國立政治大學 115 學年度 碩士班暨碩士在職專班 招生考試試題

考試科目	個體經濟學	系所別	經濟學系	考試時間	2月5日(星期四)第二節

1. （25 分）$U(X_1,X_2,X_3,X_4) = [10 + \min(X_1^{-\alpha},X_2^{-\alpha})(1+X_3)^{1-\alpha}]^{-\beta}$為某一消費者的效用函數，其中 $1 > \alpha > \beta > 0$。
   (A) （10 分）此效用函數所代表的偏好是否滿足完整性、遞移性、越多越好以及連續性。
   (B) （15 分）請推導此消費者的需求函數。

2. （25 分）$Q = f(x) = x^2$，為某一廠商的生產函數，其中 Q 為產出，x 為要素投入，$f(.)$為生產函數。同時，要素的市場價格為 W，產出的市場價格則為 P。
   (A) （5 分）何謂生產函數？請寫下生產函數的定義。
   (B) （5 分）請推導此廠商的邊際成本函數。
   (C) （15 分）在完全競爭市場之下，請推導此廠商利潤極大化的最適產量。

3. （20 分，每小題 10 分）True/False/Uncertain
   (A) The slope of a labor demand curve is always negative.
   (B) $\frac{\partial PS}{\partial P}$ is always equal to $\frac{\partial \pi}{\partial P}$ for an individual firm in a perfect competitive market. But it is not true for monopoly. PS is producer surplus, π is profit and P is output price.

4. （10 分）What are homogeneous and homothetic functions? What properties do they have in common? How do they differ? When would an economist prefer one over the other when building an economic model?

5. （10 分）A consumer has initial wealth \$90,000 and faces the risk to lose \$30,000 with probability 0.1. An insurance company offers the following insurance contract: The consumer gets a payment P > 0, if she suffers a loss, and zero otherwise. The premium for this contract is 0.1P. Assume that the consumer has an expected utility function and is strictly risk averse. Will the consumer buy the insurance? How much premium will the consumer pay in order to maximize his/her expected utility?

6. （10 分，每小題 5 分）解釋下列名詞，並說明其在經濟學上的重要性。
   (A) strictly dominated strategy
   (B) conditional factor demand function

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	總體經濟學	系所別	經濟學系	考試時間	2月5日(四) 第三節

1. (25%)近期英國<<經濟學人>>的封面故事，指出台灣為維持出口競爭力，長期壓低新台幣兌美元的匯率，使得新台幣長期被低估，對消費者形成隱形的稅賦，民眾因此無法感受到經濟成長所帶來生活水準的提升。

   (a)(10%) <<經濟學人>>文中採用大麥克指數暗示新台幣被嚴重低估。請問，甚麼是大麥克指數？大麥克指數如何能夠判別新台幣和美元之間，新台幣被低估，背後的理論基礎為何?，試以名目匯率與實質匯率的關係說明之。

   (b)(10%) 台灣央行反擊<<經濟學人>>的論點，認為雖然使用大麥克指數計算，新台幣是被低估，但若是使用“iphone 指數”計算，新台幣反而是被高估。你覺得“大麥克指數”和“iphone 指數”何者比較能夠判別兩國之間的實質購買力？理由為何? 兩種指數各有甚麼優缺點?

   (c) (5%) 貨幣貶值為何能夠提高一國的出口競爭力? 對進口有何影響？試說明之。

2. (25%)近來，人工智慧的發展正在改變勞動投入與資本投入的替代關係。有些人認為人工智慧會取代勞力，造成勞工失業，也有些人認為人工智慧會提升勞工的生產效率，提高勞動的邊際收益。假設，勞動(L)與資本(K)的替代彈性(σ)可定義為

$$\sigma = \frac{\partial \ln(K/L)}{\partial \ln(MPL/MPK)}$$

   MPL為勞動邊際產出，MPK為資本邊際產出

   (a)(5%)假設人工智慧使得資本可逐漸取代勞動力，σ會如何隨時間變化？請說明之。

   (b)(10%)假設生產函數為$Y = AK^{\alpha}L^{1-\alpha}$。請問$MPL=$? $MPK=$?勞動與資本的替代彈性σ為多少?

