國立政治大學 114 學年度 碩士班暨碩士在職專班 招生考試試題

考試科目	個體經濟學	系所別	經濟學系	考試時間	2月11日(星期二) 第二節

1. （20分）在完全競爭市場中，某一廠商的生產函數為 $Q = L^{\alpha}K^{\beta}$，其中 Q 為產量，L 和 K 為要素投入，$\alpha > 0$，$\beta > 0$。市場的產品價格為 P，要素 L 的價格為 W，要素 K 的價格為 R。
   a. （15分）假設 $\alpha + \beta < 1$。請推導此廠商的要素需求函數 $L(P,W,R)$ 和 $K(P,W,R)$ 以及供給函數 $Q(P,W,R)$。
   b. （5分）假設 $\alpha + \beta > 1$。請推導此廠商的要素需求函數 $L(P,W,R)$ 和 $K(P,W,R)$ 以及供給函數 $Q(P,W,R)$。

2. （20分）$U(X_1, X_2, X_3, X_4) = \min\{X_1^{\alpha}X_2^{1-\alpha}, \beta X_3 + \gamma X_4\}$ 為某一消費者的效用函數，其中 $1 > \alpha > 0$，$\beta > 0$，$\gamma > 0$。
   a. （5分）此效用函數所代表的偏好是否滿足完整性、遞移性、越多越好、嚴格凸性以及連續性。
   b. （15分）請推導此消費者的需求函數。

3. （10分）$U(X_1, X_2, X_3, X_4) = \alpha X_1 + F(X_2, X_3, X_4)$ 為某一消費者的效用函數，其中 $\alpha > 0$，F 函數的一階偏微分皆大於 0。請用數學詳細推導計算這四個財貨的所得需求彈性。

4. （20分）在一個一般均衡模型中，有兩個消費者（A 和 B）和兩個消費財貨（X 和 Y）。
   a. （10分）請寫下競爭均衡（competitive equilibrium）的定義。
   b. （10分）在哪種情形之下，此一般均衡模型有可能不存在競爭均衡。請使用 Edgeworth box 舉例說明。

5. （30分）解釋下列名詞，並說明其在經濟學上的重要性。
   a. （5分）The second welfare theorem
   b. （5分）Elasticity of substitution
   c. （5分）Market mechanism
   d. （5分）Vickrey auction
   e. （5分）Nash equilibrium
   f. （5分）Certainty equivalent

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

國立政治大學 114 學年度 碩士暨碩士在職專班 招生考試試題

考試科目	總體經濟學	系所別	經濟學系	考試時間	2 月 11 日(二) 第 三 節

1. The following system of equations describes a Keynesian macroeconomic framework: $C^d = 300 + 0.75(Y - T) - 250r$ is desired consumption function, where Y is real output and r is real interest rate. $T = 300$ is lump sum tax. $I^d = 500 - 350r$ is desired investment function. $G = 300$ is government purchase. $L = 0.75Y - 500i$ is real money demand function, where i is nominal interest rate. $M = 2370$ is nominal money supply. $\bar{Y} = 3260$ is full-employment output. $\pi^e = 0.05$ is expected inflation rate. Answer the following questions:

(1). What are the values of the real interest rate, the price level, consumption, and investment for the economy in general equilibrium. (8%)

(2). Now suppose the autonomous investment decreases to 450 due to a negative shock to investment confidence, with no change in the expected inflation. What will be the real interest rate, output, consumption, and investment in the short run (in which the price level is fixed)? (4%)

(3). Continued from (2), assume that the Okun's coefficient is 1.334 and the natural rate of unemployment is 0.05. What is the actual rate of unemployment in the short run? (5%)

(4) Continued from (2), what happens in the long run to the real interest rate, the price level, consumption, and investment, in which the expected inflation changes to 0.043? (8%)

2. Assume that firms with perfect foresight are certain about future profitability, interest rate, tax policies, etc. Let K_{t-1} denote stocks of physical capital at the end of period $t-1$. Moreover, a representative firm owns the stock of physical capital K_{t-1}, and pays a wage w_t to the L_t workers hired at the begin of period t. The single goods Y_t of the firm is produced by using K_{t-1} and L_t, according the following Cobb-Douglas production technology: $Y_t = K_{t-1}^{\alpha} L_t^{1-\alpha}$, $0 < \alpha < 1$, at time t. Assume that the goods market is perfectly competitive. Both labor and capital markets are also perfectly competitive. For simplicity, the price of the single goods is assumed to be 1. Moreover, the purchase price of capital goods is also assumed to be 1. The firm expends I_t to invest equipment and machinery at time t. An equation that shows capital accumulation over time is required.

