考試科目	計算機數學	系所別	資訊科學系	考試時間	2月12日(三) 第2節

I. 離散數學：60% (第 1~11、17~19 題)

II. 線性代數：40% (第 12~16 題)

請書寫必要的解題過程，僅提供答案而無必要過程，將無法獲得該題滿分。可使用中文或英文作答，力求書寫工整，如字跡潦草，無法閱讀，將影響評分。

第一部分、單選題 (第 1~14 題)，每題 3 分，無需提供解題過程。

1. Which of the following pairs of propositions is logically equivalent?
   (a) $(p\rightarrow q)\vee(p\rightarrow r)$ and $p\rightarrow(q\wedge r)$
   (b) $(p\rightarrow q)\vee(p\rightarrow r)$ and $p\rightarrow(q\vee r)$
   (c) $(p\rightarrow q)\vee(p\rightarrow r)$ and $p\rightarrow(q\wedge\neg r)$
   (d) $(p\rightarrow q)\vee(p\rightarrow r)$ and $p\rightarrow(\neg q\wedge r)$

2. Let $A=\{a,b,c,d,e,f,g\}$ and $R=\{(a,a),(b,b),(c,c),(c,d),(c,g),(d,g),(e,e),(f,f),(g,g)\}$ be a relation on A. Then R is
   (a) reflexive and symmetric
   (b) antisymmetric and not reflexive
   (c) reflexive and antisymmetric
   (d) symmetric and not reflexive

3. Which of the following statements is NOT valid:
   (a) $(A-C)\cap(C-B)=A-B$
   (b) $(B-C)\cup(C-B)\cup(\bar{B}\cap\bar{C})=(\overline{B\cap C})$
   (c) $(A\oplus B)\oplus B=A$
   (d) $(A\cap B)\cup(A\cap B\cap C)=(A\cup C)\cap(A\cap B)$

4. In how many different ways can 10 identical balls distributed among four distinguishable children if each child receives at least two balls?
   (a) 8
   (b) 9
   (c) 10
   (d) 11

考試科目	計算機數學	系所別	資訊科學系	考試時間	2月12日(三) 第2節

5. For $n \geq 1$, how many functions are there from the set $\{1, \ldots, n\}$ to the set $\{1, 2, 3, 4, 5\}$?
   (a) n^5
   (b) $5n$
   (c) 5^n
   (d) $5n!$

6. Following Q.5, how many of the functions are injective when $n = 3$?
   (a) 20
   (b) 50
   (c) 60
   (d) 120

7. What is $191^{45} \mod 13$?
   (a) 3
   (b) 9
   (c) 5
   (d) 1

8. What is $(-12)^{36} \cdot 50^{19} \mod 7$?
   (a) 2
   (b) 1
   (c) 4
   (d) 3

9. What is the probability that a fair die, rolled repeatedly until the number 6 appears for the first time, requires exactly four rolls? Choose the closest value:
   (a) 8%
   (b) 10%
   (c) 12%
   (d) 14%

考試科目	計算機數學	系所別	資訊科學系	考試時間	2月12日(三) 第2節

10. At a university with students from exactly 50 states, what is the smallest number of students needed to ensure that at least 100 students are from the same state?
    (a) 4951
    (b) 4949
    (c) 4957
    (d) 4954

11. How many of the following cases correspond to the existence of a planar graph?
    (1) 7 vertices and 13 edges; (2) 6 regions and 5 vertices; (3) 8 vertices and 20 edges; (4) 10 regions and 5 edges.
    (a) 1
    (b) 2
    (c) 3
    (d) 4

12. For an $n \times n$ matrix A, what can you say about the eigenvalues of A^{-1}, assuming A is invertible?
    (a) They are the reciprocals of the eigenvalues of A
    (b) They are the negatives of the eigenvalues of A
    (c) They are equal to the eigenvalues of A
    (d) They are zero.

