

考試科目	經濟學	系所別	商學院共同科	考試時間	2 月 12 日(三) 第二節
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### I. Multiple Choice (1 points each)

Identify the letter of the choice that best completes the statement or answers the question.

- Holding all other forces constant, when the price of gasoline rises, the number of gallons of gasoline demanded would fall substantially over a ten-year period because
  - buyers tend to be much less sensitive to a change in price when given more time to react.
  - buyers tend to be much more sensitive to a change in price when given more time to react.
  - buyers will have substantially more real income over a ten-year period.
  - the quantity supplied of gasoline increases very little in response to an increase in the price of gasoline.
- Suppose that Mike is filling out a survey. The survey asks him what he would do if the price of his favorite shaving gel increased. Mike reports that he would switch to a different brand. The survey then asks what he would do if the price of all shaving gels increased. Mike reports that he must use shaving gel, so he would have to adjust his spending elsewhere. These examples illustrate the importance of
  - changes in total revenue in determining the price elasticity of demand.
  - the definition of a market in determining the price elasticity of demand.
  - a necessity versus a luxury in determining the price elasticity of demand.
  - the time horizon in determining the price elasticity of demand.
- Suppose that Elmer's hourly wage increases, and he decides to work fewer hours. For Elmer, the substitution effect of the wage change is
  - greater than the income effect.
  - equal to the income effect.
  - smaller than the income effect.
  - We do not have enough information with which to answer the question.
- In the work-leisure trade-off analysis, if tax reductions induce a consumer to work more, an explanation for this occurrence is that
  - leisure is an inferior good.
  - leisure is a Giffen good.
  - the substitution effect dominates the income effect.
  - the income effect dominates the substitution effect.

備

註

- 作答於試題上者，不予計分。
- 試題請隨卷繳交。

考試科目	經濟學	系所別	商學院共同科	考試時間	2月12日(三)第二節
------	-----	-----	--------	------	-------------

5. Which of the following production functions exhibits constant returns to scale?

- A.  $Q = K \cdot L$ .
- B.  $Q = K^{0.4} \cdot L^{0.4}$ .
- C.  $Q = K + L - 2$ .
- D.  $Q = 2K + 3L$ .

6. Consider a market demand function  $Q = e^{3600 + \ln \frac{1}{(p+8)^3}}$ . There is a monopolist in the market with no fixed cost, and its marginal cost is \$8 at all levels of output. What price will it charge to maximize its profits?

- A. 20
- B. 12
- C. 10
- D. 16

7. Josh sells mojos as a monopolist in a market with two consumers, Lyle and Yu-Min. Lyle's demand function for mojos is  $Q_L = 20 - P_L$ . Yu-Min's demand function is  $Q_Y = 10 - P_Y$ . Josh only knows there are two consumers but does not know who has a higher demand. Therefore, he decides to offer two options  $O_1 = (q_1, m_1)$  and  $O_2 = (q_2, m_2)$ . If a consumer chooses  $O_i$ , he pays  $m_i$  dollars to buy  $q_i$  mojos. Consumers can only choose one offer, and resale is not allowed. If Josh wants to practice 2nd-degree price discrimination, which of the following offers should he use?

- A.  $O_1 = (20, 200)$  and  $O_2 = (10, 50)$
- B.  $O_1 = (20, 125)$  and  $O_2 = (8, 48)$
- C.  $O_1 = (20, 140)$  and  $O_2 = (6, 42)$
- D.  $O_1 = (20, 165)$  and  $O_2 = (4, 32)$

8. In the game matrix below, the first payoff in each pair goes to player A who chooses the row, and the second payoff goes to player B, who chooses the column. Let  $a, b, c$  and  $d$  be positive constants.

Player B

		Left	Right
Player A	Top	$a, 1$	$b, 0.5$
	Bottom	$1, c$	$2, d$

If the game only has a mixed-strategy Nash equilibrium, then we can conclude that

- A.  $b > a$  and  $d > c$ .
- B.  $a > 1$  and  $d < 0.5$ .
- C.  $a < 1$  and  $b > 2$ .
- D. None of the above is correct.

