

考試科目	計算機數學	系所別	資訊科學系	考試時間	2 月 6 日(二) 第二節
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本次考試共 25 題單選題，每題 4 分。

選擇題請在答案卡上作答，否則不予計分。

1. $\begin{bmatrix} 162 & 5045 \\ 7 & 218 \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} 1 & c \\ 0 & 1 \end{bmatrix}$ and $a, b, c \in \mathbb{R}$, then $a + b + c = ?$

- (A)58 (B)59 (C)60 (D)61

2. How many of the following statements are true?

- If the matrix A is a 7×4 matrix and $\text{rank}(A) = 3$, then the dimension of the row space of A is 4.
- Let A be an $m \times n$ matrix. Then $\text{nullity}(A) \geq n - m$.
- For two squares $n \times n$ matrices A and B , $\text{rank}(AB) = \text{rank}(A)$ if and only if B is nonsingular.
- If the columns of a matrix are dependent, so are the rows of the matrix.

- (A)0 (B)1 (C)2 (D)3 (E)4

3. Which matrix is non-singular?

(A) $\begin{bmatrix} 2 & 4 & 6 & 9 \\ 1 & 3 & 5 & 10 \\ 1 & 3 & 5 & 7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 3 & 7 \\ 3 & 6 & 18 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 5 & 7 & 9 & 1 \\ 3 & 3 & 9 & 12 \\ 4 & 4 & 8 & 12 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 4 & 8 & 8 & 12 \end{bmatrix}$

4. How many of the following statements are true?

- Every $n \times n$ matrix has n eigenvalues and n eigenvector.
- If A is an $n \times n$ matrix, then the rank of A is equal to its nonzero eigenvalues.
- Two eigenvectors corresponding to the same eigenvalue are always linearly dependent.
- If a triangular matrix is similar to a diagonal matrix, it is already diagonal.

- (A)0 (B)1 (C)2 (D)3 (E)4

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5. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Find the eigenvalues of $A^3 + A^2 + A + I$.

(A) $5 + 2\sqrt{5}, 5 - 2\sqrt{5}$

(B) $0, 1$

(C) $3 + \sqrt{5}, 3 - \sqrt{5}$

(D) $\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

6. How many of the following statements are true?

• $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ is similar to $\begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$.

• If A is idempotent, then $I - A$ is idempotent.

• If A is idempotent, then $(I - 2A)^{-1} = (I - 2A)$.

• A square matrix with linearly independent column vectors is diagonalizable.

(A)0 (B)1 (C)2 (D)3 (E)4

7. How many of the following statements are true?

• If u is orthogonal to v and w , then u is orthogonal to $v + w$.

• If u is orthogonal to $v + w$, then u is orthogonal to v and w .

• If $\{u, v\}$ is an orthonormal set in V , then $\|u - v\| = \sqrt{2}$.

• Every orthonormal set of V must be linear independent.

(A)0 (B)1 (C)2 (D)3 (E)4

8. How many of the following statements are true?

• If A^T is orthogonal, A is also orthogonal.

• If A is an orthogonal matrix, then A is symmetric.

• If A has orthonormal columns, then $AA^T = I$.

• Let A be a positive-definite matrix in $R^{n \times n}$. Then $A + I_n$ is a positive-definite matrix.

(A)0 (B)1 (C)2 (D)3 (E)4

9. Let bases $E = \{v_1, v_2\} = \left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 13 \end{bmatrix} \right\}$ and $F = \{u_1, u_2\} = \left\{ \begin{bmatrix} 6 \\ 1 \end{bmatrix}, \begin{bmatrix} 17 \\ 3 \end{bmatrix} \right\}$. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the transition

matrix from E to F and $a, b, c, d \in \mathbb{R}$. Please determine the value of $a + b + c + d$.

(A)-182 (B)-184 (C)-190 (D)-194

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10. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$. What is the sum of the eigenvalues of A ?

- (A)0 (B)2 (C)4 (D)6

11. Let set $A = \{5,6,7,8\}$. Define a relation R on A where $R = \{(5,7), (5,8), (6,7), (6,8), (7,5), (7,8)\}$, which of the following properties does this relation have?

- (1) Symmetric
 (2) Asymmetric
 (3) Antisymmetric
 (4) Reflexive
 (5) Irreflexive
 (6) Transitive

- (A)(1), (4), (6) (B)(2), (5), (6) (C)(3), (5) (D)(5)

12. $29^{322} \equiv a \pmod{17}$, find a .

- (A)8 (B)9 (C)11 (D)12

Questions 13-16 below constitute a set. Please consider them as one large question and answer accordingly.