13. A and B are any matrices with the same number of rows. What can you say about the comparison of $Rank(A)$ and $Rank([A\ B])$ without any additional conditions, where $[A\ B]$ is the block matrix formed by concatenating A and B column-wise.
    (a) $Rank(A) \geq Rank([A\ B])$
    (b) $Rank(A) \leq Rank([A\ B])$
    (c) $Rank(A) > Rank([A\ B])$
    (d) $Rank(A) = Rank([A\ B])$
    (e) $Rank(A) < Rank([A\ B])$

考試科目	計算機數學	系所別	資訊科學系	考試時間	2月12日(三) 第2節

14. Which one is false for the following statements:
(a) If A is diagonalizable, then there is a basis for eigenvectors of A
(b) Not all 2×2 matrices are diagonalizable
(c) If A does not have n distinct eigenvalues, then A is not diagonalizable
(d) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \\ 4 & 2 & 3 & 1 \\ 3 & 1 & 4 & 2 \end{bmatrix}$, then $[1,1,1,1]^T$ is an eigenvector of A.
(e) Eigenvectors corresponding to different eigenvalues are linearly independent.

第二部分、計算題 (第 15~19 題)

15. Suppose A is 3 by 4, and $Ax = 0$ has exactly 2 special solutions:

$$x_1 = \begin{bmatrix} 3 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

(a) (6%) Find A's row reduced echelon form R.
(b) (8%) Find the dimensions of all four fundamental subspaces $C(A)$, $N(A)$, $C(A^T)$, $N(A^T)$.
(c) (6%) Following (b), find bases for two of these subspaces (choose two subspaces for which sufficient information is available).

16. (11%) Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$, perform a singular value decomposition on A, as $A = U\sum V^T$.

17. (7%) In the multinomial expansion of $(2x - y + 3z)^9$, what is the coefficient of xy^6z^2?

18. (10%) Use the inclusion-exclusion principle to count the number of integers between 1 and 5,000 that are not divisible by 10,12, or 21.

19. (10%) Fibonacci numbers are defined by $f_n = f_{n-2} + f_{n-1}$ for $n \geq 2$, with the initial values $f_0 = 0$ and $f_1 = 1$, prove $f_0f_1 + f_1f_2 + f_{2n-1}f_{2n} = f_{2n}^2$ by induction.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	作業系統	系所別	資訊科學系 資訊科學與工程組、智慧計算組	考試時間	2月12日(三) 第三節

Note: All the questions are **single choice**. Please select **the best answer**. (100%; 2.5% for each question)

選擇題請在答案卡上作答，否則不予計分

1. Which of the following contains the addresses of all the service routines? (A) page table (B) system call table (C) interrupt vector (D) file descriptor table
2. Which of the following clears the interrupt by servicing the device? (A) interrupt request line (B) interrupt service routine (C) interrupt vector (D) interrupt-controller hardware
3. Microkernels use _____ for communication. (A) message passing (B) shared memory (C) system calls (D) virtualization
4. When a process fails, the operating system takes a _____ which can then be probed by a debugger for failure analysis? (A) capture of network statistic information (B) crash dump (C) core dump (D) capture of CPU usage information
5. Which of the following process state will be switched from "running" state when an interrupt occurs? (A) ready (B) terminated (C) waiting (D) new
6. Which of the following system calls is used to have a new program loaded into the new process' s memory space? (A) fork() (B) exit() (C) wait() (D) exec()
7. Which of the following IPC mechanism is easier to implement in a distributed system? (A) shared memory (B) message passing (C) socket communication (D) ordinary pipe
8. Which is not considered as a challenge when designing applications for multicore systems? (A) Deciding which activities can be run in parallel (B) Determining if data can be separated so that it is accessed on separate cores (C) Ensuring there is a sufficient number of cores (D) Identifying data dependencies between tasks.
9. The _____ model multiplexes many user-level threads to a smaller or equal number of kernel threads. (A) two-level (B) many-to-many (C) one-to-one (D) many-to-one
10. Which of the following is a synchronous signal? (A) illegal memory access (B) terminating a process with specific keystrokes (C) having a timer expire (D) none of the above
11. Assume process P0 and P1 are the process before and after a context switch, and PCB0 and PCB1 are respectively their process control block. Which of the following time units are included inside the dispatch latency? (A) P0 executing (B) P1 executing (C) save state into PCB0, and restore state from PCB1 (D) none of the above
12. Which of the following scheduling algorithm may suffer from convoy effect? (A) SJF (B) FCFS (C) RR (D) Multilevel queue
13. _____ is a significant problem with priority scheduling algorithms. (A) Complexity (B) Determining the length of the next CPU burst (C) Determining the length of the time quantum (D) Starvation
14. Processes P1~P4 arrival at P1=0ms, P2=1ms, P3=2ms, P4=3ms, and with the burst time P1=8ms, P2=4ms, P3=10ms, P4=5ms. What is the average waiting time by Shortest-Remaining-Time-First scheduling? (A)5ms (B)5.5ms (C)6ms (D)6.5ms
15. What scheduling algorithm does Round Robin degenerate to when the time quantum is excessively large? (A) SJF

