國立政治大學　112　學年度 碩士班暨碩士在職專班 招生考試試題

考試科目	計算機數學	系所別	資訊科學系 資訊安全碩士學位學程	考試時間	2 月 3 日(五) 第 二 節

本次考試共 25 題單選題，每題 4 分。　　選擇題請在答案卡上作答，否則不予計分。

1. If $\begin{bmatrix}11 & 5\\35 & 16\end{bmatrix} = \begin{bmatrix}1 & 0\\a & 1\end{bmatrix}\begin{bmatrix}1 & b\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\c & 1\end{bmatrix}$ and $a, b, c \in R$, then $a+b+c = ?$

(A) 7　(B) 8　(C) 9　(D) 10

2. How many of the following statements are true?

- If E is an elementary matrix, then $\det(E) = \pm 1$.
- For any $A, B \in M^{n\times n}(F)$, $\det(AB) = \det(A)\cdot\det(B)$.
- A matrix $A \in M^{n\times n}(F)$ has rank n if and only if $\det(A) \neq 0$.
- For any $A \in M^{n\times n}(F)$, $\det(A^t) = -\det(A)$.

(A)0　(B)1　(C)2　(D)3　(E)4

3. Let A be an $m \times n$ matrix whose null space has dimension k. Which conclusion is correct?

(A) The dimension of $NULL(A^T)$ is k.
(B) The dimension of row space of A is $m-k$.
(C) The dimension of column space of A is $m-k$.
(D) The dimension of row space of A is $n-k$.

4. How many of the following vector functions are linear transformations?

- $T_1\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}x^2\\x+y\\y^2\end{bmatrix}$
- $T_2\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right) = \begin{bmatrix}x+y\\x+y+z\\0\end{bmatrix}$
- $T_3\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right) = \begin{bmatrix}e^{x+y}\\\sqrt{y}\end{bmatrix}$
- $T_4\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}x+y\\10\end{bmatrix}$

(A)0　(B)1　(C)2　(D)3　(E)4

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5. How many of the following statements are true?

- The Gram–Schmidt orthogonalization process allows us to construct an orthonormal set from an arbitrary set of vectors.
- An orthonormal basis must be an ordered basis.
- Every orthogonal set is linearly independent.
- Every orthonormal set is linearly independent

(A)0　(B)1　(C)2　(D)3　(E)4

6. Let $A = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$, please find A^{100}

(A) $\begin{bmatrix} -4^{100} & 1-4^{100} \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4^{100} & 1-4^{100} \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 4^{100} & 1-4^{100} \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 4^{100} & 1+4^{100} \\ 0 & 1 \end{bmatrix}$

7.How many of the following statements are true?

- Every linear operator on an n-dimensional vector space has n distinct eigenvalues.
- Any two eigenvectors are linearly independent.
- Similar matrices always have the same eigenvalues.
- Similar matrices always have the same eigenvectors.

(A)0　(B)1　(C)2　(D)3　(E)4

For problems 8-10, please find a singular value decomposition for the following matrix.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = U\Sigma V$$

8. $U = ?$

(A) $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ (C) $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

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9. $\Sigma = ?$

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$ (B) $\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$ (C) $\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$ (D) $\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

10. $V = ?$

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} \\ 0 & \frac{3}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{-3}{\sqrt{2}} \end{bmatrix}$

11. Determine whether each of these compound propositions is satisfiable.

(1) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$.

(2) $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$.

(3) $(p \vee q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$.

(A)(1)　(B)(1), (2)　(C)(2), (3)　(D) (1), (3)

12. Let $S = \{a, \{a\}, \phi, \{\phi\}\}$, and $P(S)$ denote the power set of S. How many of the following statements are true?

- $a \in S$
- $\{a\} \subseteq S$
- $\{\{a\}\} \subseteq S$
- $\phi \in S$
- $\phi \subseteq S$
- $\phi \in P(S)$
- $\phi \subseteq P(S)$
- $\{\phi\} \in P(S)$
- $\{\phi\} \subseteq P(S)$

(A)6　(B)7　(C)8　(D) 9

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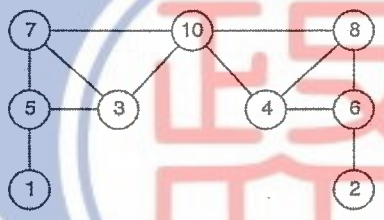
13. Let set $A = \{1,2,3,4\}$. Define a relation of R of A as $R =$ $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$. Which of the following properties does this relation have?

