

|      |             |    |     |      |              |     |
|------|-------------|----|-----|------|--------------|-----|
| 考試科目 | 資料結構<br>演算法 | 所別 | 資科系 | 考試時間 | 3月14日<br>星期六 | 第1節 |
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可用中文或英文回答

1. (20%) True or False (Please write T or F as an answer to each of the statement)

- (1) Given two sorted linked list, it is possible to merge them into one sorted linked list in linear time.
- (2) Hashing can be used to implement the key insertion and finding in constant average time.
- (3) Stack is helpful to improve the execution time of Dijkstra's shortest path algorithm.
- (4) Dynamic programming, like the divide and conquer method, solves problems by combining the solutions to subproblems.
- (5) A topological ordering is not possible if the graph is cyclic.
- (6) Greedy method always leads to an optimal solution.
- (7) The Floyd's all pair shortest path algorithm is a divide and conquer algorithm.
- (8) If we have proved that the lower bound of an NP-complete problem is polynomial, then we have proved that  $NP = P$ .
- (9) If an NP-complete problem X is polynomial reducible to a problem Y, then Y is an NP-complete problem.
- (10) Every NP-complete problem must be a NP-hard problem.

2. (20%) Single selection

- (1) A logical expression contains three types of operators, namely, **in order of precedence**,  $\sim$  (not),  $\vee$  (or),  $\wedge$  (and). Which is the postfix expression of the logical expression

$$G \wedge (A \vee B) \vee (C \vee (F \vee \sim D \wedge E))$$

- (a)  $GAB \vee CFD \sim \vee E \wedge \vee \vee \wedge$
  - (b)  $GAB \vee \wedge CFD \sim E \vee \vee \vee \wedge$
  - (c)  $GAB \vee \wedge CFD \sim E \wedge \vee \vee \vee$
  - (d)  $GAB \vee CF \sim D \vee E \wedge \vee \vee \wedge$
- (2) Which of the following data, inserted in the input order, will produce a complete binary search tree ?
  - (a) (Bill, Grace, James, John, Lily, Mary)
  - (b) (John, Mary, Grace, Bill, Lily, James)
  - (c) (Mary, Lily, John, James, Grace, Bill)
  - (d) (James, Grace, John, Bill, Lily, Mary)
  - (e) (Grace, Bill, James, John, Mary, Lily)

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(3) Which of the following is a heap ?

- (a) 

|    |    |    |   |    |    |    |    |   |    |    |    |
|----|----|----|---|----|----|----|----|---|----|----|----|
| 50 | 30 | 46 |   | 18 | 25 | 42 | 60 |   |    |    |    |
| 1  | 2  | 3  | 4 | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 |
- (b) 

|    |    |    |    |    |    |    |   |   |    |    |    |
|----|----|----|----|----|----|----|---|---|----|----|----|
| 18 | 25 | 42 | 40 | 46 | 30 | 50 |   |   |    |    |    |
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 |
- (c) 

|   |   |   |   |   |   |   |   |   |    |    |    |
|---|---|---|---|---|---|---|---|---|----|----|----|
| 1 | 2 | 3 |   | 5 | 6 | 7 |   |   | 10 | 11 | 12 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
- (d) 

|    |    |    |    |    |    |    |    |   |    |    |    |
|----|----|----|----|----|----|----|----|---|----|----|----|
| 18 | 25 | 23 | 40 | 46 | 42 | 53 | 50 |   |    |    |    |
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 |

(4) Which of the following sorting algorithm takes the least number of comparisons for sorting of the following sequence of data (6, 16, 66, 78, 95, 100, 180, 229) ?

- (a) selection sort  
 (b) quick sort  
 (c) insertion sort  
 (d) merge sort  
 (e) heap sort

(5) Which of the following formula is the worst case time complexity in terms of comparison operations for quick sort of  $n$  records ?