備

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考 試 科 目	經 濟 學	系 所 別	商 學 院 共 同 科	考 試 時 間	2 月 12 日(三) 第 二 節
---------	-------	-------	-------------	---------	-------------------

9. Concerts in Taipei Dome cause noise and earthquake-like vibrations for surrounding residents. According to an estimation, there are 1,000 residents affected, and the average mental damage per concert for each resident is  $D_R = 5 + Q$ , where  $Q$  stands for the number of concerts per month. On the other hand, the monthly concert revenue of Taipei Dome is  $R_T = 10,000Q - 1,000Q^2$ . The marginal cost of holding each concert is 1,000. What is the socially optimal number of concerts?

- A. 1
- B. 3
- C. 5
- D. 7

10. Which of the following is not a characteristic of a public good?

- A. It is not excludable.
- B. It is not diminished or depreciated as additional people consume the good.
- C. Its benefits cannot be withheld from anyone.
- D. Because it is a free good, there is no opportunity cost.

11. Which of the following events would decrease the unemployment rate?

- A. A decrease in the labor force combined with an increase in the total population.
- B. A decrease in the labor force, with no change in the number of employed workers.
- C. A decrease in the number of employed workers, with no change in the labor force.
- D. An increase in the labor force, with no change in the number of employed workers.

12. Which of the following policies is considered quantitative easing?

- A. The central bank prints money and directly transfers it to financial institutions.
- B. The central bank purchases short-term government bonds.
- C. The central bank sells short-term government bonds.
- D. The central bank purchases long-term government bonds.

13. In the long run, what is the primary cause of inflation?

- A. Increase in the minimum wage.
- B. Government's currency manipulation.
- C. Growth in the money supply.
- D. Protective trade policies and high tariff rates.

14. All else being equal, which of the following outcomes would we expect from a decrease in Taiwan's interest rates?

- A. Citizens decide to hold more foreign bonds.

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考試科目	經濟學	系所別	商學院共同科	考試時間	2月12日(三)第二節
------	-----	-----	--------	------	-------------

- B. Citizens choose to hold more currency.  
 C. Domestic business owners decide to invest in new equipment for their factory.  
 D. All of the above are correct.

15. If inflation expectations rise, the short-run Phillips curve shifts:

- A. to the right, so that at any inflation rate, unemployment is higher in the short run than before.  
 B. to the left, so that at any inflation rate, unemployment is higher in the short run than before.  
 C. to the right, so that at any inflation rate, unemployment is lower in the short run than before.  
 D. to the left, so that at any inflation rate, unemployment is lower in the short run than before.

16. In a world described by the classical dichotomy, assuming GDP and the price level are given, if President Trump raises U.S. import tariffs while all other factors remain unchanged, how will this policy affect Taiwan's loanable funds market?

- A. The real interest rate will increase.  
 B. The real interest rate will decrease.  
 C. The real interest rate will stay the same.  
 D. It will shift both the demand and supply of loanable funds simultaneously.

17. Consider a partial equilibrium model for good *A* in Taiwan. If President Trump raises U.S. import tariffs on that good, thereby reducing Taiwan's export prices, and assuming Taiwan is a small country and the U.S. is a large country in the market for good *A*, how will this affect Taiwan's consumer surplus?

- A. It will increase.  
 B. It will decrease.  
 C. It will stay the same.  
 D. The change will be ambiguous.

18. If President Trump raises U.S. import tariffs, assuming all other factors remain unchanged, how will this policy affect Taiwan's economy according to the classical Keynesian model?

- A. The output will decrease but the price level will increase.  
 B. Both the output and price level will decrease.  
 C. The output will decrease but the price level will stay the same.  
 D. The output will increase but the price level will stay the same.

19. If President Trump raises U.S. import tariffs and leads to workers in the associated industries in Taiwan losing their jobs and other things being equal, how will this affect the natural rate of unemployment in Taiwan?