   (c)(10%)假設生產函數為常替代生產函數(CES)：$Y = A(\alpha K^{\rho} + (1-\alpha)L^{\rho})^{1/\rho}$。請問$MPL=$? $MPK=$?勞動與資本的替代彈性σ為多少?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考 試 科 目	總體經濟學	系 所 別	經濟學系	考 試 時 間	2月5日(四) 第三節

3. (15%)考慮一個封閉經濟，人口總量不隨時間成長: $N_t = N$。假設社會總投資為 $I_t = sY_t$，其中 $s \in [0,1]$ 為邊際儲蓄傾向，Y_t 為社會總產出。若資本 K_t 的折舊率為 δ，且社會總資本累積滿足以下的資本累積動態方程: $K_{t+1} = (1-\delta)K_t + I_t$。請問:

(a)(5%)假設人均產出為人均資本的函數: $\frac{Y_t}{N} = f(\frac{K_t}{N})$，其中函數 f 滿足 $f'' < 0 < f'$ 的條件，請以函數 f 表示此經濟體到達穩定態(steady state)的條件

(b)(10%)承上題，請以人均資本 $\frac{K_t}{N}$ 為橫軸，人均產出 $\frac{Y_t}{N}$ 為縱軸，作圖標示出穩定態的位置，並以此圖說明，邊際儲蓄傾向 s 增加時，會如何影響穩定態下的人均資本量 $\frac{K^*}{N}$?

4. (35%)考慮一個兩期的封閉經濟，其中代表性家計單位的終身效用函數為

$$U(C_1, C_2, L_1, L_2) = \ln(C_1) - \frac{(L_1)^\gamma}{\gamma} + \beta[C_2^\alpha(1-L_2)^{1-\alpha}]$$

代表性家計單位第一期的薪資為 w_1，第二期的薪資為 w_2。因此，第一期的薪資所得為 w_1L_1，第二期的薪資所得為 w_2L_2。家計單位的所得僅用於消費和儲蓄 S，儲蓄的跨期利率為 $(1+r)$。請問：

(a)(10%)根據上述的資訊，請寫下代表性家計單位的跨期最適化問題，並求一階條件(first-order conditions) 包含消費、勞動和儲蓄的最適選擇。

(b)(10%)假設 $\alpha = 0.5,\ \beta = 0.8, \gamma = 2,\ w_1 = 1.01,\ w_2 = 0.25$，$(1+r) = 1.25$，請問代表性個人的最適儲蓄 $S =$? (當 $S < 0$ 則為借貸)兩期的最適消費 $(C_1, C_2) =$? 兩期的最適勞動 $(L_1, L_2) =$?

(c)(10%)同(b)小題的參數，假設政府對個人的兩期薪資所得皆課徵線性的薪資所得稅，稅率為 $\tau_L = 0.1$，請問此時，代表性個人的最適儲蓄 $S =$? (當 $S < 0$ 則為借貸)，兩期的最適消費 $(C_1, C_2) =$? 兩期的最適勞動 $(L_1, L_2) =$?

(d)(5%)請比較(b)、(c)的結果，解釋為何政府課稅後，個人的最適選擇如消費、儲蓄和勞動發生你所計算出的變化？請以所得效果和替代效果的角度逐一說明背後的經濟直覺。

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

國立政治大學 115 學年度 碩士班暨碩士在職專班 招生考試試題

考 試 科 目	統計學	系所別	經濟學系	考試時間	2 月 5 日（四）第四節

Instructions:

- Please label the question numbers and answer them in numerical order.
- A table of the cumulative standard normal distrbution is provided on the last page.
- You must show your work and provide complete answers for full credit. Answers with no explanations or derivations, or incorrect ones, will receive a score of zero.

1. (30%) Suppose X and Y have the following joint probability density function (p.d.f.):

$$f_{XY}(x,y;\mu) = \begin{cases} \frac{\mu^2}{2}\exp\left(-\mu x - \frac{\mu y}{2}\right) & \text{if } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

for $\mu > 0$, where $\exp(\cdot)$ denotes the exponential function.

(a) (5%) What is the coverage probability of the interval $\left[X, X + \frac{Y}{2}\right]$ for μ?

(b) (5%) Suppose $\{(X_i, Y_i)\}$, $i = 1, \ldots, n$, is a random sample from the joint p.d.f. $f_{XY}(x, y; \mu)$. Find the maximum likelihood estimator $\hat{\mu}_{ML}$ for μ.

(c) (5%) Find the Cramér-Rao Lower Bound for unbiased estimators of μ.

(d) (5%) Is $\hat{\mu}_{ML}$ the minimum variance unbiased estimator? Why or why not?

(e) (10%) Find the limiting distribution of $\sqrt{n}(\hat{\mu}_{ML} - \mu)$ as $n \to \infty$.