- (a)  $T(n)=2T(n/2)+cn$   
 (b)  $T(n)=T(n/2)+cn$   
 (c)  $T(n)=2T(n/2)+n^2$   
 (d)  $T(n)=T(n-1)+cn$

(6) Which of the following formula is the worst case time complexity in terms of comparison operations for merge sort of  $n$  records ?

- (a)  $T(n)=2T(n/2)+cn$   
 (b)  $T(n)=T(n/2)+cn$   
 (c)  $T(n)=2T(n/2)+n^2$   
 (d)  $T(n)=T(n-1)+cn$

(7) Which of the following is true for the minimum spanning tree of a graph with  $n$  vertices?

- (a) The minimum spanning tree is cyclic.  
 (b) Prim's algorithm is a greedy algorithm while Kruskal's algorithm is not.  
 (c) Prim's algorithm starts from any vertices in the graph.  
 (d) The minimum spanning tree of this graph consists of  $n+1$  edges

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- (8) Which of the following is **FALSE** concerning a graph with  $v$  vertices and  $e$  edges?
- (a) There exists  $O(e+v)$  algorithm for single source shortest path of acyclic graph.
  - (b) There exist algorithms in which finding the shortest path from the source to another vertex is any faster (by more than a constant factor) than finding the shortest paths from the source to all the other vertices.
  - (c) In critical path analysis, the longest path is the critical path.
  - (d) It is possible to depth first traversal a graph in linear time.
- (9) The worst case time complexity of finding the second minimum key in an  $n$ -key heap is,
- (a)  $O(1)$  (b)  $O(\log n)$  (c)  $O(n)$  (d)  $O(n \log n)$  (e)  $O(n^2)$  (f)  $O(n^2 \log n)$
- (10) Which of the following is **NOT** an NP-complete problem ?
- (a) Traveling salesman problem.
  - (b) Knapsack problem
  - (c) Closest pair problem
  - (d) Clique problem
  - (e) Satisfiability problem

3. (10%) Please show the result of sorting 56, 6, 15, 100, 51, 38, 82 using radix sort with 7 buckets. The result of each pass must be listed.

4. (10%)

- (1) Figure 1 shows the array implementation for an AVL tree  $T$ . Please produce the result after inserting 75, 99 and 32 successively into the AVL tree  $T$ .
- (2) Please give the time complexity of insertion a key into an  $n$ -node AVL tree
- (3) Please give the time complexity of a single rotation, LL, in an  $n$ -node AVL tree.
- (4) Please give the time complexity of a double rotation, LR, in an  $n$ -node AVL tree.
- (5) Please give the time complexity of finding the maximum in an  $n$ -node AVL tree.

|    |    |    |    |    |    |    |   |   |    |    |    |    |    |    |
|----|----|----|----|----|----|----|---|---|----|----|----|----|----|----|
| 50 | 29 | 76 | 12 | 35 | 63 | 88 |   |   |    |    | 60 | 70 |    | 95 |
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |

Figure 1: An AVL tree  $T$ .

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5. (10 %) Given the following algorithm, please analyze and give
- (1) the best time complexity of the comparison operation.
  - (2) the best time complexity of the exchange operation.
  - (3) the worst time complexity of the comparison operation.
  - (4) the worst time complexity of the exchange operation.

```

void X-Algorithm(int A[], int N)
{
    int j, p, m;
    for (p=0; p < N-2; p++)
    {
        m=p;
        for (j=p+1; j < N-1 ; j++)
            if (A[j] < A[m])
                m=j;
        exchange (A[p], A[m]);
    }
}
    
```

6. (10%) There is a town with  $n$  citizens. It is known that some pairs of people are friends. According to the famous saying that "The friends of my friends are my friends, too" it follows that if A and B are friends and B and C are friends then A and C are friends, too.
- (1) Please design and illustrate an  $O(n^3)$  algorithm, using the example matrix  $X$  shown in Figure 2, to determine whether two citizens are friends for each pair of people. In this matrix  $X$ , there are 7 citizens. If two citizens  $i$  and  $j$  are friends, then the cells  $x_{ij}$  and  $x_{ji}$  are denoted as '1'. Otherwise, the cells are denoted as '0'.
  - (2) Please design and illustrate an  $O(n)$  algorithm, using the example matrix  $X$ , to count how many people there are in the largest group of friends.