(1) symmetric
(2) asymmetric
(3) antisymmetric
(4) reflexive
(5) irreflexive
(6) transitive

(A)(1), (4), (6)　(B)(2), (5), (6)　(C)(3), (5)　(D)(5)

14. Consider a graph



How many of the following statements are true?

- It is bipartite.
- It has the longest simple path of length 8.
- It has an Euler circuit.
- It doesn't have an Euler circuit.

(A)0　(B)1　(C)2　(D)3

15. How many of the following statements are true?

- A graph G has a spanning tree if G is connected.
- A graph $G = (V, E)$ with $|E| = m$ satisfying $2m = \sum_{v \in V} \deg(v)$.
- A graph $G = (V_1, V_2, E)$ is bipartite, when G has a Hamilton cycle, $|V_1| = |V_2|$.
- A graph $G = (V_1, V_2, E)$ is bipartite, when G has a Hamilton cycle, $\big||V_1| - |V_2|\big| \leq 1$.

(A)1　(B)2　(C)3　(D)4

For problems 16-18, please solve the linear recurrence relation $a_n + 6a_{n-1} + 9a_{n-2} = (-3)^n$ with $a_0 = 2$ and $a_1 = 3$, and let $a_n = (i + jn + kn^2) \cdot (-3)^n$.

16. i =? (A)1 (B)2 (C)-2 (D)3

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17. j =? (A)$-\frac{7}{2}$ (B) $\frac{7}{2}$ (C) $-\frac{5}{2}$ (D) $\frac{5}{2}$

18. k =? (A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

For problems 19-20, please find $X = 101^{-1}$ modulo 4620.
Let $X = 100 \cdot a + b$.
19. a =? (A)15 (B)16 (C)17 (D)18
20. b =? (A)0 (B)1 (C)2 (D)3

For problems 21-22, suppose E and F are events in a sample space with $p(E) = \frac{1}{3}$, $p(F) = \frac{1}{2}$, and $p(E|F) = \frac{2}{5}$. Find $p(F|E) = \frac{a}{b}$.

21. a =? (A)1 (B)2 (C)3 (D)4
22. b =? (A)2 (B)3 (C)4 (D)5

23. Which the following statement is false?
(A) If NFA with k states accepts any character at all, then it cannot accept a string of length $< k$
(B) The set for all the string that does not belong to a particular regular language L, is also a regular language
(C) The result of subset operation of a regular language set can still be regular
(D) Any kind of NFA can always convert to a DFA

24. Let N be an NFA with n states, let k be the number of states of a minimal DFA which is equivalent to N. Which one of the following is necessarily true?
(A) $k \geq n^2$
(B) $k \geq 2^n$
(C) $k \leq n^2$
(D) $k \leq 2^n$

25. Which of the following is not context-free language?
(A) $L1 : \{ 0^p 1^q 0^r \mid p = q \text{ and } pqr \geq 0\}$
(B) $L2 : \{ 0^p 1^q 0^r \mid p = q = r \text{ and } pqr \geq 0\}$
(C) $L1 : \{ 0^p 1^q 0^r \mid p = q \text{ or } q = r \text{ and } pqr \geq 0\}$
(D) all of above are context-free language

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	作業系統	系所別	資訊科學系/資訊科學與工程組、智慧計算組	考試時間	2 月 3 日(五) 第 3 節

選擇題請在答案卡上作答，否則不予計分。

Note: All of the questions are single choice, please select the best answer.

Part A (25%; 2.5% for each question)

1. The amount of memory accessible from the TLB is called (a) TLB scope (b) TLB domain (c) TLB coverage (d) TLB reach

2. A _____ occurs when a process does not have a logical mapping to a page, yet that page is in memory. (a) major page fault (b) minor page fault (c) TLB miss (d) page invalid

3. The violation of unauthorized destruction of data is called (a) Breach of confidentiality (b) Breach of integrity (c) Breach of availability (d) Denial of service

4. _____ is a technique in which the guest OS is modified to work in cooperation with the VMM to optimize performance. (a) Para-virtualization (b) Emulator (c) Type-1 Hypervisor (d) Type-2 Hypervisor

