國立政治大學 115 學年度 碩士班暨碩士在職專班 招生考試試題

考試科目	資料結構及演算法	系所別	資訊科學系	考試時間	2月5日(四) 第四節

For the first 10 questions, select the best answer: 選擇題請在答案卡上作答，否則不予計分。

1. (5%) What is the upper bound on the height of a red-black tree with n internal nodes?
   (A) $n/2$
   (B) $\log_2(n+1)$
   (C) n
   (D) $2\log_2(n+1)$
2. (5%) A queue is implemented using two stacks, S_{in} and S_{out}. The implementation ensures the FIFO property by defining the operations as follows:
   - Enqueue: The new element is simply pushed onto S_{in}.
   - Dequeue:
     - If S_{out} is not empty, the top element is popped from S_{out}.
     - If S_{out} is empty, all elements are first transferred from S_{in} to S_{out} (by repeatedly popping from S_{in} and pushing onto S_{out}, and then the top element is popped from S_{out}.

   What is the amortized time complexity of the dequeue operation?
   (A) $O(1)$
   (B) $O(\log n)$
   (C) $O(n)$
   (D) $O(n\log n)$
3. (5%) In a binary search tree, if an element is searched for, and the search path visits nodes with keys 50, 75, 60, 68. What is the relationship between the key 65 and the node with key 68?
   (A) 65 must be in the left subtree of 68.
   (B) 65 must be in the right subtree of 68.
   (C) 65 could be either in the left or right subtree of 68.
   (D) 65 must be a child of 68.
4. (5%) The Floyd-Warshall algorithm is used to find which of the following?
   (A) Single-Source Shortest Path (SSSP)
   (B) All-Pairs Shortest Path (APSP)
   (C) Minimum Spanning Tree (MST)
   (D) Longest Common Subsequence (LCS)
5. (5%) What is the recurrence relation for the time complexity of the merge sort algorithm, assuming n is a power of 2?
   (A) $T(n) = T(n-1) + O(1)$
   (B) $T(n) = 2T(n/2) + O(1)$
   (C) $T(n) = 2T(n/2) + O(n)$
   (D) $T(n) = T(n/2) + O(n)$

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6. (5%) For two decision problems A and B, A is said to be *polynomial-time reducible* to B, denoted A $\leq_P$ B, if there exists a function f that is computable in polynomial time, such that for every input x:

$$x \in A \iff f(x) \in B$$

Consider two decision problems A and B. It is given that problem A is NP-Complete. Which of the following statements regarding the direction of reduction is true?

(A) If there exists a reduction A $\leq_P$ B, and B belongs to the class NP, then B must be NP-Complete.

(B) If there exists a reduction B $\leq_P$ A, and B belongs to the class NP, then B must be NP-Complete.

(C) If there exists a reduction A $\leq_P$ B, then B must be verifiable in polynomial time.

(D) If there exists a reduction B $\leq_P$ A, then B must be strictly harder than A.

7. (5%) Which of the following statements regarding the characteristics of several common shortest path algorithms is incorrect?

(A) Dijkstra's algorithm: It is applicable to graphs where all edge weights are non-negative, and it is typically implemented using a priority queue to achieve a time complexity of $O((|V| + |E|) \lg|V|)$.

(B) Bellman-Ford algorithm: It can handle graphs with negative edge weights, but its time complexity is $O(|V||E|)$, making it slower than Dijkstra's Algorithm.

(C) Floyd-Warshall algorithm: It is a single-source shortest path algorithm, suitable for dense graphs, and solves the problem using the concept of Dynamic Programming.

(D) Negative weight cycles: If a graph contains a negative weight cycle, the shortest path is undefined, and the Bellman-Ford Algorithm can be used to detect the existence of such a cycle.

8. (5%) A topological sort is possible if and only if the graph is a:

(A) Complete Graph

(B) Directed Acyclic Graph

(C) Bipartite Graph

(D) Connected Graph

9. (5%) Suppose we have a queue Q containing the sequence 1, 2, 3, 4, 5. We also have an empty stack S. The only allowable operations are:

- Dequeue an element from Q and print it.
- Dequeue an element from Q and push it onto S.
- Pop an element from S and print it.

Which of the following permutations is impossible to generate as an output sequence?