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(B) Shortest-remaining-time-first (C) FCFS (D) Multilevel queue

16. Which of the following critical-section problem's requirements ensures programs will cooperatively determine what process will next enter its critical section? (A) mutual exclusion (B) progress (C) bounded waiting (D) none of the above
17. Which of the following is true for the solutions to critical-section problems? (A) Bounded waiting implies progress, and progress implies no deadlock (B) No deadlock implies progress, and progress implies bounded waiting (C) Progress implies no deadlock, and no deadlock implies bounded waiting (D) Bounded waiting implies no deadlock, and no deadlock implies progress
18. Which of the following variables are shared between the processes in Peterson's solution? (A) int turn (B) boolean flag[2] (C) both of the above (D) none of the above
19. In the monitor solution for dining-philosophers problem, a philosopher may start eating (A) at the end of the pickup( ) function before exiting the function. (B) in the beginning of the putdown( ) function (C) after exiting the pickup( ) function and before entering the putdown( ) function. (D) All of the above.

```
monitor DiningPhilosophers
{
   enum {THINKING, HUNGRY, EATING} state[5];
   condition self[5];

   void pickup(int i) {
      state[i] = HUNGRY;
      test(i);
      if (state[i] != EATING)
         self[i].wait();
   }

   void putdown(int i) {
      state[i] = THINKING;
      test((i + 4) % 5);
      test((i + 1) % 5);
   }

   void test(int i) {
      if ((state[(i + 4) % 5] != EATING) &&
       (state[i] == HUNGRY) &&
       (state[(i + 1) % 5] != EATING)) {
          state[i] = EATING;
          self[i].signal();
      }
   }

   initialization_code() {
      for (int i = 0; i < 5; i++)
         state[i] = THINKING;
    }
}
```