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 3 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 5 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 6 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

Figure 2: An example matrix  $X$

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7. (10%) A sequence  $x_1, x_2, \dots, x_n$  is said to be cyclically sorted if the smallest number in the sequence is  $x_i$  for some unknown  $i$ , and the sequence  $x_i, x_{i+1}, \dots, x_n, x_1, \dots, x_{i-1}$  is sorted in increasing order. Please design and illustrate an  $O(\log n)$  algorithm to find the position of the minimal element in a cyclic sorted sequence of  $n$  elements by using the example sequence 45, 52, 66, 72, 3, 29, 38.

8. (10%) Given the following structures and type specification of AVL trees,

- (1) please write the routine in C Language to count the number of nodes. The prototype of the function is defined as `int count(AVLTree root)`.
- (2) please write the routine in C Language to list the element of each node in descending order. The prototype of the function is defined as `void descending(AVLTree root)`.

```

typedef int DATA;
struct node {
    DATA    element;
    struct node *left;
    struct node *right;
    int     height;
};
typedef struct node  AVLNODE;
typedef AVLNODE *AVLTree;
    
```

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可用中文或英文作答

There are 11 problems for this examination and the weights for each (sub)problem is indicated.

1. Please answer the following question:
  - a. (5%) What is *context switch*?
  - b. (5%) What are the necessary conditions for the happening of a *deadlock*?
2. (10%) Please draw the *Diagram of Process State Transition* and write in the diagram the name of a transition. Explain the reasons why a transition occurs.
3. Please answer the following question:
  - a. (5%) What is *thrashing*?
  - b. (5%) How to prevent it?
4. Please answer the following question:
  - a. (4%) What is the purpose of *system call*?
  - b. (6%) What are three general methods used to pass parameters to the operating system in the system calls during different circumstance?
5. Given a reference string: 4, 3, 2, 1, 4, 3, 5, 4, 2, 1.
  - a. (5%) How many page faults will occur using FIFO page-replacement algorithm? For 4 page frame.
  - b. (5%) How many page faults will occur using LRU (Least Recently Used) page-replacement algorithm? For 4 page frame.

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6. Please answer the following question:

- (5%) Why we have to guard against race condition and synchronize the *critical section*?
- (5%) Please justify whether the following two concurrent serializable transaction  $T_0$  and  $T_1$ , schedule  $S$  can be transformed into a serial schedule  $S'$

| $T_0$    | $T_1$    |
|----------|----------|
| read(A)  |          |
| write(A) |          |
|          | read(B)  |
|          | write(B) |
| read(B)  |          |
| write(B) |          |
|          | read(B)  |
|          | write(B) |

7. Consider a *multi-level feedback queue* in a single-CPU system. The first level (queue 0) is given a quantum of 8 ms, the second one a quantum of 16 ms, the third is scheduled FCFS. Assume six jobs ( $J_1 \sim J_6$ ) arrive all at time zero with the following job times (in ms): 4, 7, 12, 15, 25 and 30.

- (4%) Show the *Gantt chart* for this system.
- (3%) Compute the *turnaround time*.
- (3%) Compute the *response time*.

8. (5%) Which of the following algorithms are preemptive scheduling?

