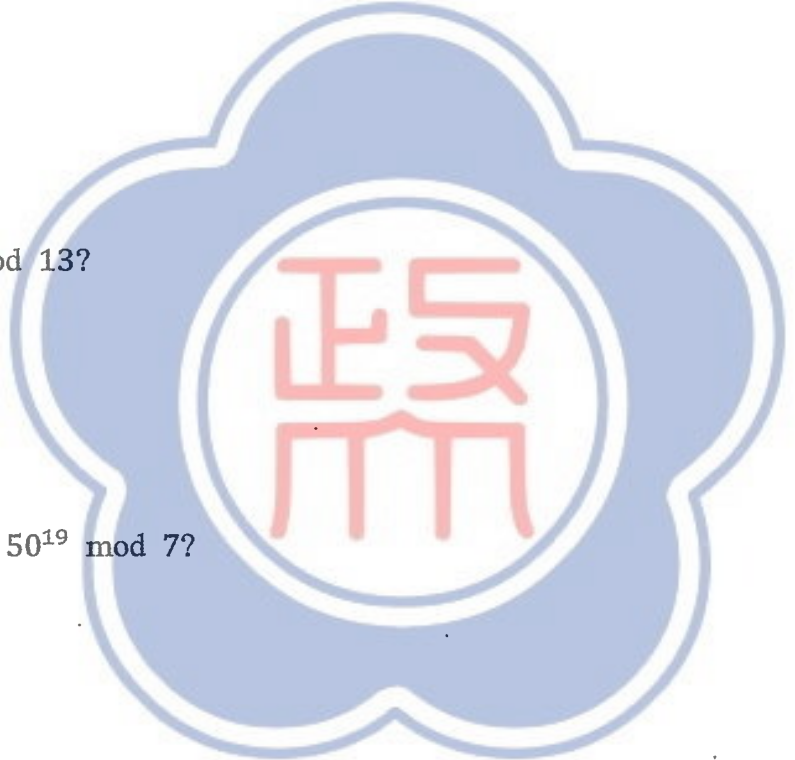
3. Which of the following statements is NOT valid:

- (a) $(A - C) \cap (C - B) = A - B$
- (b) $(B - C) \cup (C - B) \cup (\overline{B} \cap \overline{C}) = \overline{(B \cap C)}$
- (c) $(A \oplus B) \oplus B = A$
- (d) $(A \cap B) \cup (A \cap B \cap C) = (A \cup C) \cap (A \cap B)$

4. In how many different ways can 10 identical balls distributed among four distinguishable children if each child receives at least two balls?

- (a) 8
- (b) 9
- (c) 10
- (d) 11

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5. For $n \geq 1$, how many functions are there from the set $\{1, \dots, n\}$ to the set $\{1, 2, 3, 4, 5\}$?
- (a) n^5
(b) $5n$
(c) 5^n
(d) $5n!$
6. Following Q.5, how many of the functions are injective when $n = 3$?
- (a) 20
(b) 50
(c) 60
(d) 120
7. What is $191^{45} \pmod{13}$?
- (a) 3
(b) 9
(c) 5
(d) 1
8. What is $(-12)^{36} \cdot 50^{19} \pmod{7}$?
- (a) 2
(b) 1
(c) 4
(d) 3
9. What is the probability that a fair die, rolled repeatedly until the number 6 appears for the first time, requires exactly four rolls? Choose the closest value:
- (a) 8%
(b) 10%
(c) 12%
(d) 14%
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10. At a university with students from exactly 50 states, what is the smallest number of students needed to ensure that at least 100 students are from the same state?
- (a) 4951
(b) 4949
(c) 4957
(d) 4954
11. How many of the following cases correspond to the existence of a planar graph?
(1) 7 vertices and 13 edges; (2) 6 regions and 5 vertices; (3) 8 vertices and 20 edges; (4) 10 regions and 5 edges.
- (a) 1
(b) 2
(c) 3
(d) 4
12. For an $n \times n$ matrix A , what can you say about the eigenvalues of A^{-1} , assuming A is invertible?
- (a) They are the reciprocals of the eigenvalues of A
(b) They are the negatives of the eigenvalues of A
(c) They are equal to the eigenvalues of A
(d) They are zero.
13. A and B are any matrices with the same number of rows. What can you say about the comparison of $\text{Rank}(A)$ and $\text{Rank}([A B])$ without any additional conditions, where $[A B]$ is the block matrix formed by concatenating A and B column-wise.
- (a) $\text{Rank}(A) \geq \text{Rank}([A B])$
(b) $\text{Rank}(A) \leq \text{Rank}([A B])$
(c) $\text{Rank}(A) > \text{Rank}([A B])$
(d) $\text{Rank}(A) = \text{Rank}([A B])$
(e) $\text{Rank}(A) < \text{Rank}([A B])$

考 試 科 目	計算機數學	系 所 別	資訊安全碩士學位學程 一般生	考 試 時 間	2 月 12 日 (三) 第 2 節
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14. Which one is false for the following statements:

- (a) If A is diagonalizable, then there is a basis for eigenvectors of A
- (b) Not all 2×2 matrices are diagonalizable
- (c) If A does not have n distinct eigenvalues, then A is not diagonalizable

(d) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \\ 4 & 2 & 3 & 1 \\ 3 & 1 & 4 & 2 \end{bmatrix}$, then $[1, 1, 1, 1]^T$ is an eigenvector of A .

(e) Eigenvectors corresponding to different eigenvalues are linearly independent.

第二部分、計算題 (第 15~19 題)

15. Suppose A is 3 by 4, and $Ax = 0$ has exactly 2 special solutions:

$$x_1 = \begin{bmatrix} 3 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

- (a) (6%) Find A 's row reduced echelon form R .
- (b) (8%) Find the dimensions of all four fundamental subspaces $C(A)$, $N(A)$, $C(A^T)$, $N(A^T)$.
- (c) (6%) Following (b), find bases for two of these subspaces (choose two subspaces for which sufficient information is available).

16. (11%) Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$, perform a singular value decomposition on A , as $A = U\Sigma V^T$.

17. (7%) In the multinomial expansion of $(2x - y + 3z)^9$, what is the coefficient of xy^6z^2 ?

18. (10%) Use the inclusion-exclusion principle to count the number of integers between 1 and 5,000 that are not divisible by 10, 12, or 21.

19. (10%) Fibonacci numbers are defined by $f_n = f_{n-2} + f_{n-1}$ for $n \geq 2$, with the initial values $f_0 = 0$ and $f_1 = 1$, prove $f_0f_1 + f_1f_2 + f_{2n-1}f_{2n} = f_{2n}^2$ by induction.

備

註

- 一、作答於試題上者，不予計分。
- 二、試題請隨卷繳交。