(A) 1, 5, 4, 3, 2

(B) 3, 4, 5, 2, 1

(C) 1, 3, 2, 5, 4

(D) 4, 3, 1, 2, 5

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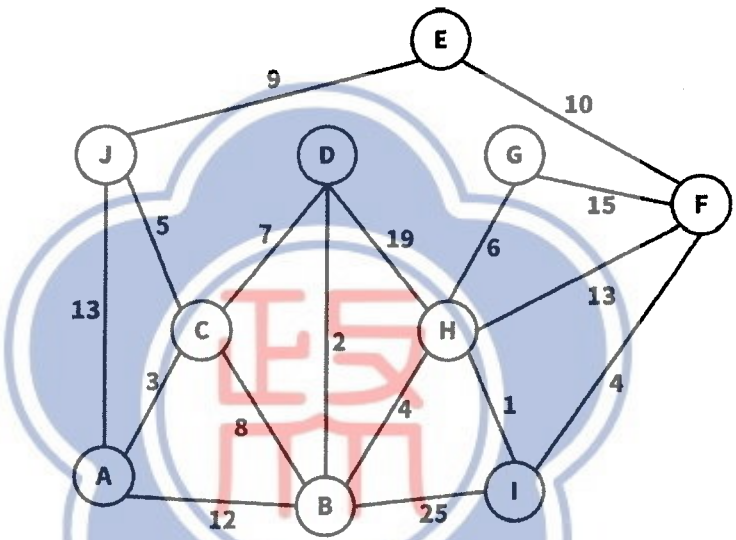
10. (5%) The Master Theorem case that applies to the recurrence $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$ results in which time complexity?
    (A) $O(n)$
    (B) $O(n\log n)$
    (C) $O(n\log^2 n)$
    (D) $O(n^2)$

11. Consider the following graph:



    (a) (5%) Apply Prim's algorithm, starting from node A, to construct the minimum spanning tree. List the edges of the minimum spanning tree in the exact order they are added by the algorithm. Use the format (node1, node2).
    (b) (5%) State the final total weight of the constructed minimum spanning tree.

12. The following code defines the structure of a binary search tree. Please answer the questions below.

```
#include <iostream>
#include <vector>
using namespace std;

struct TreeNode {
    int data;
    TreeNode* left;
    TreeNode* right;
    TreeNode(int val) : data(val), left(nullptr), right(nullptr) {}
};
```

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```
TreeNode* insertNode(TreeNode* root, int data) {
    if (root == nullptr) {
        return new TreeNode(data);
        }
    if (data < root->data) {
        root->left = blank1;}
    else if (data > root->data) {
        root->right = blank2;}
    return root;
}

void Traversal(TreeNode* root) {
    if (root != nullptr) {
        Traversal(root->left);
        cout << root->data << " ";
        Traversal(root->right);
    }
}

int main() {
    TreeNode* root = nullptr;
    vector<int> initial_data = {20, 30, 70, 15, 40, 55, 80};
    for (int val : initial_data) {
        root = insertNode(root, val);
    }

