考試科目	資料結構及演算法	所別	8141 資訊科學系	考試時間	3 月 6 日(六) 第 一 節

說明：1. 請書寫必要之解題過程。過程正確但答案錯誤，可能有部分分數。如題目之解答非顯而易見者，僅書寫答案而缺乏必要之過程，亦無法獲得該題之滿分。

2. 可使用中文或英文作答。

1. (16%, 2% for each problem) 是非題(True of False): (本大題僅回答 T 或 F 即可，不需理由)

子題號	題目內容
(1)	Hashing is a technique to achieve an O(1) expected search time. However, its worst-case search time is O(log n).
(2)	Adjacency matrix is good for representing a sparse graph.
(3)	$2^{2n} = O(2^n)$, where O is the notation for asymptotic upper bound.
(4)	Membership test in a linked list requires O(log n) time, in the worst case, for input size n.
(5)	We need two pointers to implement either queue or stack using linked lists.
(6)	The best case time complexity of a selection sort algorithm is the same as that of a insertion sort algorithm.
(7)	The worst case of insertion operation on a binary search tree takes O(log n).
(8)	A binary tree can not be used to represent a parent with three children.

2. (24%, 3% for each problem) 選擇題(select the correct answer): (本大題僅選擇答案即可，不需理由)

子題號	題目內容
(1)	What is the complexity of the recurrence equation $T(n) = 5\,T(\frac{n}{2}) + \Theta(n^2)$? (A) $\Theta(n^{\log_2 5})$ (B) $\Theta(n^2)$ (C) $\Theta(n^{\log_5 2})$ (D) $\Theta(n^2 \log n)$
(2)	If the complexity of the recurrence equation $T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + f(n)$ has been determined as $\Theta(n^2 \log_2 n)$, what will be the complexity of $T(n) = T(\frac{n}{10}) + T(\frac{9n}{10}) + f(n)$? (A) $\Theta(n^2 \log_2 n)$ (B) $f(n)$ (C) $\Theta(n^2 \log_9 n)$ (D) Can't be determined.

備註	試題隨卷繳交

考試科目	資料結構及演算法	所別	資訊科學系	考試時間	3 月 6 日(六) 第 一 節

2. (接續前頁題 2)

子題號	題目內容
(3)	How many edges will be a minimum spanning tree of a n-node graph? (A) n (B) n^2 (C) O(log n) (D) n - 1
(4)	The time complexity of the inorder-tree- traversal algorithm is (A) O(n) (B) O(n log n) (C) O(log n) (D) $O(n^2)$
(5)	Which of the following algorithm has the stack property? (We generally call the last in first out property as the stack property.) (A) breadth-first search (B) depth-first search (C) preorder-tree-traversal (D) 以上皆非
(6)	There are two most popular algorithms in finding the minimum spanning trees, the Kruskal's algorithm and the Prim's algorithm. Which of the following statement about these algorithms is correct? (A) They both are greedy algorithms. (B) During the MST finding process, the intermedium solutions of both algorithms are always a tree. (C) The time complexity of both algorithms, in the worst case, is O(E log E), where E is the number of edges in the given graph. (D) The time complexity of both algorithms, in the worst case, is $O(E^2 \log E)$, where E is the number of edges in the given graph.
(7)	In the following list of problems, not all of them have known polynomial time solutions: (1) the shortest path problems, (2) the longest path problems, (3) 2-CNF satisfiability, (4) 3-CNF satisfiability, (5) Euler tour (traverses each edge of a directed graph exactly once), (6) Hamiltonian cycle (traverses each vertex of a directed graph exactly once). Which of the following statement is correct? (A) Problems (1), (3), and (5) are NPC problems. (B) Problems (1), (3), and (5) can be solved in polynomial time. (C) Problems (1), (3), and (6) can be solved in polynomial time. (D) They all are NPC problems.

備註	試題隨卷繳交

考試科目	資料結構及演算法	所別	資訊科學系 8141	考試時間	3 月 6 日(六) 第 一 節

2. (接續前頁題 2)

子題號	題目內容
(8)	Which of the following statements is correct? (A) One of the most important reasons to use hashing technique is to obtain O(log n) search time. (B) One of the most important reasons to use tree data structure is to obtain O(log n) tree manipulation time. (C) Binary search trees are always balanced. (D) Heap can only be implemented by linked lists.

3. (10%) NP-completeness

(a) (4%) Give the definition of the class of the "NP-complete" problems.
(b) (6%) Prove the statement: "If a NP-complete problem has a polynomial time solution, then all the NP-complete problems can be solved in polynomial time."

