

考 試 科 目	計算機數學	系 所 別	資訊安全碩士學位學程 一般生	考 試 時 間	2 月 6 日(二) 第二節
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本次考試共 25 題單選題，每題 4 分。

選擇題請在答案卡上作答，否則不予計分。

1.  $\begin{bmatrix} 162 & 5045 \\ 7 & 218 \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} 1 & c \\ 0 & 1 \end{bmatrix}$  and  $a, b, c \in \mathbb{R}$ , then  $a + b + c = ?$

- (A)58 (B)59 (C)60 (D)61

2. How many of the following statements are true?

- If the matrix  $A$  is a  $7 \times 4$  matrix and  $\text{rank}(A) = 3$ , then the dimension of the row space of  $A$  is 4.
- Let  $A$  be an  $m \times n$  matrix. Then  $\text{nullity}(A) \geq n - m$ .
- For two squares  $n \times n$  matrices  $A$  and  $B$ ,  $\text{rank}(AB) = \text{rank}(A)$  if and only if  $B$  is nonsingular.
- If the columns of a matrix are dependent, so are the rows of the matrix.

- (A)0 (B)1 (C)2 (D)3 (E)4

3. Which matrix is non-singular?

(A)  $\begin{bmatrix} 2 & 4 & 6 & 9 \\ 1 & 3 & 5 & 10 \\ 1 & 3 & 5 & 7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

(B)  $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 3 & 7 \\ 3 & 6 & 18 \end{bmatrix}$

(C)  $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 5 & 7 & 9 & 1 \\ 3 & 3 & 9 & 12 \\ 4 & 4 & 8 & 12 \end{bmatrix}$

(D)  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 4 & 8 & 8 & 12 \end{bmatrix}$

4. How many of the following statements are true?

- Every  $n \times n$  matrix has  $n$  eigenvalues and  $n$  eigenvector.
- If  $A$  is an  $n \times n$  matrix, then the rank of  $A$  is equal to its nonzero eigenvalues.
- Two eigenvectors corresponding to the same eigenvalue are always linearly dependent.
- If a triangular matrix is similar to a diagonal matrix, it is already diagonal.

- (A)0 (B)1 (C)2 (D)3 (E)4

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5. Let  $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ . Find the eigenvalues of  $A^3 + A^2 + A + I$ .

- (A)  $5 + 2\sqrt{5}, 5 - 2\sqrt{5}$
- (B)  $0, 1$
- (C)  $3 + \sqrt{5}, 3 - \sqrt{5}$
- (D)  $\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

6. How many of the following statements are true?

- $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$  is similar to  $\begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$ .
  - If  $A$  is idempotent, then  $I - A$  is idempotent.
  - If  $A$  is idempotent, then  $(I - 2A)^{-1} = (I - 2A)$ .
  - A square matrix with linearly independent column vectors is diagonalizable.
- (A)0 (B)1 (C)2 (D)3 (E)4

7. How many of the following statements are true?

- If  $u$  is orthogonal to  $v$  and  $w$ , then  $u$  is orthogonal to  $v + w$ .
  - If  $u$  is orthogonal to  $v + w$ , then  $u$  is orthogonal to  $v$  and  $w$ .
  - If  $\{u, v\}$  is an orthonormal set in  $V$ , then  $\|u - v\| = \sqrt{2}$ .
  - Every orthonormal set of  $V$  must be linear independent.
- (A)0 (B)1 (C)2 (D)3 (E)4

8. How many of the following statements are true?

- If  $A^T$  is orthogonal,  $A$  is also orthogonal.
  - If  $A$  is an orthogonal matrix, then  $A$  is symmetric.
  - If  $A$  has orthonormal columns, then  $AA^T = I$ .
  - Let  $A$  be a positive-definite matrix in  $R^{n \times n}$ . Then  $A + I_n$  is a positive-definite matrix.
- (A)0 (B)1 (C)2 (D)3 (E)4

9. Let bases  $E = \{v_1, v_2\} = \left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 13 \end{bmatrix} \right\}$  and  $F = \{u_1, u_2\} = \left\{ \begin{bmatrix} 6 \\ 1 \end{bmatrix}, \begin{bmatrix} 17 \\ 3 \end{bmatrix} \right\}$ . If  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is the transition matrix from  $E$  to  $F$  and  $a, b, c, d \in \mathbb{R}$ . Please determine the value of  $a + b + c + d$ .

- (A)-182 (B)-184 (C)-190 (D)-194

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10. Let  $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ . What is the sum of the eigenvalues of  $A$ ?