考試科目	作業系統	系所別	資訊科學系 資訊科學與工程組、智慧計算組	考試時間	2月12日(三) 第三節

20. Which of the following statements is true? (A) A counting semaphore can never be used as a binary semaphore. (B) A binary semaphore can never be used as a counting semaphore. (C) Counting semaphores can be used to control access to a resource with a finite number of instances. (D) Spinlocks can be used to prevent busy waiting in the implementation of semaphore
21. Which of the following is true? (A) No deadlock implies no starvation; (B) Deadlock doesn't imply starvation ; (C) No starvation implies no deadlock; (D) Starvation implies deadlock.
22. One necessary condition for deadlock is _____, which states that a resource can be released only voluntarily by the process holding the resource. (A) hold and wait (B) mutual exclusion (C) circular wait (D) no preemption
23. In a system resource-allocation graph, _____. (A) a directed edge from a process to a resource is called an assignment edge (B) a directed edge from a resource to a process is called a request edge (C) a directed edge from a process to a resource is called a request edge (D) None of the above
24. In a resource allocation graph, when a process releases a resource, _____. (A) a request edge is inserted. (B) an assignment edge is inserted. (C) a request edge is removed. (D) an assignment edge is removed.
25. In a system that uses deadlock detection algorithm, _____. (A) a deadlock is detected sometime after it has occurred but not necessarily immediately. (B) a deadlock is detected as soon as it occurs. (C) a deadlock is detected just before it occurs. (D) a deadlock is detected sometime before it occurs, but not necessarily just before.
26. Which of the following statement is correct? (A) Base register holds the size of a process. (B) Limit register holds the size of a process. (C) Base and limit registers can be loaded by the standard load instructions in the instruction set. (D) Any attempt by a user program to access memory at an address higher than the base register value results in a trap to the operating system.
27. Given the logical address 0xAEF9 (in hexadecimal) with a page size of 256 bytes, what is the page number? (A) 0xAE (B) 0xF9 (C) 0xA (D) 0x00F9
28. Suppose we have the following page accesses: 1 2 3 4 2 3 4 1 2 1 1 3 1 4 and that there are three frames within our system. Using the FIFO replacement algorithm, what will be the final configuration of the three frames following the execution of the given reference string? (A) 4, 1, 3 (B) 3, 1, 4 (C) 4, 2, 3 (D) 3, 4, 2
29. Memory compression is a useful alternative to paging (A) when the system has slow magnetic disks but not faster SSDs. (B) when the compression algorithm is can achieve the best possible compression ratio. (C) when the speed of the compression algorithm fastest. (D) even when the system has faster SSDs.
30. What is the speedup gain for the application which is 2/3 parallel with 4 processing cores by Amdahl's Law ? (A)2 (B)1.6 (C)1.5 (D)1.2
31. Consider a disk queue holding requests to the following cylinders in the listed order: 210, 67, 10, 99, 87, 90, 19, 149. Using the C-SCAN scheduling algorithm, what is the order that the requests are serviced, assuming the disk head is at cylinder 89 and moving upward through the cylinders?
    (A) 87 – 90 – 99 – 67 – 19 – 10 – 149 – 210 (B) 210 – 67 – 10 – 99 – 87 – 90 – 19 – 149
    (C) 90 – 99 – 149 – 210 – 10 – 19 – 67 – 87 (D) 90 – 99 – 149 – 210 – 87 – 67 – 19 – 10

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32. What is the minimum number to build a RAID 0+1 system? (A) 2 (B) 3 (C) 4 (D) 5
33. A Flash Translation Layer (FTL) can (A) translate user data to a format that can be written on the NVM. (B) tracks which physical block contains the most number of consecutive invalid pages. (C) track which physical block contains only valid pages. (D) track which physical block contains only invalid pages.
34. Which of the following is TRUE about NAS and Cloud storage? (A) Cloud storage is accessed like a file system while NAS is API based. (B) In an event of temporary network disconnection, an application using NAS will typically hang while an application using cloud storage will typically pause. (C) Cloud storage is typically accessed over a LAN while NAS is accessed over a WAN. (D) All of the above.
35. In polling I/O, the main inefficiency comes from (A) the hardware controller when it notifies the CPU that the device is ready for service. (B) the basic polling operation that may be comprised of several CPU instructions. (C) polling when it is attempted repeatedly yet rarely finds a device ready for service. (D) the slow data transfer rate between a device and the host.
36. In DMA-based I/O, (A) CPU is relieved from data transfer and is interrupted after every word has been transferred to initiate the next word transfer. (B) cycle stealing can slow down the CPU computation, but off-loading the data-transfer work to a DMA controller generally improves the total system performance. (C) interrupt mechanism is not used. (D) data is transferred to/from a single block of memory, but not to/from multiple blocks.
37. After the instruction "chmod 741 a.out" (A) the owner's permissions is rwx (B) the group's permissions is rx (C) other's permission is w (D)none of above
38. Mounting a file systems means that (A) the file system is removed from the disk space. (B) the file system is being created in the disk space. (C) the file system is becoming available within the file system name space. (D) the file system is becoming unavailable within the file system name space.
39. Which is not symmetric-key cryptography? (A) AES (B) DES (C)IDEA (D)RSA
40. What are the corresponding port numbers for the protocols, respectively: SSH, FTP, Telnet, and DNS (A) 21, 22, 23, 25 (B) 22, 21, 23, 53 (C) 21, 23, 22, 37 (D) 23, 21, 22, 123

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	資料結構及演算法	系所別	資訊科學系	考試時間	2 月 12 日(三) 第四節