4. (15%) Quicksort and recurrence

For the following problems, assume the input size is n.
(a) (3%) Formulate the recurrence equation for Quicksort. (Explain every parameter you used.)
(b) (6%) Using the recurrence equation, find the best case time complexity of Quicksort.
(c) (6%) Using the recurrence equation, find the worst case time complexity of Quicksort.

5. (15%) Sorting lower bound

(a) (8%) Prove the lower bound of the comparison based sorting algorithms.
(b) (2%) The radix sort algorithm can be summarized as follows:

RADIX_SORT(A, m)
1 **for** $i \leftarrow 1$ **to** m
2 **do** use a stable sort to sort array A on digit i

What is the time complexity of this algorithm? (Assume that the maximal number of digits of all the inputs is m, each digit ranges from 0 to k, and the total number of inputs is n.)
(c) (5%) Is there any conflict on the results in (a) and (b)? Justify your answer.

備註	試題隨卷繳交

考試科目	資料結構及演算法	所別	資訊科學系 8141	考試時間	3 月 6 日(六) 第 一 節

6. (8%) Order of magnitude

One can generally use the big-O, big-Ω, little-o and the little-ω notations for the asymptotic upper bounds or lower bounds. The definitions of these notations are:

$$O(g(n)) = \{f(n) \mid \exists c, n_0 \text{ s.t. } 0 \le f(n) \le cg(n) \ \forall n \ge n_0\}$$
$$\Omega(g(n)) = \{f(n) \mid \exists c, n_0 \text{ s.t. } 0 \le cg(n) \le f(n) \ \forall n \ge n_0\}$$
$$o(g(n)) = \{f(n) \mid \forall c, \exists n_0 \forall n \ge n_0, 0 \le f(n) \le cg(n)\}$$
$$\omega(g(n)) = \{f(n) \mid \forall c, \exists n_0 \forall n \ge n_0, 0 \le cg(n) \le f(n)\}$$

(a) (4%) Give two functions that is in $O(n^2)$ but not in $o(n^2)$.

(b) (4%) Give two functions that is in $\Omega(n^2)$ but not in $\omega(n^2)$.

7. (12%) Graph Algorithms

The Ford-Fulkerson algorithm can be used to find the Maximum Flow (MF) of a given graph.

The simplest version of the Ford-Fulkerson algorithm is:

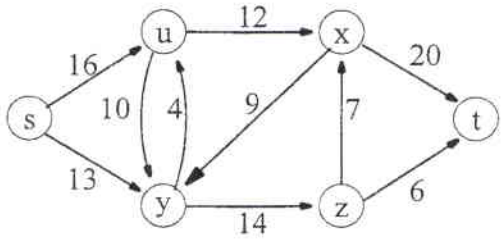
initialize flow f to 0
while there exists an augmenting path p
  do augment flow f along p
return f

and can be refined as:

FORD-FULKERSON(G,s,t)
for each edge $(u,v) \in E[G]$
  do $f[u,v] \leftarrow 0$ and $f[v,u] \leftarrow 0$
while there exists a path p from s to t in the residual network G_f
  do $c_f(p) \leftarrow \min\{c_f(u,v) : (u,v) \text{ is in } p\}$
    for each edge (u,v) is in p
      do $f[u,v] \leftarrow f[u,v] + c_f(p)$

(a) (4%) What is the complexity of this algorithm in terms of the graph size V and E.

(b) (4%) Using this algorithm to find the maximum flow of the following graph.



(c) (4%) Justify your answer in (b)

考試科目	作業系統	所別	8141 資訊科學	考試時間	3月6日(六) 第二節

1. **[Process and Thread Concepts]**

   (a) (4%)Describe the actions taken by a kernel to context-switch between processes.

   (b) (6%)Which of the following components of program state are shared across threads in a multithreaded process?

      i. Register values
      ii. Heap memory
      iii. Global variables
      iv. Stack memory

2. **[Process Synchronization]**

   (a) (6%)A semaphore S is an integer variable that, apart from initialization, is accessed only through two standard atomic operations: `wait()` and `signal()`. Please define these two standard semaphore atomic operations.

   (b) (6%)To overcome the need for busy waiting, we can modify the definition of (a)'s `wait()` and `signal()` standard semaphore atomic operations. To implement semaphores under this definition, we define a semaphore as a "C" struct:

```
typedef struct{
    int  value;
    struct process *list;
}semaphore;
```

   Each semaphore has an integer value and a list of processes `list`. When a process must wait on a semaphore via `wait()` operation, it is added to the list of processes. A `signal()` operation removes from the list of waiting processes and awakens that process. In terms of this idea, please define the `wait()` and `signal()` semaphore operations.

   (c) (6%) A file is to be shared among different processes, each of which has a unique number. The file can be accessed simultaneously by several processes, subject to the following constraint: The sum of all unique numbers associated with all the processes currently accessing the file must be less than n. Write a monitor pseudo code to coordinate access to the file.