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考試科目	經濟學	系所別	商學院共同科	考試時間	2 月 12 日(三) 第二節
------	-----	-----	--------	------	-----------------

- A. It will increase.
- B. It will decrease.
- C. It will stay the same.
- D. It depends on how the cyclical unemployment rate changes.

20. In the long run, assuming GDP and price level are given, if investors in Taiwan fear that President Trump will raise the U.S. import tariff in the near future and other things equal, how will this affect Taiwan's economics?

- A. An increase in net export of Taiwan and a rise in the real interest rate
- B. A drop in net export of Taiwan and a rise in the real interest rate
- C. An increase in net export of Taiwan and a drop in the real interest rate
- D. A drop in net export of Taiwan and a drop in the real interest rate

## II. Problems and Short-essay Questions

1. A competitive market currently has 120 firms, all of which has a total cost function of  $TC(q) = 9 + 11q + q^2$ , where  $q$  denotes the quantity produced by each firm.

a. (10 points) Currently, the market price  $p = 19$ . What is the total quantity supplied in the market? How much profit does each firm make in the short-run equilibrium?

b. (5 points) What is the long-run equilibrium price? Suppose the market demand is given by  $p = 35 - \frac{1}{30}Q$ , where  $Q$  denotes the market quantity demanded. How many firms are there in the long-run equilibrium?

c. (5 points) Suppose now the government levies a \$2 specific tax on each unit of the output produced. How many firms are there in the long-run equilibrium?

2. An industry has two firms,  $A$  and  $B$ , producing output at constant unit costs of 1 and  $C > 1$  per unit, respectively. The demand function for the industry is

$$Q = \frac{V}{P}$$

with  $V > C$ .

a. (10 points) Suppose two firms are Cournot competitors. Find the equilibrium price, quantities produced, and profits for two firms.

b. (10 points) Suppose two firms are Bertrand competitors. Moreover, when both firms set identical prices, all consumers purchase from the more cost-efficient firm. Find the equilibrium price, quantities produced, and profits for two firms.

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考 試 科 目	經 濟 學	系 所 別	商 學 院 共 同 科	考 試 時 間	2 月 12 日 (三) 第 二 節
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3. Consider consumers in Country *A* and Country *B*, who each consumes 2 units of bananas and 5 units of pizza every year. The currency unit of Country *A* is AD, and the currency unit of Country *B* is BD. Please answer the following questions in detail:

a. (10 points) Assume that in year 2000, the prices of bananas and pizza in Country *A* were 3 AD and 5 AD, respectively, and the prices of these items in Country *B* were 2 BD and 2 BD. Suppose the purchasing power parity holds between the two countries. What was the nominal exchange rate of AD in terms of BD in year 2000?

b. (5 points) Does the law of one price hold according to the information in (a)? Why or why not?

c. (5 points) In 2001, the price of bananas and pizza in Country *A* became 2 AD and 6 AD, respectively. Please calculate the CPI index of Country *A* in 2001, using 2000 as the base year.

4. In this question, we will examine the monetary and fiscal policies of Country *Z*. The economy of Country *Z* is described by the following equations:

$$Y = C + I + G$$

$$C = 100 + 0.5(Y - T)$$

$$I = 300 - 50r$$

where *Y* is GDP, *C* is consumption, *I* is investment, *G* is government spending, *T* is taxes, and *r* is the real interest rate (for example, if the real interest rate is *q*%, then  $r = q$ ).

Country *Z* has recently experienced a significant negative economic shock. The president plans to implement policies aimed at stimulating the economy. At first, the president considers an expansionary fiscal policy and the corresponding multiplier effect.

a. (2 points) What is the marginal propensity to consume (MPC) in this economy?

b. (3 points) Calculate the fiscal multiplier.

c. (5 points) Suppose Country *Z* receives foreign aid, which enables an increase in government spending without raising taxes. Given a change in government spending ( $\Delta G = 50$ ) and no change in taxes ( $\Delta T = 0$ ), what is the resulting increase in aggregate demand?