Solve the linear recurrence relation $a_n - 3a_{n-1} + 2a_{n-2} = -5n + 3$ with $a_0 = 2$ and $a_1 = 8$, and let $a_n = b * 2^n + cn^2 + dn + e$.

13. $b = ?$ (A)-3 (B)-4 (C)-5 (D)-6

14. $c = ?$ (A) $\frac{3}{2}$ (B) $\frac{5}{2}$ (C) $\frac{7}{2}$ (D) $\frac{9}{2}$

15. $d = ?$ (A) $\frac{15}{2}$ (B) $\frac{17}{2}$ (C) $\frac{19}{2}$ (D) $\frac{21}{2}$

16. $e = ?$ (A)8 (B)9 (C)10 (D)11

17. Find the number of ways to make change for \$100 using \$10, \$20 and \$50 bills.

- (A)8 (B)9 (C)10 (D)11

Questions 18-20 below constitute a set. Please consider them as one large question and answer accordingly.

Let $X = \{2,3,4,5,7\}$, $Y = \{4,5,6\}$. For a function f and a set S , define $f(S) = \{f(i) \mid i \in S\}$. Please find the answers for the following questions.

18. How many functions $f: X \rightarrow Y$ are one-to-one?

- (A)0 (B)4 (C)12 (D)24

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19. How many functions $f: X \rightarrow Y$ are there such that $|f(X)| = 3$?

- (A)76 (B)95 (C)150 (D)211

20. How many functions $f: X \rightarrow X$ are there such that $f(\{2,4\}) = \{3,4\}$ and for all $x \in X$, $f(x) \neq 5$?

- (A)64 (B)128 (C)160 (D)240

Questions 21-22 below constitute a set. Please consider them as one large question and answer accordingly.

Suppose the A and B are events in a sample, such that $p(A) = \frac{1}{3}$, $p(B) = \frac{1}{5}$, $p(A|B) = \frac{2}{7}$, and

$$p(B|A) = \frac{a}{b}.$$

21. $a = ?$ (A)3 (B)4 (C)5 (D)6

22. $b = ?$ (A)35 (B)36 (C)37 (D)38

23. Let A, B and C be sets. Which of the following statements are true?

- (A) If $A \in B$ and $B \in C$ then $A \in C$.
 (B) If $A \in B$ and $B \in C$ then $A \subseteq C$.
 (C) If $A \in B$ and $B \subseteq C$ then $A \in C$.
 (D) If $A \in B$ and $B \subseteq C$ then $A \subseteq C$.

24. How many of the following statements are true? Note that by $(a,b)=(c,d)$ we mean $a=c$ and $b=d$.

- $f: N \rightarrow N$, $f(x) = 3^x$ is one-to-one
- $f: N \rightarrow N$, $f(x) = 3^x$ is onto
- $f: N \rightarrow N$, $f(x) = 3^x$ is bijective
- $f: R \rightarrow R^2$, $f(x) = (x^2, -x^2)$ is one-to-one
- $f: N \rightarrow N \times N$, $f(x) = (x, x + 1)$ is injective

- (A)0 (B)1 (C)2 (D)3 (E)4

25. Which of the following function ϕ from $(Z, +)$ to $(Z, +)$ is a homomorphism?

- (A) $\phi(n) = -n$
 (B) $\phi(n) = n^2$
 (C) $\phi(n) = 2n + 1$
 (D) $\phi(n) = 2^n$

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註

- 一、作答於試題上者，不予計分。
 二、試題請隨卷繳交。

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Note: All the questions are single choice, please select the best answer.

Part A: Please select the best term for the description (25%; 2.5% for each question)

選擇題請在答案卡上作答，否則不予計分。

- An algorithm for which can be shown that the set of pages in memory for n frames is always a subset of the set of pages that would be in memory with $n + 1$ frames; never exhibit Belady's anomaly; (a) Queue algorithm (b) Frame algorithm (c) Stack algorithm (d) FILO algorithm
- A technique in which the guest OS is modified to work in cooperation with the VMM to optimize performance (a) Semi-virtualization (b) Emu-virtualization (c) Mono-virtualization (d) Paravirtualization
- A set of properties that a system must satisfy to ensure that processes make progress during their execution life cycle. (a) Safety (b) Liveness (c) Responsiveness (d) Progressiveness
- A computer is overworked with excessive demand paging activities. This situation arises when a system does not have enough frame to make progress. (a) Trashing (b) Stack overflow (c) Page fault (d) Fragmentation
- Parent process terminated before calling `wait()` will leave its child process in which status? (a) Zombie (b) Orphan (c) Daemon (d) Empty
- Perform more than one task simultaneously (a) Concurrency (b) Parallelism (c) Splitting (d) Batching
- Perform the same task on different data at the same time (a) Task Parallelism (b) Data Parallelism (c) Task Splitting (d) Data Batching
- Creation and management of threads done by compilers and run-time libraries rather than programmers (a) Automatic Scheduling (b) Dispatching (c) Implicit Threading (d) Explicit Threading
- A register to indicate the size of the page table (a) PTBR (b) PTSW (c) PTLR (d) PTSR
- A type of page table has one entry for each frame of real memory (a) Hierarchical Page Table (b) Inverted Page Table (c) TLB (d) Hashed Pate Table