    cout << "Order 1: ";
    Traversal(root);
    cout << endl;
    int new_value = 45;
    root = insertNode(root, new_value);
    cout << "Order 2: ";
    Traversal(root);
    cout << endl;
    return 0;
}
```

(a) (5%) What should be filled in **blank1** and **blank2**?

(b) (5%) What is the output of the code?

(c) (5%) Which traversal order of the output is used?

(d) (5%) Based on the final binary search tree (after the insertion of 45), what is the output sequence for the post-order traversal?

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13. Consider a hash table of size $M = 20$ (slots indexed from 0 to 19). The primary hash function is defined as: $h_1(k) = k \ mod \ 20$. The secondary hash function is defined as: $h_2(k) = (k \ mod \ 7) + 1$. Assume the hash table is initially empty before each part of the question. We will insert the following sequence of keys S in the order given:

$$S = [8, 38, 3, 5, 28, 18, 65, 83, 25]$$

(a) (5%) Assuming we use double hashing for collision resolution, determine the final structure of the hash table after all keys in S have been inserted. Note: The probe sequence is given by:

$$h(k, i) = \left(h_1(k) + i \cdot h_2(k)\right) mod \ M$$

where i is the probe number.

(b) (5%) If we search for 48, what is the total number of probes made during the search?

(c) (10%) Suppose we are given n keys, m hash table slots, and two uniform hash functions h_a and h_b. Further suppose hashing scheme uses h_a for the odd keys and h_b for the even keys. What is the expected number of keys in a slot?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	作業系統	系所別	資訊科學系 資訊科學與工程組、智慧計算組	考試時間	2月5日(四) 第三節

Note: All the questions are **single choice**. Please select **the best answer**. (100%; 2.5% for each question)

選擇題請在答案卡上作答，否則不予計分

1. Which of the following is the only gateway between user space and kernel space? (A) user interface (B) operating system (C) system call (D) hardware drivers
2. The close() system call in UNIX is used to close a file. What is the equivalent system call in Windows? (A) close() (B) CloseHandle() (C) CloseFile() (D) Exit()
3. Which of the following contains the executable code in the memory layout of a process? (A) data section (B) heap section (C) stack section (D) text section
4. Which of the following process state will be switched from "ready" state? (A) terminated (B) ready (C) waiting (D) running
5. Which of the following refers to the capability to allow multiple tasks make progress on a single processor system? (A) concurrency (B) data parallelism (C) parallelism (D) task parallelism
6. Which of the following is a method for implicit threading? (A) OpenMP (B) thread pools (C) grand central dispatch (D) all of the above
7. Which of the following options to deliver signals in multithreaded program should be applied to a synchronous signal? (A) deliver the signal to every thread in the process (B) deliver the signal to the thread to which the signal applies (C) deliver the signal to certain threads in the process (D) assign a specific thread to receive all signals for the process (E) all of the above
8. LWP is ____. (A) short for lightweight processor (B) placed between user and kernel threads (C) placed between system and kernel threads (D) common in systems implementing one-to-one multithreading models
9. Which of the following circumstances can preemptive scheduling take place? (A) when a process switches from the running state to the waiting state (B) when a process terminates (C)when a process switches from the waiting state to the ready state (D) none of the above
10. Shortest-remaining-time-first scheduling is the preemptive version of _____? (A) SJF (B) FCFS (C) RR (D) Multilevel queue
11. Which of the following criteria is more important for an interactive system? (A) CPU utilization (B) Turnaround time (C) Response time (D) Throughput
12. What is the speedup gain for the following application with 40 percent parallel and sixteen processing cores? (A) 1.5 (B) 2 (C) 2.5 (D) 4 (E) none of the above
13. Which of the following actions should be performed among cooperating processes? (A) process synchronization (B) coordination (C) none of the above (D) both of the above
14. In _____, the process requests permission to access and modify variables shared with others. (A) critical section (B) entry section (C) exit section (D) remainder section
15. Which of the following is true for the solutions to critical-section problems? (A) No deadlock implies progress, and progress implies bounded waiting (B) Progress implies no deadlock, and no deadlock implies bounded