For the first 15 questions, select the best answer.選擇題請在答案卡上作答，否則不予計分。

1. (4%) Let n an m be the sizes of two sets, assume $n < m$, if the sets are implemented using sorted lists, what is the time complexity to find the intersection of the two sets?
   A. $O(n+m)$
   B. $O(n \times m)$
   C. $O(n \log m)$
   D. $O(m \log n)$
   E. $O(\log n \times \log m)$
2. (4%) Consider again the above question, what if the sets are implemented using AVL trees, instead of sorted lists?
   A. $O(n+m)$
   B. $O(n \times m)$
   C. $O(n \log m)$
   D. $O(m \log n)$
   E. $O(\log n \times \log m)$
3. (4%) If an array is used to implement a binary search tree with n nodes, where index 1 (the first position) of the array stores the root, what is the maximal index i that stores the smallest key in the tree?
   A. 1
   B. n
   C. $\lceil \log n \rceil$
   D. 2^{n-1}
   E. 2^n+1
4. (4%) Which of the following statement concerning hashing is false?
   A. While hash function is designed based on arithmetic operation, hashing is also suitable for strings.
   B. If there is no collision, the worst time complexity for searching an element in a hash table is O(1).
   C. A good hash function should produce the same hash value for similar keys.
   D. Hashing is typically used for symbol tables in compilers.
   E. The size of the hash table should ideally be a prime number to reduce the likelihood of collisions.
5. (4%) Given a logical expression represented as an expression tree, along with the Boolean values assigned to the operands, which algorithm is best suited to evaluate the logical expression?
   A. Pre-order Traversal
   B. In-order Traversal
   C. Post-order Traversal
   D. Breadth-First Search
   E. Depth-First Search

考試科目	資料結構及演算法	系所別	資訊科學系	考試時間	2 月 12 日(三) 第 四 節

6. (4%) Given the following algorithm, which is likely to be the result if the input variable N is -1?

```
Boolean Magic (int N)
{    if (N==1)
        Return True
     else if (N==0)
        Return False
     else { N= N-2;
        Return Magic(N);
           };
}
```

A. -3
B. True
C. False
D. Stack Overflow
E. Syntax Error

7. (4%) Which sorting algorithm will produce the following results of pass 0, 1 and 2 for 33, 7, 63, 50, 31, 20?

Pass 0: 7, 33, 63, 50, 31, 20
Pass 1: 7, 20, 63, 50, 31, 33
Pass 2: 7, 20, 31, 50, 63, 33

A. Insertion Sort
B. Selection Sort
C. Bubble Sort
D. Merge Sort
E. Quick Sort

8. (4%) If an array is used to implement a max-heap, where index 1 (the first position) of the array stores the root, what is the maximal index *i* that stores the 3rd ranked element?

A. 3
B. 4
C. 5
D. 6
E. 7

考試科目	資料結構及演算法	系所別	資訊科學系	考試時間	2 月 12 日(三) 第四節

9. (4%) Given an undirected connected graph G of n vertices and m edges, which of the following statement concerning spanning trees is **false**?
   A. The minimum spanning tree should not include the heaviest edge in any cycle of G.
   B. The maximum spanning tree can be found by modifying Kruskal's algorithm to sort edges in descending order by weight.
   C. Each spanning tree contains exactly n-1 edges.
   D. Both Prim's and Kruskal's algorithms are greedy algorithms.
   E. Heap is commonly used to implement Kruskal's Algorithm for finding minimum spanning tree efficiently.
10. (4%) Given an undirected graph of n vertices, m edges and p connected components, what is the minimum number of edges required to make this graph connected?
   A. n-p+1
   B. p-1
   C. n-1
   D. p
   E. n
11. (4%) Which of the following statements concerning topological ordering is **true**?
   A. It is impossible to give the topological ordering for a direct cyclic graph.
   B. The shortest paths from a single source in a weighted DAG can be found using an O(V+E)-time greedy algorithm based on topological ordering, where V is the number of vertices and E is the number of edges.
   C. In a DAG, if there is a path from vertex u to vertex v, then v appears after u in topological ordering.
   D. The longest paths from a single source in a weighted DAG can be found using an O(V+E)-time greedy algorithm based on topological ordering, where V is the number of vertices and E is the number of edges.
   E. All of the above.
12. (4%) Which of the following statements concerning time complexity is false?
   A. The time complexity of rotation operation of an AVL tree of size n is O(log$n$).
   B. The worst-case time complexity of merge sort is O($n$log$n$).
   C. The time complexity of KMP algorithm to search for a pattern of length m in a text of length n is O(n+m).
   D. The time complexity of the dynamic programming approach to find the longest common subsequence between two sequences of length n and m is O(n×m).
13. (4%) Which of the following statements concerning reduction is **false**?
   A. The bipartite graph matching problem can be reduced to the network flow problem.
   B. If a problem X is polynomial reducible to a problem Y, then the lower bound of X can be applied to Y.
   C. A problem X is NP-Complete if X is polynomial reducible to some NP-Complete problem Y.
   D. A problem X is an NP-hard problem if every problem in NP is polynomial reducible to X.