- First-in-first-out
- Round-robin
- Shortest-job-first
- Multilevel Feedback Queue Scheduling

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9. Please answer the following question about memory management:

- a. (3%) Consider a two-level page table scheme, in which the outer page table itself is also paged with the page table store in memory. If a memory reference takes 90 nanoseconds, how long does a paged memory reference take?
- b. (2%) If we add TLBs, and 95 percent of all page-table references are found in the TLBs, what is the effective memory reference time? (Assume that finding a page-table entry in the TLBs take 9 nanoseconds, if the entry is there)

10. Please answer the following question:

- a. (5%) What is *starvation*?
- b. (3%) Among the FCFS(first-come-first-serve), SSTF(shortest-serve-time-next), LOOK, SCAN, C-LOOK, C-SCAN, and N-Step SCAN disk-head scheduling policies, which are subject to starvation at high loads?
- c. (2%) Response times are more predictable in preemptive systems than in non-preemptive systems. *True or False?*

11. Consider the following hardware configuration. Virtual address = 32 bits, page size = 4Kbytes, and a page table entry occupies 4 bytes. How many pages should the OS allocate for the pages tables of a 12Mbyte process under the following paging mechanisms?

- a. (5%) one-level paging.
- b. (5%) two-level paging. (Assuming that the number of entries in a first-level page table is the same as that in a second-level page table)

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1. (15%) (a). (5%) Find the least positive integer  $x$  satisfying the congruence:

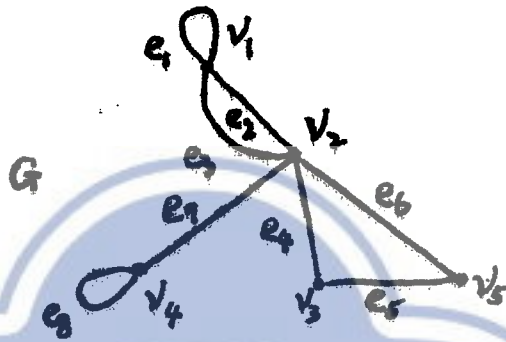
$$x \equiv 325^{4329} \pmod{11}.$$

(b). (10%) Find the least positive integer  $x$  satisfying the congruence:

$$531x \equiv 1 \pmod{1769}.$$

2. (10%) (a). (4%) How many vertices and how many edges are there in the complete bipartite graphs  $K_{m,n}$ .

(b). (6%) Present the pseudograph  $G$  using an incidence matrix.



3. (5%) Prove that there are infinitely many primes.

4. (10%) Find the state diagram for the nondeterministic finite-state automaton with the following state table. The final states are  $s_2$  and  $s_3$ .

| State Table |                 |            |
|-------------|-----------------|------------|
| State       | Input           |            |
|             | 0               | 1          |
| $s_0$       | $s_0, s_1$      | $s_3$      |
| $s_1$       | $s_0$           | $s_1, s_3$ |
| $s_2$       |                 | $s_0, s_2$ |
| $s_3$       | $s_0, s_1, s_2$ | $s_1$      |

5. (10%)  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 37$ ,  $x_1 + x_2 + x_3 = 6$  and  $x_1, x_2, x_3 > 0$ . Find the number of nonnegative integer solutions.

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6. (10%) Let  $v, k$  and  $\lambda$  be positive integers such that  $v > k \geq 2$ . A  $(v, k, \lambda)$ -balanced incomplete block design (abbreviate to  $(v, k, \lambda)$ -BIBD) is a pair  $(X, A)$  such that the following properties are satisfied:

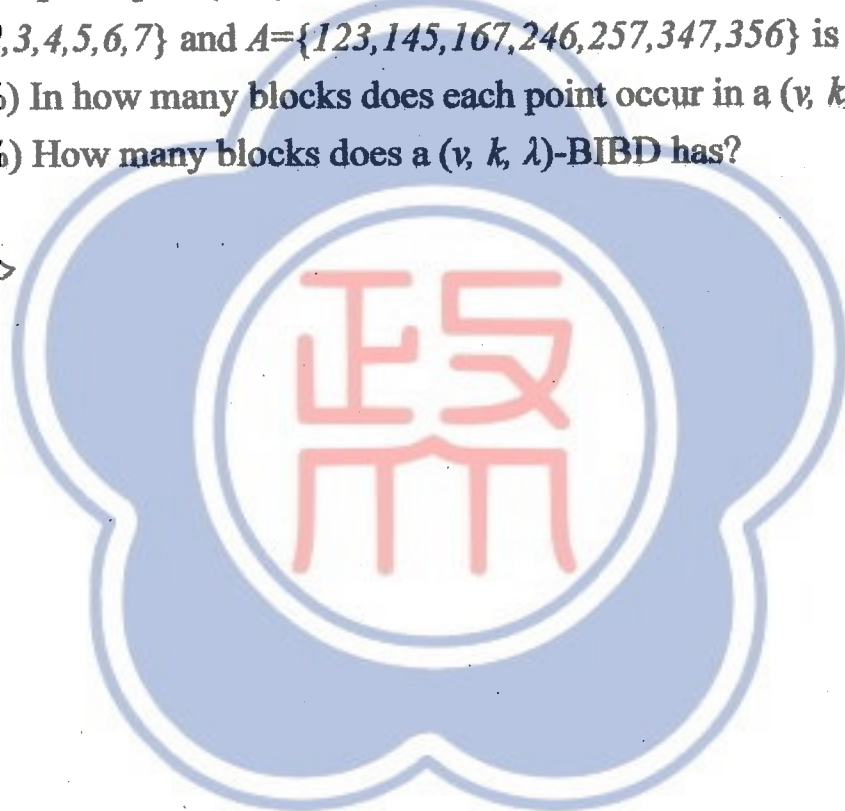
- (1).  $X$  is a set of  $v$  elements called points,
- (2).  $A$  is a collection of subsets of  $X$  called blocks,
- (3). each block contains exactly  $k$  points, and
- (4). every pair of distinct points is contained in exactly  $\lambda$  blocks.

For example, a pair  $(X, A)$  with

$X = \{1, 2, 3, 4, 5, 6, 7\}$  and  $A = \{123, 145, 167, 246, 257, 347, 356\}$  is a  $(7, 3, 1)$ -BIBD.

- (a). (5%) In how many blocks does each point occur in a  $(v, k, \lambda)$ -BIBD?
- (b). (5%) How many blocks does a  $(v, k, \lambda)$ -BIBD has?

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| 考試科目 | 計算機數學 | 所別 | 資訊科學系 | 考試時間 | 3月14日<br>星期六 | 第3節 |
|------|-------|----|-------|------|--------------|-----|

7. (10%) Solve the following system using its augmented matrix  $M$ :

$$x + 2y - z = 3$$

$$x + 3y + z = 5$$

$$3x + 8y + 4z = 17$$

8. (10%)

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 7 & 7 & 4 \\ 1 & 2 & 2 & 5 & 5 & 6 \\ 3 & 6 & 6 & 15 & 14 & 15 \end{bmatrix}$$

(a) Find  $\text{rank}(M_k)$ , for  $k = 1, 2, \dots, 6$ , where  $M_k$  is the submatrix of  $A$  consisting of the first  $k$  columns  $C_1, C_2, \dots, C_k$  of  $A$ .

(b) Which columns  $C_{k+1}$  are linear combinations of preceding columns  $C_1, C_2, \dots, C_k$ ?

9. (10%) Consider the following subspaces of  $\mathbb{R}^5$ :

$$U = \text{span}(u_1, u_2, u_3) = \text{span}\{(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)\}$$

$$W = \text{span}(w_1, w_2, w_3) = \text{span}\{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3), (2, 5, 3, 2, 1)\}$$

Find a basis and the dimension of  $U + W$ .

10. (10%)

$$\text{Let } A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

(a) Find all eigenvalues and corresponding eigenvectors.

(b) Find a nonsingular matrix  $P$  such that  $D = P^{-1}AP$  is diagonal, and  $P^{-1}$ .

備 考 試 題 隨 卷 繳 交

命 題 委 員 :

( 簽 章 )

命題紙使用說明：1. 試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。  
2. 書寫時請勿超出格外，以免印製不清。  
3. 試題由郵寄遞者請以掛號寄出，以免遺失而示慎重。