5. _____ is usually used to deal with the noncontiguous external fragmentations problem. (a) TLB walk (b) PTBR (c) Compaction (d) MMU

6. _____ is to move entire processes between main memory and a backing store (a) Paging (b) Swapping (c) Context Switching (d) Thrashing

7. While a process is in its critical section, any other process that tries to enter its critical section must loop continuously in the call to acquire the lock is called a _____. (a) Trap (b) Interrupt (c) Spinlock (d) Monitor

8. Which of the following is not a necessary conditions of deadlock? (a) mutual exclusion (b) circular wait (c) preemptive (d) Hold and Wait

9. _____ is a mechanism used to address the difficulties of multithread programming by transferring the creation and management of threading from developer to run-time libraries. (a) Kernel Thread (b) Implicit Threading (c) Thread Local (d) Asynchronous Threading

10. Which of the following are shared among all threads belonging to the same process? (a) files (b) stack (c) program counter (d) registers

Part B (75%; 3% for each question)

11. _____ never exhibits Belady's anomaly. (a) FIFO Algorithm (b) Stack Algorithm (c) LRU-Approximation Page Replacement Algorithm (d) MFU Algorithm

12. Which of the following technique is used to prevent unnecessary write back when page replacement

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occurs? (a) TLB (b) DMA (c) valid bit (d) modify bit

13. Which of the following statement is false? (a) Mobile operating systems typically do not support swapping. Instead, they use an alternative technology called compressed memory. (b) vfork() does not use copy-on-write (c) The effective access time of Demand Paging is directly in proportional to the page-fault rate (d) Typically, counting-based page preplacement algorithms approximate optimal algorithm well.

14. Why the Linux NOOP scheduler use FCFS policy for the read request for NVM devices? (a) The read service time is uniform for NVM devices. (b) Most of the read requests will be cached (c) Advanced policy is too complex to be implemented as hardware (d) none of the above

15. Which of the following statement is correct (a) The main drawback of Inverted Page Table is the overhead of pointers in its entries (b) The main drawback of Hierarchical Paging is hard to support shared paging (c) Inverted Page Table reduces the amount of memory needed to store the page table; but it increase the searching time of the page table. (d) Hierarchical Paging is appropriate for 64-bit architectures.

16. Consider a computer with 64-bit logical address with single-level paging. Assuming that the page size is 16M and the memory is byte-addressed. The physical memory is 64G. Which of the following is wrong? (a) The maximum number of pages a process can have is 2^40 (b) The number of bits for physical memory is 36 (c) The maximum number of frames is 2^24 (d) the number of offset bits in page table is 24

17. Assuming that there are totally 6 processes in memory; their working-set sizes are represented as a tuple: (2,5,6,4,4,3). Currently, there are 200 available frames. Which of the following is true? (a) OS will increase the degree of multi-programming (b) Σ WSSi=4 (c) thrashing will happen (d) WSSi=24

18. If the base register is loaded with value 12345 and limit register is loaded with value 1001, which of the following memory address access will result in a trap to the operating system? (a) 13346 (b) 12345 (c) 13345 (d) 12346

19. Given that physical memory is 256KB and is partitioned into 8 frames. If logical memory is 5MB, what is the number of pages needed in virtual memory? (a) 160 (b) 320 (c) 640 (d) 80

20. Which of the following is a correct sequence of using semaphore? (a) signal → signal (b) wait →signal (c) signal →wait (d) wait →wait

21. Which of the following statement is false? (a) Race conditions can result in corrupted values of shared data (b) The preemptive kernel may be more responsive than non-preemptive kernel. (c) Peterson's solution works on modern computer architectures (d) Semaphores and mutex locks both provide mutual exclusion.