In Country *Z*, the nominal interest rate is fixed at 8%. The current growth rate of the money supply is 7%, and the current inflation rate is 5%. To stimulate investment, the central bank increases the growth rate of the money supply to 9%.

d. (2 points) What is the inflation rate after the central bank's monetary policy?

e. (3 points) Referring to the Fisher effect (Fisher equation), calculate the new real interest rate after the monetary policy.

f. (5 points) Given the change in the real interest rate, calculate the change in investment ( $\Delta I$ ). What is the resulting increase in aggregate demand?

備

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考試科目	保險法	系所別	風險管理與保險學系 法律組	考試時間	2月12日(週三)第2節
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一、試依據我國保險法、學理與相關立法例，分析並評論：1.我國保險契約之當事人架構與改進方向；2.受益人之條件或資格；3.財產保險有無受益人規定之適用？（25%）

二、A 以自己為要保人與被保險人，向 B 保險公司投保醫療險。A 雖曾因呼吸道問題住院，但對書面詢問事項：「近一年是否有任何住院？」仍回答「否」。近年 A 腰部漸感不適，導致行動緩慢。A 經 C 醫師診治後，建議住院兩天並施打高濃度自體血小板血漿（platelet-rich plasma, PRP）。A 以相關費用向 B 請求理賠，B 則認為 A 違反告知義務、破壞最大善意原則；且徵詢其理賠與醫療顧問後，認為此處置不符合醫療常規、欠缺住院必要性而拒賠。試依保險法與學理、我國實務見解論述之。（25%）

三、A 以自己為要保人，其妻 B 為被保險人，向 C 保險公司投保人壽保險附加醫療險。保險契約並載明若欠繳保費時可以保單價值準備金墊繳。後 AB 因個性不合而分居，但未離婚。A 之 2023 年應繳保費於該年 11 月 1 日到期，但 A 並未繳納。C 乃對 A 之居所發出催告，並以保單價值準備金墊繳保費。由於該墊繳之保單價值準備金將於 2024 年 6 月 1 日耗盡，C 乃於該年 5 月 24 日再次向 B 之居所發出催告，敘明如再不繳納保費則契約將於送達後 30 日失效，該催告於 5 月 27 日抵達。8 月 1 日 B 因肺疾住院欲向 C 請求理賠，C 則以保險契約已經失效而拒賠。請問是否有理由？（25%）

四、A 與 B 為夫妻，由 A 出資購置房屋一幢，向 C 銀行辦理房貸；並由 A 為要保人與被保險人向 D 保險公司投保人壽保險，並指定 C 為受益人。之後 A 經商失敗，積欠廠商 E 貨款 300 萬元，且與 B 生活陷入困境。E 乃訴請法院代為解除該人壽保險契約，以解約金清償貨款。試依據我國法規、實務見解與相關理論，分析本題之保險法律關係。（25%）

備

註

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考試科目	公司法	系所別	風險管理與保險學系法律組	考試時間	2月12日(三) 第四節
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## 第一題

A 股份有限公司為依中華民國法律設立登記之非公開發行公司，章程授權資本額新台幣(下同)5 億元，迄至 2024 年 12 月 31 日止，已完成發行 1000 萬股，全數均為普通股，面額每股 10 元，第一次發行 500 萬股，發行價格每股 20 元，第二次發行 500 萬股，每股價格 35 元。甲為 A 公司之自然人股東，持有股份總數為 80 萬股。A 公司董事會 2025 年 1 月 3 日決議辦理現增 5000 萬股，並決定本次現增普通股之每股面額設為 20 元，以 50 元為每股發行價格，其中 10%保留給員工認購，其他額度由原股東認購。

問(答案均應附理由):

- 1、在 2024 年 12 月 31 日止，假設 A 公司無其他項目列計於資本公積，也未曾動用帳上資本公積，問 A 公司有多少資本公積?(10 分)
- 2、上開董事會現增決議在不變更發行價格之前提下，應如何修正決議內容始適法?若有不同修正方式可以詳細說明之(20 分)
- 3、若甲在 A 公司本次可以發行數額的最高限度內，扣除員工 10%認股之後，甲依其自有持股最多可以認購多少股?應繳納多少股款?(20 分)