Part B: Please select the best answer (75%; 3% for each question)

- Considering the algorithms for page replacement, which of the following is true? (a) the second chance algorithm uses an ordered pair (used, modified) to select the victim (b) the additional-reference-bits algorithm gains additional ordering information by recording the reference bits at regular intervals (c) LRU algorithm is optimal (d) Belady's algorithm suffers from Belady's anomaly
- Which of the following statement is true? (a) Xen is type 1 hypervisor (b) Microsoft HyperV is type 2

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<p>hypervisor (c) Oracle Virtual Box is type 3 hypervisor (d) IBM LPARs is type 0 hypervisor</p>					
<p>13. Apache Kafka is a high-performance message broker, capable of processing messages at a rate of 200MB per second. Consider a system equipped with 32GB of memory. In this setup, Kafka allocates 2GB for the JVM heap, while the remaining 30GB is utilized as a cache for message logs. The memory access time is specified as 200ns, and the combined miss penalty and disk I/O time for the message cache is 8ms (where 1ms equals 10^6 ns). Given these parameters, identify which of the following statements is false:</p>					
<p>(a) The optimal effective message access time for the overall system is 200ns. (b) The effective access time of the system is 8000ns if the cache hit rate is 99.9%. (c) The effective access time of the system is 8ms if no messages are stored in the memory. (d) Generally, increasing the available memory improves the access time.</p>					
<p>14. Which of the following statements is false? (a) Copy-On-Write (COW) allows a parent and its child process to share identical pages until any page is modified. (b) The <code>vfork()</code> function is an implementation of COW strategy. (c) The <code>vfork()</code> function suspends the parent process until the child process is created. (d) The primary purpose of COW is to optimize memory usage.</p>					
<p>15. Which of the following statements about the implementation of processes in Linux is false? (a) The Process Control Block (PCB) is implemented using <code>mm_struct</code>. (b) The pointers <code>brk</code> and <code>start_brk</code> store the addresses marking the start and end of a process's heap segment. (c) The decompression logic for the Linux kernel code is included within the kernel's archive bundle. (d) "systemd" is the first process executed following system boot.</p>					
<p>16. Which of the following is not a programming model of GPGPU? (a) CUDA (b) OpenCL (c) OpenACC (d) Open ML</p>					
<p>17. Which of the following is correct about Direct Communication? (a) processes must name each other explicitly (b) Messages are exchanged via mailboxes (c) Link established only if processes share a common mailbox (d) Mailbox can be owned either by OS or processes</p>					
<p>18. Which of the following statement about Interrupt is false? (a) Interrupt chaining refers to the design that some interrupt vector items points to the head of an ISR list (b) Modern OS tends to keep the interrupt vector small (c) Maskable interrupts can be turned off by CPU temporarily (d) Bottom half interrupt should be processed as soon as possible</p>					
<p>19. When considering Docker's layered file system, which of the following statements is true? (a) Each modification to an existing image creates a new layer in the file system, but all layers are merged into a</p>					