$$K_t = I_t + (1 - \delta)K_{t-1},$$

where $\delta \in (0,1)$ is the rate of capital depreciation. The firm chooses the amount of investment and physical capital and the number of workers to use every period in production in order to maximize the sum of future discounted cash flows (in which the firm has taken wage w_t and real interest rate r_t as given):

$$\max_{\{I_t, K_t, L_t\}} \quad \sum_{t=1}^{\infty} \frac{CF_t}{(1+r_1)(1+r_2)\dots(1+r_{t-1})}$$

where CF_t denotes firms' cash flows and $r_t > 0$ is real interest rate. The cash flow is defined as:

$$CF_t = Y_t - w_t L_t - I_t - \frac{\psi}{2}\frac{I_t^2}{K_t},$$

where $\frac{\psi}{2}\frac{I_t^2}{K_t}$ represents the adjustment costs and $\psi > 0$ is constant. Answer the following questions:

(1) Derive the optimal intertemporal decisions of the firm's I_t, L_t, and K_t for maximizing the sum of future discounted cash flows and explain the economic intuition for each decision. (12%)

(2) Continued from (1), derive the Tobin's q. (5%)

(3) Draw diagrams to analyze the effects of a permanent decrease in the real interest rate on physical capital,

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investment, and the Tobin's q. (8%)

3. Answer the following questions:
(1) Describe three alternative responses available to policymakers when the economy is in recession. (5%)
(2) Continued from (1), draw diagrams to analyze the effects of three alternative responses on employment, the price level, and the composition of output. (12%)
(3) What are some of the practical difficulties in using macroeconomic stabilization policies to fight recessions? (8%)

4. Suppose the central bank dislikes inflation variability around a target level π^*. It also prefers to keep unemployment stable around an unemployment target u^*. These objectives can be represented in terms of minimizing

$$V = \lambda(u - u^*)^2 + \frac{1}{2}(\pi - \pi^*)^2,$$

where π is the inflation rate and u is the unemployment rate. The economy is described by

$$u = u_n - a(\pi - \pi^e) + v, a > 0$$

where u_n is the natural rate of unemployment and π^e is expected inflation. Expectations are formed by the public before observing the disturbance v. The central bank can set inflation after observing v. Assume $u^* < u_n$. Answer the following questions:
(1) What is the equilibrium rate of inflation under discretion? What is the equilibrium unemployment rate? (10%)
(2) What is the equilibrium rate of inflation under commitment? What is the equilibrium unemployment rate under commitment? How are they affected by u^*? Explain. (15%)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

國立政治大學 114 學年度 碩士班暨碩士在職專班 招生考試試題

考試科目	統計學	系所別	經濟學系	考試時間	2 月 11 日（二）第四節

Instructions:

- *Please label the question numbers and answer them in numerical order.*
- *You must show your work and write complete answers. Answers with no explanations or derivations, or incorrect ones, will receive a score of zero.*

1. (25%) Let $X_1, \ldots, X_n$ be a random sample from the $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $0 < \sigma < \infty$. Consider the estimation of σ^2 using the squared error loss. Let S^2 be the sample variance of $X_1, \ldots, X_n$.

   (a) (4%) Find $E[S^2]$ and $\text{Var}[S^2]$.

   (b) (5%) Find $E[S^2]$ if the population distribution is unknown with mean μ and variance σ^2.

   (c) (8%) Define a class of estimators of the form cS^2 with a nonrandom c. Compute the risk function in this class and find the best estimator that minimizes the risk.

   (d) (8%) Suppose that $\sqrt{n}(S^2 - \sigma^2)$ converges to $N(0, V)$ in distribution as $n \to \infty$. Using the Delta method find the limiting distribution of $\sqrt{n}(S - \sigma)$, where S is the sample standard deviation.

2. (25%) Suppose that $X_1, \ldots, X_n$ is a random sample from $U[0, \theta]$, where U denotes a uniform distribution.

   (a) (5%) Find the maximum likelihood estimator of θ, denoted as $\hat{\theta}$.

   (b) (5%) Find the sampling distribution of $\hat{\theta}$. (Provide the probability density function.)