## 第二題

A 公司為依我國公司法設立之上市公司，設立於 2000 年，章程授權資本額新台幣(下同)60 億元，迄至 2024 年 12 月 31 日止已完成發行：(1)、普通股 3 億股，為 5 次發行完成後之普通股總股數；(2)私募甲種可轉換特別股 1 億股，2020 年 7 月 1 日發行，發行價格每股 20 元，甲特章程規定發行滿五年後之次日起一個月期間內，甲特股東可以行使轉換權，以一股甲特轉換為一股普通股，超過一個月期限後，甲特之可累積可再參加分配盈餘權利即行停止，自期限屆滿後與普通股有相同的盈餘分配權利；(3)、2021 年 7 月 1 日發行無擔保公司債 10 億元，每單位公司債票面金額 100 萬元，依照票面金額十足發行，共計發行 1000 單位，發行期間 4 年，每年於發行滿一年之次日起一個月內應依照票面金額 6%年利率給付現金利息，發行期間四年屆滿後次日起算之一個月內，公司債持有人可以行使認股權，每一單位公司債可以每股 15 元認購 10 萬股普通股；(4)私募乙種不可轉換、附認股權永續特別股 1000 萬股，2022 年 7 月 1 日完成發行，發行價格每股 30 元，乙特發行期滿三年後次日起算之一個月內，乙特股東可以行使一股乙特對一股普通股之認股權，認股價格每股 20 元，未於期限內認購者，其認股權消除。上述股權衍生性有價證券之額度並已於章程為相應合法之額度保留。問(答案均應附理由):

- 1、A 公司董事會於 2025 年 2 月 1 日決議第一項記載:於 2025 年 6 月 1 前，辦理現增 2 億股，每股面額定為 20 元，但發行價格定為每股 40 元，董事會決議日之普通股收盤價為每股 45 元，請說明董事會決議之適法性與爭點，本題若涉有證券交易法爭點，請一併說明之?(20 分)
- 2、上開董事會決議第二項記載: 甲特依其持股數、乙特及無擔保公司債依其可認購普通股數量，均有權

考 試 科 目	公司法	系 所 別	風險管理與保險學系法律組	考 試 時 間	2 月 12 日(三) 第四 節
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依比率與普通股共同參與本次現增。請說明本項決議之適法性與爭點。(20 分)

- 3、上開董事會決議第三項記載：擬修正乙種特別股章程，新增贖回權之規定，規定乙特發行滿十年後次日起算一個月內，股東有權要求公司以每股發行價格 3 倍之價格贖回之，本項並提報股東會決議。請說明不論股東會是否通過本項修正，其董事會決議修正章程議案之適法性及爭點。(10 分)



備

註

- 一、作答於試題上者，不予計分。
- 二、試題請隨卷繳交。

考試科目	保險學	系所別	風險管理與保險學系法律組	考試時間	2 月 12 日(三)第 4 節
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1. 試說明政府高度監理保險業的理由。(25%)
2. 共保 (Coinsurance) 的目的為何? 保單內有共保條款時, 實務上可能會發生什麼問題?(25%)
3. 金管會推動外匯價格變動準備金新制的目的為何? 此制度的內容為何? 試說明你對此制度的瞭解。(25%)
4. 何謂不可抗辯條款 (Incontestable clause)? 不可抗辯條款的目的為何?(25%)



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考試科目	統計學	系所別	風險管理與保險學系	考試時間	一月 12 日(三) 第 4 節
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選擇題請在答案卡上作答，否則不予計分。(每題 4 分)