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waiting (C) Bounded waiting implies progress, and progress implies no deadlock (D) Bounded waiting implies no deadlock, and no deadlock implies progress

16. Which of the following is not true about *test_and_set* instruction? (A) Returns the original value of passed parameter (B) It is executed atomically (C) It is a hardware instruction (D) Returns the new value of passed parameter (E) Set the new value of passed parameter to "TRUE"
17. Which of the following is not true about *compare_and_swap* instruction? (A) It is a hardware instruction (B) It is executed atomically (C) Set the new value of passed parameter to "TRUE" (D) Returns the original value of passed parameter
18. When mutex lock is implemented as a binary semaphore, what should its value be initialized to be? (A) 1 (B) 0 (C) -1 (D) none of the above
19. The counting semaphore is initialized to _____. (A) 0 (B) 1 (C) the number of resources available (D) none of the above
20. Atomic integers in Linux are useful when (A) several variables are involved in a race condition. (B) an integer variable needs to be updated. (C) a single process accesses several variables involved in a race condition. (D) All of the above.
21. A(n) _____ is a sequence of read-write operations that are atomic. (A) atomic integer (B) semaphore (C) mutex lock (D)memory transaction
22. In a system resource-allocation graph, ____. (A) a directed edge from a resource to a process is called an assignment edge (B) a directed edge from a resource to a process is called a request edge (C) a directed edge from a process to a resource is called an assignment edge (D) None of the above
23. Deadlock prevention using preempting allocated resources cannot be used for (A) memory (B) CPU registers. (C) database transactions (D) mutexes
24. If execution time binding is used, (A) logical addresses of process may change over time but physical addresses remain the same. (B)both physical and logical addresses may change over time (C) physical addresses of process may change over time but logical addresses remain the same (D) both physical and logical addresses remain the same over time.
25. An address generated by a CPU is referred to as a ____. (A) physical address (B) logical address (C) post relocation register address (D) Memory-Management Unit (MMU) generated address
26. A frame table stores (A) which frames are allocated. (B) which frames are free. (C) total number of frames. (D) All of the above.
27. The protection bit in a page table (A) marks a page table as read-only or read-write. (B) provides protection against unauthorized updates in the page table. (C) marks a frame as read-only or read-write. (D) All of the above.
28. The roll out, roll in variant of swapping is used ____. (A) for priority-based scheduling algorithms (B) for the round-robin scheduling algorithm (C) when a backing store is not necessary (D) when the load on the system

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has temporarily been reduced

29. If an instruction modifies several different locations, a page fault can be handled by (A) terminating the process. (B) loading multiple pages in advance. (C) incorporating special hardware. (D) using temporary registers to hold the values of overwritten locations.
30. Optimal page replacement ____. (A) is the page-replacement algorithm most often implemented (B) can suffer from Belady's anomaly (C) is used mostly for comparison with other page-replacement schemes (D) requires that the system keep track of previously used pages
31. A drawback of equal or proportional allocation is that (A) they are very expensive to compute. (B) a high-priority process is treated the same as a low-priority process. (C) the allocation varies according to the degree of multiprogramming. (D) the processes that arrive earlier get more pages than the processes arriving later.
32. The surface of a magnetic disk platter is divided into ____. (A) tracks (B) arms (C) sectors (D) cylinders
33. Which of the following disk head scheduling algorithms does not take into account the current position of the disk head? (A) C-SCAN (B) FCFS (C) SCAN (D) All scheduling algorithms take into account the current position of the disk head
34. A control register in an I/O device control (A) is written by the host to send output. (B) is written by the host to choose half-duplex communication. (C) contains status bits that can be read by the host. (D) is read by the host to get input.
35. In a blocking system call, the execution of a process is suspended (A) until the process is woken up by some other process. (B) until a fixed amount of time has elapsed. (C) the I/O has completed. (D) the kernel raises the priority of that process.