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<p>single layer after a container is restarted. (b) Docker images are composed of multiple layers, and when a container is run from an image, a new writable layer is added on top of the existing layers. (c) All file system layers in Docker are read-only, and changes made in one container are immediately reflected in all other containers using the same image. (d) the layered file system of Docker originates from HDFS.</p>					
<p>20. Which of the following component cannot be accessed by the CPU directly? (a) register (b) disk (c) L1/L2 cache (d) Main Memory</p>					
<p>21. Which of the following CPU scheduling algorithm may suffer from convoy effect? (a) SJF (b) FCFS (c) RR (d) Multilevel queue</p>					
<p>22. In demand paging, (a) a page loaded in memory may never be accessed. (b) all pages that a program will access during execution are loaded in memory in the beginning. (c) a page is loaded in memory only when it is needed during execution. (d) a page is removed from memory when it is not cached.</p>					
<p>23. Which of the following statement is correct? (a) Base register holds the size of a page table. (b) Limit register holds the size of a process. (c) Base and limit registers can be loaded in user mode. (d) Any attempt by a user program to access memory at an address higher than the base register value results in a trap to the OS.</p>					
<p>24. A large page size results in (a) lower external fragmentation (b) more page faults (c) lower internal fragmentation (d) efficient disk I/O</p>					
<p>25. Consider a computer with 64-bit logical address with single-level paging. Assuming that the page size is 8M and the memory is byte-addressed. The physical memory is 32G. Which of the following is wrong? (a) The maximum number of pages a process can have is 2^{41} (b) The number of bits for physical memory is 35 (c) The maximum number of frames is 2^{23} (d) the number of offset bits in page table is 23</p>					
<p>26. Which of the following is not true about Monitor? (a) a high-level construct for synchronization (b) only the process in the wait set can get the object lock (c) Only one process may be active within the monitor at a time (d) Every Java object has associated with it a single lock</p>					
<p>27. The CFS scheduler uses which data structure to keep track of runnable task (a) hash table (b) queue (c) merkle tree (d) red-black tree</p>					
<p>28. In Linux, the number of locks being held by a task is called (a) mutex_lock_count (b) preempt_count (c) lock_count (d) none of the above</p>					
<p>29. What is false about the concept of a monitor? (a) A monitor is a low-level synchronization mechanism that relies on the use of semaphores. (b) When a thread calls notify(), an arbitrary thread is selected and move</p>					

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<p>from the wait set to the entry set (c) Similar to a class in OO language (d) Only one process may be active within the monitor at a time</p> <p>30. Which of the following technology reduces the overhead of CPU for bulk data movement? (a) DMA (b) NUMA (c) SMP (d) SAN</p> <p>31. Banker's algorithm is use do detect safe sequences of threads. It belongs to which of the following strategies? (a) Deadlock avoidance (b) Deadlock prevention (c) Deadlock recovery (d) Deadlock handing</p> <p>32. Which of the following is false? (a) When the CPU receives a non-maskable interrupt, it stops what it is doing and immediately transfers execution to the interrupt handler (b) I/O controllers emit interrupts via interrupt-request lines (c) In critical sections, the CPU may disable the maskable interrupts permanently (d) In an Intel processor, the interrupt No. 32 is a maskable interrupt</p> <p>33. Which of the following is false about Java Callable and Executor mechanisms? (a) Callable allows for threads to return results (b) Executor provides a mechanism for communication between concurrent tasks (c) Executor integrates thread creation and thread execution (d) The execution of Executor based on the producer/consumer model.</p> <p>34. In a computer system with a 32-bit virtual address space utilizing single-level paging, assume the page size is 4KB and memory is byte-addressable. Which of the following statements is false? (a) If the maximum physical memory size is 32GB, then the number of bits required for physical addresses is 35. (b) A process can have up to 2^{20} pages. (c) The maximum number of frames in the system is 2^{23}. (d) Given a TLB hit rate of 79% and memory access and TLB access times of 100ns and 20ns respectively, the effective memory access time can be less than 140ns.</p> <p>35. Which of the following true? (a) The binder framework is an IPC mechanism supported by iOS (b) In Mach Message Passing, the Notify port is used to send messages to the kernel (c) In Mach Message Passing, the section object is used to transfer large objects (d) In Mach Message Passing, the Bootstrap server is used to host a set of registered bootstrap ports.</p>					
備註	<p>一、作答於試題上者，不予計分。 二、試題請隨卷繳交。</p>				

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For the first 10 questions, there is no need to explain your answer.

- (4%) Which of the following statements is wrong:
 - $n \log n = \Omega(n^{1.001})$
 - $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{n} = O(\log n)$
 - $\log(n!) = \Theta(n \log n)$
 - $1 + 2 + 4 + 8 + 16 + \dots + 2^k = \Theta(2^k)$
- (4%) Which of the following sorting algorithms has a best-case time complexity of $O(n)$?
 - Merge sort
 - Insertion sort
 - Quick sort
 - Heap sort
- (4%) Compare singly linked lists and doubly linked lists, assuming both maintain pointers to the first and last nodes. Which of the following operations is faster on doubly linked lists?
 - Add a node to the front of the list (the new node becomes the first node).
 - Add a node to the end of the list (the new node becomes the last node).
 - Delete the first node on the list.
 - Delete the last node on the list.
- (4%) There are various designs for min heaps (e.g., binary heap, binomial heap, and Fibonacci heap), and they all support ExtractMin, DecreaseKey, and Insert. Which of the following statements about min heaps is correct:
 - DecreaseKey always takes $\Theta(\log n)$ time in a binary heap.
 - Insert always takes $O(1)$ time in a binary heap.
 - It is possible to design a min heap such that both DecreaseKey and Insert always take $O(1)$ time.
 - It is possible to design a min heap such that both ExtractMin and Insert always take $O(1)$ time.
- (4%) Consider an undirected graph stored in an adjacency matrix. Given a vertex v in the graph, what is the time complexity of computing v 's degree (i.e., the number of v 's neighbors in the graph)?
 - $O(1)$
 - $\Theta(\log n)$
 - $\Theta(n)$
 - $\Theta(v$'s degree)