- 樣本平均數的標準誤 (Standard Error of the Mean, SEM) 是用來衡量：
    - 樣本平均數的變異程度
    - 母體平均數的變異程度
    - 樣本與母體平均數之間的差異
    - 樣本標準差
  - 在假設檢定中，顯著水準 ( $\alpha$ ) 代表：
    - 接受虛無假設的機率
    - 拒絕虛無假設時犯第一類錯誤的機率
    - 樣本平均數的標準差
    - 假設檢定的統計功效
  - 中央極限定理指出：
    - 母體必須服從常態分配
    - 樣本越大，樣本平均數的分佈越接近常態分配
    - 樣本標準差等於母體標準差
    - 樣本的範圍會趨於穩定
  - 在迴歸分析中，決定係數 ( $R^2$ ) 的含義是：
    - 迴歸模型的顯著性
    - 自變數解釋因變數變異的比例
    - 迴歸係數的統計顯著性
    - 自變數之間的相關性
  - 若 p-value 小於顯著水準  $\alpha$ ，則應：
    - 接受虛無假設
    - 拒絕虛無假設
    - 減少樣本數
    - 增加顯著水準
  - 當使用多元線性回歸模型時，若發現自變數之間存在高度共線性 (Multicollinearity)，則最可能的結果是：
    - 模型的 ( $R^2$ ) 值下降
    - 迴歸係數的標準誤增加
    - 模型解釋力增強
    - 顯著性水平提高
- 

考試科目	統計學	系所別	風險管理與保險學系	考試時間	2 月 12 日(三) 第 4 節
7. 假設有兩個獨立樣本，樣本大小分別為 ( $n_1 = 50$ ) 和 ( $n_2 = 60$ )，變異數未知但被認為相等。要比較兩個平均數，應使用：					
A) 單樣本 t 檢定					
B) 配對樣本 t 檢定					
C) 雙樣本獨立 t 檢定 (假設變異數相等)					
D) 卡方檢定					
8. 在時間序列分析中，ARIMA 模型中的參數 ( $p, d, q$ ) 分別表示：					
A) 自相關項數量、趨勢差分次數、移動平均項數量					
B) 樣本數、平均數、標準差					
C) 變異數、期望值、中位數					
D) 統計功效、顯著性水平、樣本標準差					
9. 若相關係數 ( $r = 0.85$ )，其決定係數 ( $R^2$ ) 的值為：					
A) 0.7225 B) 0.85 C) 0.9225 D) 0.95					
10. 在迴歸分析中，若殘差圖呈現漏斗形狀，可能表示：					
A) 殘差具有常態性					
B) 存在異方差性 (Heteroscedasticity)					
C) 存在多重共線性					
D) 自變數未顯著					
11. 當多元迴歸模型存在多重共線性 (Multicollinearity) 時，可能的結果是：					
A) 迴歸係數標準誤減小					
B) 變異數分析結果更穩定					
C) 迴歸係數的估計不穩定					
D) 決定係數 ( $R^2$ ) 降低					
12. 下列哪種統計檢定可用來判斷整體回歸模型是否顯著？					
A) t 檢定					
B) F 檢定					
C) 卡方檢定					
D) Z 檢定					

考試科目	統計學	系所別	風險管理與保險學系	考試時間	2 月 12 日(三) 第 4 節
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13. 在時間序列分析中，白噪聲 (White Noise) 過程的特徵是：

- A) 資料有明顯趨勢
- B) 自相關係數為 0
- C) 資料存在季節性變動
- D) 資料呈現長期記憶效應

14. 在時間序列分析中，ADF (Augmented Dickey-Fuller) 檢定的主要用途是：

- A) 檢測自相關性
- B) 檢測異方差性
- C) 檢測時間序列是否為平穩序列
- D) 檢測多重共線性

15. 在 GARCH 模型中，主要用於建模哪種特性？

- A) 趨勢性
- B) 季節性
- C) 異方差性
- D) 自相關性

16. 若隨機變數  $X$  服從均勻分布  $U(0, 1)$ ，則隨機變數  $Y = -\ln(X)$  的分布為：

- A) 均勻分布
- B) 指數分布
- C) 正態分布
- D) 卡方分布

17. 在主成分分析 (PCA) 中，特徵值 (Eigenvalues) 的含義是什麼？

- A) 表示資料集的維度
- B) 表示每個主成分的變異量
- C) 表示資料點之間的距離
- D) 表示特徵向量的方向

18. 在 PCA 中，如何選擇主要的主成分？

- A) 按特徵向量的長度排序
- B) 按特徵值從小到大排序
- C) 按特徵值從大到小排序
- D) 隨機選擇特徵向量

考試科目	統計學	系所別	風險管理與保險學系	考試時間	2 月 12 日(三) 第 4 節
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19. 如果在 PCA 中，某個特徵值接近於零，這表示什麼？