36. File's _____ is a unique tag identifies the file within the file system. (A) type (B) identifier (C) location (D) name
37. UNIX systems employ _____ (A) acyclic-graph directory (B) general graph directory (C) single-level directory (D) two-level tree directory
38. Unified virtual memory uses _____to cache both process page and file data (A) disk block caching (B) double caching (C) page caching (D) buffer caching
39. The file owner_____ (A) is the user who cannot change the file attributes, but can execute the file. (B) is the user who can change the file attributes, but cannot grant access to the file. (C) is the user who can change the file attributes and grant access to the file. (D) is the only user who can execute the file.
40. IPSec uses _____ encryption. (A) asymmetric (B) Caesar cipher (C) one-time password (D) symmetric

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	計算機數學	系所別	資訊科學系	考試時間	2月 5日(四) 第二節

本次考試共 25 題單選題，每題 4 分。選擇題請在答案卡上作答，否則不予計分。

1. If the following matrix equation holds:

$$\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} 1 & d & e \\ 0 & 1 & f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 2 \\ -2 & 5 & 3 \end{bmatrix}$$

Please determine the value of $a+b+c+d+e+f$.

(A) 10 (B) 11 (C) 12 (D) 13

2. How many of the following statements are correct?

Let A be an $n \times m$ matrix whose null space has dimension k. Consider the following statements:

1. The dimension of $NULL(A^T)$ is $n-m+k$.
2. The dimension of $CS(A)$ is $m-k$.
3. The dimension of $RS(A)$ is $m-k$.
4. The dimension of $RS(A)$ is $n-k$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

3. Let $E=\{v_1,v_2\}=\left\{\begin{bmatrix}3\\7\end{bmatrix},\begin{bmatrix}5\\14\end{bmatrix}\right\}$ and $F=\{u_1,u_2\}=\left\{\begin{bmatrix}2\\5\end{bmatrix},\begin{bmatrix}1\\3\end{bmatrix}\right\}$ be two bases for $\mathbb{R}^2$.

If $[I]_E^F=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is the transition matrix from E to F, where $a,b,c,d\in\mathbb{R}$. Please determine the value of $a+b+c+d$.

(A) 3 (B) 5 (C) 7 (D) 9

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4. How many of the following statements are True?

(1) Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^3$ is linear and $T(1,2) = (1,0,2)$ and $T(2,3) = (1,-1,4)$. Is $\mathrm{Ker}(T) = \{0\}$? (2) The linear transformation $L : P_2 \to P_2$ defined by $L(at^2 + bt + c) = 2a + b$ is one to one. (3) Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 \\ -x_1 \\ 6x_1 \end{bmatrix}$. Is T onto?

(A) 0 (B) 1 (C) 2 (D) 3

5. How many of the following statements are False?

(1) If A is invertible and 1 is an eigenvalue of A, then 1 is also an eigenvalue of A^{-1}. (2) If A contains a row or column of zeros, then 0 is a eigenvalue of A. (3) Each eigenvector of A is also an eigenvector of A^2. (4) Each eigenvalue of A is also an eigenvalue of A^2. (5) Eigenvectors must be nonzero vectors.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

6. Find an orthonormal basis for the column space of D:

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 0 \\ 1 & 4 & 6 \\ 1 & 4 & 6 \end{bmatrix}$$

Let the orthonormal basis obtained by the Gram-Schmidt process be $\{q_1, q_2, q_3\}$, where:

$$q_1 = \frac{1}{2}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}, \quad q_2 = \frac{1}{\sqrt{12}}\begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix}, \quad q_3 = \frac{1}{\sqrt{6}}\begin{bmatrix} i \\ j \\ k \\ l \end{bmatrix}$$

Please determine the value of the sum of all numerators: $a + b + c + d + e + f + g + h + i + j + k + l$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

備註	作答於試題上者，不予計分。 試題請隨卷繳交。

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7. Given the data points $(x, y) = (0,1), (3,4), (6,5)$, find the best squares fit by a linear function $y = ax + b$.

Please determine the value of $a + b$.

(A) 1 (B) 2 (C) 3 (D) 4

For problems 8-10, please find a singular value decomposition for the following matrix:

$$B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 1 & -3 \end{bmatrix} = U\Sigma V^T$$

8. $U = ?$

$$(A)\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix} \quad (B)\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \quad (C)\begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (D)\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \end{bmatrix}$$

9. $\Sigma = ?$

$$(A)\begin{bmatrix} 16 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad (B)\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad (C)\begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (D)\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$$