- (A)0 (B)2 (C)4 (D)6

11. Let set  $A = \{5,6,7,8\}$ . Define a relation  $R$  on  $A$  where  $R = \{(5,7), (5,8), (6,7), (6,8), (7,5), (7,8)\}$ , which of the following properties does this relation have?

- (1) Symmetric  
 (2) Asymmetric  
 (3) Antisymmetric  
 (4) Reflexive  
 (5) Irreflexive  
 (6) Transitive

- (A)(1), (4), (6) (B)(2), (5), (6) (C)(3), (5) (D)(5)

12.  $29^{322} \equiv a \pmod{17}$ , find  $a$ .

- (A)8 (B)9 (C)11 (D)12

**Questions 13-16 below constitute a set.** Please consider them as one large question and answer accordingly.

Solve the linear recurrence relation  $a_n - 3a_{n-1} + 2a_{n-2} = -5n + 3$  with  $a_0 = 2$  and  $a_1 = 8$ , and let  $a_n = b * 2^n + cn^2 + dn + e$ .

13.  $b = ?$  (A)-3 (B)-4 (C)-5 (D)-6

14.  $c = ?$  (A) $\frac{3}{2}$  (B) $\frac{5}{2}$  (C) $\frac{7}{2}$  (D) $\frac{9}{2}$

15.  $d = ?$  (A) $\frac{15}{2}$  (B) $\frac{17}{2}$  (C) $\frac{19}{2}$  (D) $\frac{21}{2}$

16.  $e = ?$  (A)8 (B)9 (C)10 (D)11

17. Find the number of ways to make change for \$100 using \$10, \$20 and \$50 bills.

- (A)8 (B)9 (C)10 (D)11

**Questions 18-20 below constitute a set.** Please consider them as one large question and answer accordingly.

Let  $X = \{2,3,4,5,7\}$ ,  $Y = \{4,5,6\}$ . For a function  $f$  and a set  $S$ , define  $f(S) = \{f(i) \mid i \in S\}$ . Please find the answers for the following questions.

18. How many functions  $f: X \rightarrow Y$  are one-to-one?

- (A)0 (B)4 (C)12 (D)24

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19. How many functions  $f: X \rightarrow Y$  are there such that  $|f(X)| = 3$ ?

- (A)76 (B)95 (C)150 (D)211

20. How many functions  $f: X \rightarrow X$  are there such that  $f(\{2,4\}) = \{3,4\}$  and for all  $x \in X$ ,  $f(x) \neq 5$ ?

- (A)64 (B)128 (C)160 (D)240

Questions 21-22 below constitute a set. Please consider them as one large question and answer accordingly.

Suppose the  $A$  and  $B$  are events in a sample, such that  $p(A) = \frac{1}{3}$ ,  $p(B) = \frac{1}{5}$ ,  $p(A|B) = \frac{2}{7}$ , and

$$p(B|A) = \frac{a}{b}.$$

21.  $a = ?$  (A)3 (B)4 (C)5 (D)6

22.  $b = ?$  (A)35 (B)36 (C)37 (D)38

23. Let  $A, B$  and  $C$  be sets. Which of the following statements are true?

- (A) If  $A \in B$  and  $B \in C$  then  $A \in C$ .  
 (B) If  $A \in B$  and  $B \in C$  then  $A \subseteq C$ .  
 (C) If  $A \in B$  and  $B \subseteq C$  then  $A \in C$ .  
 (D) If  $A \in B$  and  $B \subseteq C$  then  $A \subseteq C$ .

24. How many of the following statements are true? Note that by  $(a,b)=(c,d)$  we mean  $a=c$  and  $b=d$ .

- $f: N \rightarrow N$ ,  $f(x) = 3^x$  is one-to-one
- $f: N \rightarrow N$ ,  $f(x) = 3^x$  is onto
- $f: N \rightarrow N$ ,  $f(x) = 3^x$  is bijective
- $f: R \rightarrow R^2$ ,  $f(x) = (x^2, -x^2)$  is one-to-one
- $f: N \rightarrow N \times N$ ,  $f(x) = (x, x+1)$  is injective

- (A)0 (B)1 (C)2 (D)3 (E)4.

25. Which of the following function  $\phi$  from  $(Z, +)$  to  $(Z, +)$  is a homomorphism?

- (A)  $\phi(n) = -n$   
 (B)  $\phi(n) = n^2$   
 (C)  $\phi(n) = 2n + 1$   
 (D)  $\phi(n) = 2^n$

備

註

- 一、作答於試題上者，不予計分。  
 二、試題請隨卷繳交。