- A) 該主成了解釋了大部分變異
- B) 該主成分可以被忽略
- C) 資料可能存在共線性
- D) 資料未經標準化

20. 若隨機變數  $X$  和  $Y$  獨立，則條件期望 ( $E(X|Y)$ ) 等於：

- A)  $E(X)$  B)  $E(Y)$  C)  $E(XY)$  D)  $E(Y|X)$

21. 若  $E(X|Y) = 2Y + 1$ ，且  $E(Y) = 3$ ，則  $E(X)$  等於？

- A) 7 B) 6 C) 5 D) 4

22. 隨機變數  $X$  和  $Y$  的聯合分布已知，條件期望  $E(X|Y)$  的性質是？

- A) 是一個常數
- B) 是隨機變數  $Y$  的函數
- C) 是隨機變數  $X$  的函數
- D) 與  $Y$  無關

23. 在統計學習中，模型過度擬合 (Overfitting) 的特徵是什麼？

- A) 模型在訓練集和測試集上表現一致
- B) 模型在訓練集上表現良好，但在測試集上表現差
- C) 模型無法學習訓練數據的模式
- D) 模型完全忽略數據的變異性

24. 在統計學習中，Bias-Variance 權衡 (Bias-Variance Tradeoff) 的核心概念是什麼？

- A) 降低偏差 (Bias) 可以同時降低方差 (Variance)
- B) 提高模型複雜度總是更好
- C) 偏差與方差之間存在相互權衡
- D) 減少方差不會影響偏差

25. 在統計學習中，交叉驗證 (Cross-Validation) 的主要目的為何？

- A) 減少訓練時間
- B) 增加訓練數據的數量
- C) 評估模型的泛化能力
- D) 減少模型的偏差

備

註

- 一、作答於試題上者，不予計分。
- 二、試題請隨卷繳交。

考試科目	微積分	系所別	風險管理與 保險學系	考試時間	2 月 12 日(三) 第 2 節
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Problem 1 (20 points) (10% each)

Evaluate the integrals.

(a)

$$\int_0^3 \frac{x^2}{\sqrt{x+1}} dx.$$

(b)

$$\int \frac{3x^3 - 2x - 2}{x^2(x^2 + 1)} dx.$$

Problem 2 (20 points) (10% each)

Find the following limits if they exist.

(a)

$$\lim_{x \rightarrow 0^+} (1 - \cos x)^{\frac{1}{\ln x}}.$$

(b)

$$\lim_{x \rightarrow 1} \left( \frac{1}{\ln x} - \frac{1}{x-1} \right).$$

Problem 3 (15 points)(15%)

Let

$$F(x) = \int_0^{x^2} \cos(t^2 + t) dt.$$

(a) (6%) Find  $F'(x)$ .

(b) (9%) Find

$$\lim_{x \rightarrow 0} \frac{F(x)}{x^2}.$$

備

註

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考試科目	微積分	系所別	風險管理與 保險學系	考試時間	2 月 12 日(三) 第 2 節
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Problem 4 (15 points)(5% each)

(a) Derive the MacLaurin series of

$$\tan^{-1}x.$$

(b) Find the value of  $a \in \mathbb{R}$  such that the limit

$$\lim_{x \rightarrow 0} \frac{\sin(ax) - \sin x - \tan^{-1}x}{x^3}$$

is finite.

(c) Evaluate the above limit.

Problem 5 (15 points)(15%)

Find the maximum and the minimum of the function

$$f(x, y) = 3x^2 - 2y^2$$

on the curve

$$2x^2 - 2xy + y^2 = 1.$$

Problem 6 (15 points) (5% each)

Let  $f(x) = xe^x$ .