考試科目	資料結構及演算法	系所別	資訊科學系	考試時間	2 月 12 日(三) 第 四 節

14. (4%) Which of the following statements is **false**?
    A. Every problem which can be solved in polynomial time by deterministic algorithms must be a NP problem.
    B. A problem is NP-hard if it is as least as hard as all NP problems.
    C. If any NP-Complete problem can be solved in polynomial time, then all NP-hard problems can be solved in polynomial time.
    D. Every NP-Complete problem must be NP.
15. (4%) Which of the following problems is NP-Hard?
    A. Given an undirected graph, find the maximum number of vertices that form a complete subgraph.
    B. Given a directed network, find the maximum flow from the source to the sink.
    C. Given a bipartite graph, find the maximum number of matching pairs.
    D. Given two sequences, find the common subsequence of maximal length.
16. (20%) Six Degrees of Separation is the theory that any person on the planet can be connected to any other person on the planet through a chain of acquaintances that has no more than five intermediaries on average. For example, in Figure 1, the degree of separation between Gia and Felix is five. They can be connected by at least four intermediaries. The average degree of separation in this social network is about 2.1. Given a dense social network of n persons, design a dynamic programming algorithm to compute the average degree of separation in this social network.
    (1) (10%) Give the algorithm in pseudo code clearly.
    (2) (5%) Give the recurrence relation.
    (3) (5%) Analyze the time complexity of your algorithm.

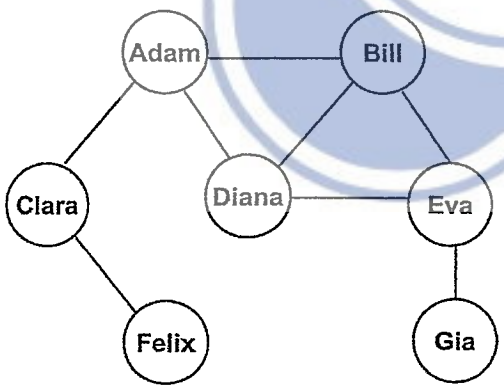

Figure 1

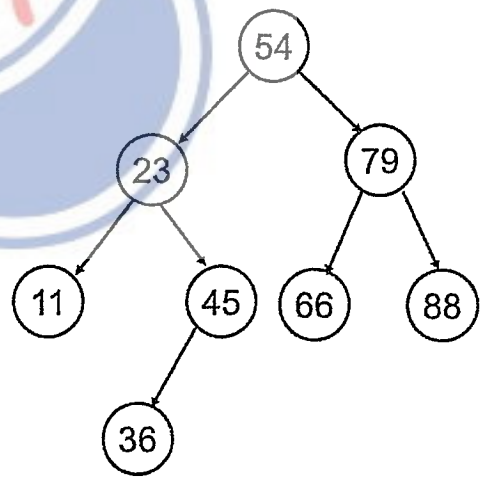

Figure 2

考試科目	資料結構及演算法	系所別	資訊科學系	考試時間	2 月 12 日(三) 第 四 節

17. (20%) Please design an O(log$n$) algorithm to search for the k-th element in an AVL tree of n elements while the time complexity of insertion operation remains O(log$n$).
    (1) (5%) How to modify the data structure of an AVL tree to support this search operation?
    (2) (10%) Give the O(log$n$) algorithm to search for the k-element.
    (3) (5%) How to modify the insertion operation to accommodate for the modification of the data structure. (Please illustrate your answer by using the AVL-tree provided in Figure 2.)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。