   (c) (10%) Find $E[\hat{\theta}]$ and $\text{Var}[\hat{\theta}]$.

   (d) (5%) Is $\hat{\theta}$ a consistent estimator for θ? (Use the definition of convergence in probability to demonstrate (in)consistency.)

3. (16%) Suppose that the true process is $Y = X + X^2$ with $X \sim U[0, 1]$. Consider the model $Y = \beta_0 + \beta_1 X + u$ with the error u satisfying $E[u] = E[Xu] = 0$.

   (a) (8%) Use the model restrictions directly to calculate β_0 and β_1.

   (b) (2%) Find $E[u|X]$.

   (c) (3%) Denote $\hat{\beta}_1$ the OLS estimator of β_1. Would you agree that $\hat{\beta}_1$ consistently estimates the average marginal effect of X on Y? Why or why not?

   (d) (3%) Assume the data are divided into two groups, each following $Y = X + X^2$ but with a distinct marginal distribution of X, resulting in their respective (β_0, β_1) calculated as in (a). Would you agree that the effect of X on Y differs across groups? Why or why not?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	統計學	系所別	經濟學系	考試時間	2 月 11 日（二）第四節

4. (34%) Using a sample of 532 workers, a researcher is interested in whether there is a tradeoff between the time spent sleeping per week and the time spent in paid work. The regression results are presented below, with standard errors in parentheses and SER denoting the standard error of the regression.

   The variable *sleep* is total minutes per week spent sleeping at night; *work* is total weekly minutes spent working; *educ* is years of schooling; $male = 1$ for men and $male = 0$ for women; $D_{age\geq 40} = 1$ for a worker aged 40 or older and $D_{age\geq 40} = 0$ otherwise; and *hrwage* is hourly wages in dollars.

	Dependent Variable: *sleep*			
	(1)	(2)	(3)	(4)
constant	3301.8	3578.73	3719.58	3725.53
	(36.5)	(44.76)	(95.51)	(97.48)
work		−0.16	−0.16	−0.16
		(0.02)	(0.02)	(0.02)
educ			−11.74	−11.30
			(6.86)	(6.86)
male	−115.8	45.90	45.35	50.15
	(48.3)	(38.86)	(42.42)	(42.64)
$D_{age\geq 40}$	−2.9			
	(56.2)			
$male \times D_{age\geq 40}$	110.2			
	(76.4)			
ln(*hrwage*)			8.63	
			(3.30)	
hrwage				−1.12
				(1.21)
$hrwage^2$				0.07
				(0.05)
Observations	532	532	532	532
SER	429	408	400	400
R^2	0.015	0.107	0.122	0.122

(a) (4%) The sample can be divided into four groups based on *male* and $D_{agd\geq 40}$. Calculate the sample mean of sleep time per week for each group based on coefficient estimates in Column (1).

(b) (4%) If *sleep* is measured in hours in Column (1), what are the coefficient estimate and standard error for *male*, as well as the SER and R^2?

(c) (3%) A student is unhappy with model (2) as "a female dummy is omitted from the model." Comment on this criticism.

(d) (3%) Given Column (2) and the estimated regression $\widehat{male} = 0.086 + 0.0002 \times work$, determine the slope coefficient estimate in the simple regression of *sleep* on *work*.

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4. (continued)

(e) (3%) Interpret the coefficient estimate of 8.63 for ln(*hrwage*) in Column (3).

(f) (3%) Following (e), does the partial effect of hourly wages on mean sleep time decrease, remain constant, or increase with each one-dollar rise? Explain your reasoning.

(g) (3%) Based on Column (3), all other factors being equal, is there a statistically significant tradeoff between working and sleeping? How strong is the evidence? (The 0.95, 0.975, 0.99, 0.995, and 0.9995 quantiles of $N(0,1)$ are 1.64, 1.96, 2.33, 2.58, and 3.29.)

(h) (3%) Based on Column (4), determine the hourly wage level at which weekly sleep time starts increasing as hourly wages rise.

(i) (4%) Adjust model specification (2) to test whether the tradeoff between sleep and work time varies by gender. State the relevant null and alternative hypotheses. Provide an appropriate test statistic (with its explicit form) and its asymptotic null distribution.

(j) (4%) Based on Column (4), a researcher wants to test the hypothesis that, holding other factors fixed, hourly wages have no effect on sleeping. State the relevant null and alternative hypotheses. Provide an appropriate test statistic (with its explicit form) and its asymptotic null distribution.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。