10. $V = ?$

$$(A)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (B)\begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix} \quad (C)\begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (D)\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

備註	作答於試題上者，不予計分。 試題請隨卷繳交。

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11. Let set $A = \{1, 2, 3, 4\}$. Define a relation R on A where $R = \{(1,1), (1,2), (2,2), (2,3), (1,3), (3,3), (4,4)\}$, which of the following properties does this relation have?

(1) Symmetric (2) Asymmetric (3) Antisymmetric (4) Reflexive (5) Irreflexive (6) Transitive

(A) (3), (4), (6) (B) (1), (4), (6) (C) (3), (5) (D) (2), (5), (6)

12.

$$47^{245} \equiv a \pmod{19}, \text{find } a.$$

(A) 4 (B) 5 (C) 9 (D) 11

13. Let $A = \{\emptyset, 1, \{1\}, \{1, \emptyset\}\}$, and $P(A)$ denote the power set of A. How many of the following statements are TRUE?

- $\{1\} \in A$
- $\{1\} \subseteq A$
- $\{\emptyset\} \in A$
- $\{\emptyset\} \subseteq A$
- $\{1, \emptyset\} \in A$
- $\{1, \emptyset\} \subseteq A$
- $\{\{1\}\} \subseteq P(A)$
- $\{\emptyset, \{1\}\} \in P(A)$
- $\{\emptyset, \{1\}\} \subseteq P(A)$

(A) 6 (B) 7 (C) 8 (D) 9

備註	作答於試題上者，不予計分。 試題請隨卷繳交。

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The following questions constitute a set. Please answer accordingly.

Suppose A and B are events in a sample space, such that $P(A) = \frac{4}{9}, P(B) = \frac{5}{11}, P(A|B) = \frac{2}{5}$, and $P(B|A) = \frac{a}{b}$ (where $\frac{a}{b}$ is an irreducible fraction).

14. $a =$?

(A) 7 (B) 8 (C) 9 (D) 10

15. $b =$?

(A) 20 (B) 21 (C) 22 (D) 23

The following questions constitute a set. Please answer accordingly.

Let $X = \{1, 2, 3, 4, 5, 6\}$ and $Y = \{a, b, c, d\}$. For a function f and a set S, define $f(S) = \{f(i) \mid i \in S\}$. Please find the answers for the following questions.

16. How many functions $f : X \to Y$ are onto (surjective)?

(A) 1024 (B) 1560 (C) 2048 (D) 4096

17. How many functions $f : X \to Y$ are there such that $|f(X)| = 3$?

(A) 540 (B) 1080 (C) 1260 (D) 2160

18. How many functions $f : X \to X$ are there such that $f(\{1, 2, 3\}) = \{1, 2\}$ and for all $x \in X$, $f(x) \neq 6$?

(A) 192 (B) 384 (C) 512 (D) 768

備註	作答於試題上者，不予計分。 試題請隨卷繳交。

考試科目	計算機數學	系所別	資訊科學系	考試時間	2月 5日(四) 第二節

For the following problems, please solve the linear recurrence relation $a_n - 4a_{n-1} + 4a_{n-2} = 2^n$ with $a_0 = 1$ and $a_1 = 2$, and let the solution be in the form $a_n = (i + jn + kn^2) \cdot 2^n$.

19. $i = ?$

(A) 0 (B) 1 (C) 2 (D) -1

20. $j = ?$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) -1

21. $k = ?$

(A) 1 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

22. Consider a graph $G = (V, E)$ with vertices $V = \{1, 2, 3, 4, 5, 6, 7\}$ and edges $E = \{(1, 2), (2, 3), (3, 1), (3, 4), (4, 5), (5, 6), (6, 4), (4, 7)\}$. How many of the following statements are TRUE?

- It is a bipartite graph.
- The length of the longest simple path is 6.
- It has an Euler path.
- It is a planar graph.

(A) 0 (B) 1 (C) 2 (D) 3

23. Let A and B be sets. Which of the following statements is TRUE?

(A) If $A \subseteq B$, then $A \cup B = A$. (B) If $A \subseteq B$, then $A \cap B = A$. (C) If $A \in B$, then $A \subseteq B$. (D) If $A \subseteq B$ and $B \in C$, then $A \in C$.

備註	作答於試題上者，不予計分。 試題請隨卷繳交。

考試科目	計算機數學	系所別	資訊科學系	考試時間	2月 5日(四) 第二節

24. Let $G = (V, E)$ be a simple undirected graph with $|V| = n$ and $|E| = m$. Let G be a bipartite graph with partition $V = V_1 \cup V_2$. How many of the following statements are ALWAYS TRUE?

- If G is connected and $m = n - 1$, then G is a tree.
- The number of vertices with odd degree in G is even.
- If G has a Hamiltonian Cycle, then $|V_1| = |V_2|$.
- If G has a Hamiltonian Path, then $|V_1| = |V_2|$.

(A) 1 (B) 2 (C) 3 (D) 4

25. How many of the following statements are TRUE? (Note: $\mathbb{Z}$ is the set of integers, $\mathbb{R}$ is the set of real numbers, and $\mathbb{N} = \{1, 2, 3, \dots\}$)

- $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, defined by $f(m, n) = 2m + 3n$, is onto (surjective).
- $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = x^3 + x$, is one-to-one (injective).
- $f : \mathbb{Z} \to \mathbb{Z}$, defined by $f(n) = \lfloor \frac{n}{2} \rfloor$, is one-to-one.
- $f : \mathbb{R}^2 \to \mathbb{R}^2$, defined by $f(x, y) = (x + y, x + y)$, is bijective.
- $f : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$, defined by $f(n) = (n, n + 1)$, is onto.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

備註	作答於試題上者，不予計分。 試題請隨卷繳交。