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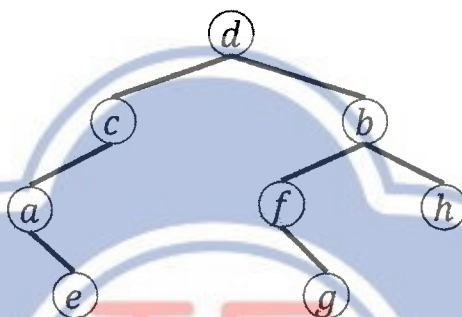
6. (4%) Which of the following statements about binary search trees is correct?
- A. Searching a data in a binary search tree can be done in $O(\log n)$ time, regardless of the tree height.
 - B. Sorting all the data in a binary search tree can be done in $O(n)$ time, regardless of the tree height.
 - C. Finding the smallest data in a binary search tree can be done in $O(\log n)$ time, regardless of the tree height.
 - D. None of the above statements is correct.
7. (4%) Compare hash tables and balanced binary search trees. Which of the following operations is faster on a balanced binary search tree?
- A. Insert a data.
 - B. Search a given data.
 - C. Range query.
 - D. None of the above.
8. (4%) In the context of computational complexity, which of the following best describes a pseudo-polynomial time algorithm?
- A. An algorithm whose running time is polynomial in the size of the input.
 - B. An algorithm whose running time is polynomial in the numeric value of the input, often involving dynamic programming.
 - C. An algorithm that runs in logarithmic time.
 - D. An algorithm with linear time complexity, solving problems efficiently regardless of the input size.
9. (4%) Assume that all edges have distinct weights. Which of the following statements about minimum spanning tree is wrong?
- A. The minimum spanning tree cannot include the edge with the largest weight.
 - B. The minimum spanning tree must include the edge with the smallest weight.
 - C. The minimum spanning tree must include the edge with the second smallest weight.
 - D. The minimum spanning tree is unique.
10. (4%) Which of the following problems is NP-hard?
- A. Find the minimum spanning tree in a connected graph.
 - B. Find the maximum spanning tree (i.e., the spanning tree that has the largest total edge weight) in a connected graph.
 - C. Find the shortest cycle in a graph (assume that every edge has a positive weight).
 - D. Find the longest cycle in a graph.

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11. (15%) Find an asymptotically tight solution to the following recurrence WITHOUT the master theorem. Explain the logic that you used to solve the problem.

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ 6T\left(\frac{n}{2}\right) + \Theta(n^2) & \text{if } n > 1 \end{cases}$$

12. (10%) Consider the following binary search tree, which stores keys a, b, c, d, e, f, g , and h . Sort these keys in non-decreasing order. For example, if you think $a \leq b \leq c \leq d \leq e \leq f \leq g \leq h$, simply write a, b, c, d, e, f, g, h .



13. (10%) Consider the following task: given a sorted doubly linked list (i.e., the i th node in the list stores the i th smallest data in the list) and a number x , determine if x is stored in the sorted doubly linked list. Is it possible to accomplish this task in $O(\log n)$ time using binary search? Please explain your answer.
14. (10%) Consider the following task: given n numbers d_1, d_2, \dots, d_n , compute the average of these n numbers. A straightforward approach is to directly compute $\frac{d_1 + d_2 + \dots + d_n}{n}$. However, this may cause overflow because $d_1 + d_2 + \dots + d_n$ may overflow. Design an algorithm to compute the average without causing overflow. You can assume that each number alone does not cause overflow.
15. (15%) A B-tree can be parameterized by the minimum degree t . Every node other than the root must store at least $t - 1$ keys.
- (5%) Explain the advantage of B-trees over red-black trees when the parameter t is set appropriately.
 - (5%) True or false: the height of a B-tree decreases as t increases. Please explain your answer.
 - (5%) True or false: the time complexity of insertion decreases as t increases. Please explain your answer.

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註

- 一、作答於試題上者，不予計分。
- 二、試題請隨卷繳交。