備註	試題隨卷繳交

考試科目	作業系統	所別	8141 資訊科學	考試時間	3月6日(六)第二節

3. [Memory-Management Strategies]

   (a) (6%)What is the purpose of paging the page tables?

   (b) (6%)Consider the following segment table:

Segment	Base	Length
0	219	600
1	2300	14
2	90	100
3	1327	580
4	1952	96

   What are the physical addresses for the following logical addresses?

   i. 0, 430

   ii. 1, 10

   iii. 4, 112

   (c) (6%)Explain why sharing a reentrant module is easier when segmentation is used than when pure paging is used.

4. [Virtual-Memory Management]

   (a) (6%)Under what circumstances do page faults occur? Describe the actions taken by the operating system when a page fault occurs.

   (b) (4%)Consider a system that uses pure demand paging.

   i. When a process first starts execution, how would you characterize the page fault rate?

   ii. Once the working set for a process is loaded into memory, how would you characterize the page fault rate?

   (c) (8%)Assume that a program has just referenced an address in virtual memory. Describe a scenario in which each of the following can occur. (If no such scenario can occur, explain why.)

   - TLB miss with no page fault
   - TLB miss and page fault
   - TLB hit and no page fault
   - TLB hit and page fault

考試科目	作業系統	所別	資訊科學(8141)	考試時間	3月6日(六)第二節

5. **[Implementation File Systems]**

   (a) (6%) Please define the following terms: File Control Block (FCB), system-wide open-file table, per-process open-file table.

   (b) (6%)Consider a system that supports the strategies of contiguous, linked, and indexed allocation. What criteria should be used in deciding which strategy is best utilized for a particular file?

   (c) (6%)Consider a file system that uses inodes to represent files. Disk blocks are 16 KB in size, and a pointer to a disk block requires 4 bytes. This file system has 12 direct disk blocks, as well as single, double, and triple indirect disk blocks. What is the maximum size of a file that can be stored in this file system?

6. **[Secondary-Storage Structure]** The term *fast wide SCSI-II* denotes a SCSI bus that operates at a data rate 20 megabytes per second when it moves a packet of bytes between the host and a device. Suppose that a fast wide SCSI-II disk drive spins at 7200 RPM, has a sector size of 1024 bytes, and holds 160 sectors per track.

   (a) (3%)Estimate the sustained transfer rate of this drive in megabytes per second.

   (b) (5%)Suppose that the average seek time for the drive is 10 milliseconds. Estimate the I/O operations per second and the effective transfer rate for a random-access workload and reads individual sectors that are scattered across the disk.

   (c) (5%)Calculate the random-access I/O operations per second and transfer rate for I/O sizes of 8 kilobytes.

   (d) (5%)If multiple requests are in the queue, a scheduling algorithm such as SCAN should be able to reduce the average seek distance. Suppose that a random-access workload is reading 8-kilobyte pages, the average queue length is 10, and the scheduling algorithm reduces the average seek time to 3 milliseconds. Now calculate the I/O operations per second and the effective transfer rate of the drive.

備註	試題隨卷繳交

國立政治大學 99 學年度研究所 博/碩 士班招生考試試題

考試科目	計算機數學	所別	8141 資訊科學系碩士班	考試時間	3月6日(星期六)第三節

I. 離散數學部分 (60 %)

(1). 是非題: (10 %,不倒扣,正確答 O 錯誤答 X)

1.1. If f is a numeric function such that $f(n) = \Theta(n^2)$, then $f(n) = O(n^3)$.

1.2. $(p \vee q) \rightarrow r$ logically implies $(p \rightarrow r) \wedge (q \rightarrow r)$

1.3. Every irreflexive and transitive relation is antisymmetic.

1.4. Every simple graph has a chromatic number ≤ 4.

1.5. Two graphs with different number of edges are not isomorphic.

1.6. There is no simple circuit of non-zero length in a tree.

1.7. $\sim\forall x\, \exists y\, P(x,y)$ logically implies $\forall y\, \exists x \sim P(x,y)$.

1.8. Every function from `int` to `int` can be implemented as a Java method or C function.

1.9. The poset (N, |) of non-negative integers N under the dividability relation | is a lattice but is not bounded since it has no greatest element.

1.10. A binary relation R is transitive if and only if $R^2 \subseteq R$.

(2). 單選題 (9%;每題 3%不倒扣)

2.1. Which of the following propositional formulas is not logically equivalent to the implication $p \rightarrow q$?

(a) $(\sim p) \vee q$ (b) $(\sim q) \rightarrow (\sim p)$ (c) $\sim(p \wedge \sim q)$ (d) $p \leftrightarrow (\sim q)$

2.2. Let G = (N, T, S, P) be a context free grammar, where N= {S,A,B} is the set of nonterminal symbols, T = {0,1} is the set of terminal symbols, S is the start symbol and the set of productions P is given as follows: { S → 0A, S → 1A, A→ 0B, B→1, B→1A }. Then which of the followings is the language generated by G ?.