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	統計學	系所別	經濟學系	考試時間	2月5日（四）第四節

2. (45%) For positive random variables X and Y, suppose

$$Y = \beta X + \sigma(X)e$$

with an unknown parameter β, $\sigma(X) = X$, $\mathbb{E}[X] = 1$, $\text{Var}(X) = 2$, $\mathbb{E}[e \mid X] = 0$, and $\mathbb{E}[e^2 \mid X] = \sigma^2$. To estimate β, you are given a random sample $\{(X_i, Y_i)\}, i = 1, \ldots, n$. Let $\overline{X}$ and $\overline{Y}$ denote the respective sample means. Consider the three estimators:

$$W_1 = \frac{1}{n}\sum_{i=1}^{n}\frac{Y_i}{X_i}, \quad W_2 = \frac{\sum_{i=1}^{n} X_i Y_i}{\sum_{i=1}^{n} X_i^2}, \quad W_3 = \frac{\sum_{i=1}^{n} Y_i}{\sum_{i=1}^{n} X_i}$$

(a) (5%) Find $\mathbb{E}\left[\frac{Y}{X}\right]$.

(b) (5%) Find $\mathbb{E}\left[\overline{Y} \mid X_1, \ldots, X_n\right]$.

(c) (5%) Find $\text{Var}\left(\overline{Y}\right)$.

(d) (6%) Are W_1, W_2, and W_3 unbiased estimators for β?

(e) (6%) Are W_1, W_2, and W_3 consistent estimators for β?

(f) (6%) Are W_1, W_2, and W_3 efficient estimators for β?

(g) (5%) Let $X = \log(Z)$, where log denotes the natural logarithm. Interpret β in terms of the association between Y and Z.

(h) (7%) Following (g), suppose the parameter of interest, γ, is the *average* marginal effect of a one-unit change in Z on the expected value of Y. Given a random sample $\{(Y_i, Z_i)\}, i = 1, \ldots, n$, consider the estimator:

$$W_4 = \frac{W_2}{\overline{Z}}$$

where $\overline{Z}$ denotes the sample mean of Z_i. Will W_4 tend to overestimate, underestimate, or consistently estimate γ? (Suppose $\beta > 0$.)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考 試 科 目	統計學	系所別	經濟學系	考試時間	2 月 5 日（四）第四節

3. (25%) The table below reports OLS and Probit regression results of children's economic outcomes on child and parental characteristics, where standard errors are in parentheses and the variables are described below:
   - Income: Child's income in thousand dollars.
   - College Grad: a dummy variable equal to one if the adult child graduated from a four-year college and zero otherwise.
   - Male: a dummy variable equal to one if the child is male and zero if female.
   - Mother Drinks: a dummy variable equal to one if mother drinks alcohol and zero otherwise.
   - Mother's Education and Father's Education: years of education of parents.
   - Log Parents' Income: the natural logarithm of parents' income in dollars.

Dependent Variable	(1) OLS Income	(2) OLS College Grad	(3) Probit College Grad
Mother's Education		0.021 (0.008)	0.060 (0.019)
Father's Education		−0.004 (0.007)	−0.010 (0.017)
Log Parents' Income		0.011 (0.027)	0.008 (0.064)
Male	−5.200 (0.590)	−0.159 (0.041)	−0.398 (0.090)
Mother Drinks	−7.800 (8.500)		
Male × Mother Drinks	−2.800 (5.100)		
Constant	75.800 (10.200)	0.766 (0.264)	0.142 (0.566)
Observations	874	897	1088

(a) (5%) Interpret all of the coefficient estimates in Column (1).

(b) (5%) Define $\theta = \mathbb{E}[\text{Income} \mid \text{Male} = 1, \text{Mother Drinks} = 1] - \mathbb{E}[\text{Income} \mid \text{Male} = 1, \text{Mother Drinks} = 0]$. Use the coefficient estimates in Column (1) to provide a point estimate of θ and describe how to obtain its standard error.

(c) (5%) Consider a female child whose parents each have 16 years of education and whose parents' log income is 12 (in dollars). Using Column (3), compute the probability that the child graduates from college.

(d) (5%) Using Column (3), what is the difference in predicted probabilities of graduating from college for the female child in (c) compared with a male child with all other characteristics identical?

(e) (5%) Now use Column (2) to estimate the change in predicted probabilities for the comparison in (d).

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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Table 2 CUMULATIVE NORMAL DISTRIBUTION

$$\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}\,dt$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

x	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.891	4.417
$\Phi(x)$	.90	.95	.975	.99	.995	.999	.9995	.99995	.999995
$2[1-\Phi(x)]$	.20	.10	.05	.02	.01	.002	.001	.0001	.00001

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。