(a) Find the Taylor series for  $f(x)$  at  $x = 0$ .

(b) Calculate

$$\int_0^x te^t dt$$

and find its Taylor series at  $x = 0$ .

(c) Find the sum

$$\sum_{n=0}^{\infty} \frac{1}{n!(n+2)}.$$

備

註

- 一、作答於試題上者，不予計分。
- 二、試題請隨卷繳交。

考試科目	統計學	系所別	風險管理與保險學系 精算科學組	考試時間	2 月 12 日 (三) 第四節
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1. (30 pts) Please explain the following items.

- (a) (10 pts) Likelihood Ratio Tests
- (b) (10 pts) Exponential memoryless
- (c) (10 pts) Convergence in distribution

2. (10 pts) An insurance company models the number of claims it receives in a year using the following distribution. Let  $X$  represent the number of claims filed for natural disasters (e.g., earthquakes, hurricanes) in a given year, where:

$$f_X(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \dots,$$

with  $\lambda = 2$ . For each claim received, there is a 50% chance that the claim is for damages caused by flooding. Let  $Y$  represent the number of flood-related claims in a year. What is the distribution of  $Y$ ?

- (a)  $f_Y(y) = \frac{e^{-1.2y}}{y!}$
- (b)  $f_Y(y) = \frac{e^{-2}}{y!}$
- (c)  $f_Y(y) = \frac{e^{-0.5} 0.5^y}{y!}$
- (d)  $f_Y(y) = \frac{e^{-1} 0.5^y}{2y!}$
- (e) None of the above.

3. (10pts) Which of the following statements about the Central Limit Theorem (CLT) is incorrect?

- (a) The CLT describes the sampling distribution of population mean.
- (b) When using the CLT, the population from which the data is drawn does not need to follow a normal distribution.
- (c) When using the CLT, the population from which the data is drawn does not need to consist of continuous data.
- (d) When using the CLT, the sample size must be sufficiently large.
- (e) None of the above.

考 試 科 目	統 計 學	系 所 別	風險管理與保險學系 精算科學組	考 試 時 間	2 月 12 日 (三) 第 四 節
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4. (10pts) Let  $X$  have a Poisson distribution with mean  $\lambda$ . Find a transformation  $u(X)$  so that  $\text{Var}[u(X)]$  is free of  $\lambda$ , for large values of  $\lambda$ .

**Hint:** Use Tylor Expansion  $u(X) \approx u(\lambda) + [u'(\lambda)](X - \lambda) + \frac{u''(\lambda)}{2}(X - \lambda)^2$ , and higher terms can be neglected when  $\lambda$  is large.

5. (20pts) Let  $X_1, X_2, \dots, X_n$  be i.i.d. random variables with cumulative distribution function  $F(\cdot)$ , and consider the statistic

$$F_n(x) = \frac{1}{n} \sum_{i=1}^n I(X_i \leq x),$$

where  $I(\cdot)$  is the indicator function.

- (a) (5 points) Prove that  $\mathbb{E}[F_n(x)] = F(x)$ .
- (b) (10 points) Derive the variance of  $F_n(x)$ .
- (b) (5 points) Determine whether  $F_n(x)$  is the UMVUE (Uniformly Minimum Variance Unbiased Estimator) of  $F(x)$ . Justify your answer.
6. (20 pts) If  $X_1, \dots, X_n$  is a random sample from a geometric distribution

$$P(X = x) = \theta(1 - \theta)^{x-1} I_{\{1,2,\dots\}}(x),$$

and assume  $\theta$  follows a uniform prior on  $(0, 1)$ .

- (a) (10 pts) Find the posterior distribution of  $\theta$ .
- (b) (10 pts) Determine the Bayes estimator of  $\theta$  under squared error loss, where the loss function is defined as

$$L(\theta, \delta(X_1, \dots, X_n)) = (\delta(X_1, \dots, X_n) - \theta)^2.$$

備

註

- 一、作答於試題上者，不予計分。  
二、試題請隨卷繳交。