(a) {01}{0,1}*{01} (b) {0,1}{01}*{01} (c) {0,1}{01}*{0,1} (d) {01}$\{0,1\}^+$

2.3. Let R = {(1,1),(2,1),(3,2),(2,3),(4,3)}. Then which of the following statements is *not* correct?

(a) $(2,2) \in R^2$ (b) $(4,3) \in R^3$ (c) $(2,4) \in R^*$ (d) $(4,1) \in R^+$

備註	試題隨卷繳交

考試科目	計算機數學	所別	8141 資訊科學系碩士班	考試時間	3月6日(星期六)第三節

I. 離散數學部分(續前頁):

(3) 填空題 :(21%; 每格 3%)

3.1. In the expansion of $(w+x+y+z)^{20}$, the coefficient of the term $w^2x^4y^6z^8$ is _____ and there are totally _____ different terms.

3.2. The sum of the degrees of all vertices of a tree with n vertices is _____.

3.3. What is the least positive integer x satisfying the following system of congruence equations: $x \equiv 5 \pmod 7$, $x \equiv 4 \pmod 9$, and $x \equiv 3 \pmod{13}$? _____

3.4. Let K_n denote a complete simple graph with $n > 0$ vertices. Then what is the length of a shortest circuit containing all edges of the graph K_{2n} ? _____

3.5. If f is an increasing function satisfying the recurrence relation: $f(n) = 9\, f(n/3) + 2\, n^2$, then the asymptotic order of $f(n) = \Theta($_____$)$.

3.6. A deterministic finite automata(DFA) M is a tuple (Q,Σ,δ,s,F), where Q is the set of states, Σ is the set of input symbols, $s \in Q$ is the initial state, $F \subseteq Q$ is the set of final states and $\delta: Q \times \Sigma \to Q$ is a state transition function. Suppose now $|Q| = n$ and $|\Sigma| = m$. Then there are _____ different DFAs in which Q is the set of states and Σ the set of input symbols.

計算與證明 (20 %)

(4). [12%] Let $\Sigma = \{0,1\}$ and $A \subseteq \Sigma^*$ be a set of bit strings defined inductively by the following rules:

Basis case: $\varepsilon \in A$, where ε is the empty string.

Inductive case: If $x \in A$ and $y \in A$, then so are 0x1, 1x0 and xy.

Moreover, for each bit string z, let f0(z) and f1(z) denote the number of 0s and 1s occurring in z respectively. So, for example, if z = '1001', then $z \in A$ and f0(z) = f1(z) = 2.

(a) According to the above definition explain briefly why the string '1001' belongs to A. [2%]

(b) Prove by structural induction that for all bit strings z, if $z \in A$ then f0(z) = f1(z). [5%]

(c) On the other hand, show that if z is a bit string such that f0(z) = f1(z), then $z \in A$. [5%]

(5). [8 %] Let G =(V,E) be a directed multigraph such that $E \neq \varnothing$ and for all vertices $x \in V$, in-degree(x) = out-degree(x). Show that there exists a simple circuit of length > 0 in G.

備註	試題隨卷繳交

考試科目	計算機數學	所別	8141 資訊科學系 碩士班	考試時間	3 月 6 日(六) 第 三 節

II. 線性代數部份(40%)

(6). [10%] Determine which of the following matrices are orthogonal matrix and find the inverse, where possible.

(a) $\begin{bmatrix} \frac{4}{5} & 0 & -\frac{3}{5} \\ -\frac{9}{25} & \frac{4}{5} & -\frac{12}{25} \\ \frac{12}{25} & \frac{3}{5} & \frac{16}{25} \end{bmatrix}$

(b) $\begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{bmatrix}$

(c) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(7). [10%] Please find a least square solution of the following inconsistent system Ax = b, and the orthogonal projection of b on the column space of A.

$$A=\begin{bmatrix} 2 & -2 \\ 1 & 1 \\ 3 & 1 \end{bmatrix}, b=\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

(8). [10%] Please use Gaussian elimination to solve the given system of equations.

3x +2y -z= -15
5x+3y+2z= 0
3x+y+3z= 11
-6x-4y+2z= 30

(9). [10%] Please find all eigenvalues and corresponding eigenvectors of Matrix A.

$$A=\begin{bmatrix} -2 & 2 & 3 \\ -2 & 3 & 2 \\ -4 & 2 & 5 \end{bmatrix}$$

備註	試題隨卷繳交