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22. Which of the following is false? (a) POSIX Mutex Lock fulfills mutual exclusion and bounded waiting condition (b) Semaphore fulfills mutual exclusion, bounded waiting, and progress condition (c) Semaphore has a waiting queue associated with it (d) When a process calls the wait() operation and finds that the semaphore value is not positive, it will suspend itself

23. Consider DISK scheduling algorithms for HDD. Which algorithm is best for heavily loaded system (a) FCFS (b) SSTF (c) SCAN (d)C-SCAN

24. Which of the following statement is true (a) Dynamic loading does not need special support from OS (b) Dynamic linking loads partial program into memory when it is needed (c) Dynamic linking is particularly useful when large amounts of code are infrequently used (d) Dynamic loading prevents duplicated code

25. The reason that the interrupt vector is indexed by numbers is to increase _____. (a) Stability (b) Scalability (c) Security (d) Speed

26. Which of the following statement is false? (a) An unsafe state may lead to a deadlocked state. (b) A deadlocked state is an unsafe state. (c) At a specific time, a system in the safe state may be in a deadlocked state. (d) A system in an unsafe state at one time may be in a safe state at another time

27. Which of the following statement is false? (a) Concurrency means more than one task is making progress (b) Even single core processor can achieve concurrency (c) Data parallelism is to perform the same task on different data (d) It is impossible to realize concurrency without parallelism

28. Given the following code segment, what is the correct execution sequence for the parent process? (a) xyw (b) xz (c) xy (d) xw

```
int main()
{
        pid_t pid
        pid = fork();  // (x)

        if (pid < 0) {
                // (y)
                return 1;
        }
        else if (pid != 0) {
                // (z)
        }
        else {
              // (w)
        }

  return 0;
}
```

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29. Which of the following statements is correct? (a) The count++ statement in a C program is atomic. (b) Race condition occurs when several processes access and manipulate the same data concurrently (c) On multiple-processor machine, we could simply disable the interrupts to prevent race condition (d) Memory barrier is a solution to the critical section problem

30. To boot a windows system, the code located at the firmware direct the system to read the boot code from ____, which is the first logical block in a hard disk or the first page in a NVM (e.g. SSD) (a) BOOTR (b) BOOTD (c) MBD (d) MBR

31. Which of the following does not affect the context switch time? (a) disk size (b) number of registers (c) special instructions (d) memory speed

32. Which of the following statements is false? (a) Darwin serves as the foundation of Mac OS and iOS (b) Mac OS uses BSD system calls (c) Mac OS uses a pure microkernel architecture (d) message passing in Darwin does not require copying of message

33. In Google Chrome, there are different types of processes, which of the following process type renders web pages, deals with HTML, JavaScript? (a) browser process (b) renderer process (c) plug-in process (d) page process

34. Given the following C code segment, which of the following technologies is (most possibly) used by this program? (a) shared memory (b) pipe (c) RPC (d) queue

```
int main() {
…
close(fd[0]);
write(fd[1], “test”, 5);
close(fd[1]);
…
}
```

35. Linux and Windows operating systems implement which threading model? (a) one-to-one (b) Many-to-one (c) Many-to-Many (d) One-to-Many

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	資料結構及演算法	系所別	資訊科學系/資訊科學與工程組、智慧計算組	考試時間	2月3日(五)第四節

1. (60%) Select the best answer. 選擇題請在答案卡上作答，否則不予計分。

Unless otherwise stated, n is the number of data in a data structure.

1) (5%) Consider a linked list in which each node stores an integer.

Which of the following statements is true?

A. If the list is a doubly linked list, then inserting a node to the front of the list takes $O(1)$ time.

B. If the list is a singly linked list, then deleting a given node from the list takes $O(1)$ time.

C. If the list is a doubly linked sorted list (i.e., the ith node contains the ith smallest integer), then searching an integer in the list takes $O(\log n)$ time.

D. If the list is a singly-linked sorted list, then searching an integer in the list takes $O(\log n)$ time.

2) (5%) Let `arr` be an array that contains n (n>3) integers. Moreover, `arr[0]=arr[1]=arr[2]=n+1` and `arr[i]=i` for every $3 \leq i \leq n-1$.

What is the time complexity of sorting `arr` in non-decreasing order using insertion sort?

A. $\Theta(n^3)$.

B. $\Theta(n^2)$.

C. $\Theta(n \log n)$.

D. $\Theta(n)$.

3) (5%) Which of the following statements is true?

A. The worst-case time complexity of counting sort is $\Theta(n \log n)$.

B. The worst-case time complexity of quick sort is $\Theta(n \log n)$.

C. The average-case time complexity of quick sort is $\Theta(n \log n)$.

D. The best-case time complexity of counting sort is $\Theta(n \log n)$.

4) (5%) Consider a variant of merge sort where merging two sorted arrays of sizes n_a and n_b $(n_a \geq n_b)$ takes $\Theta(n_a^2)$ time in the worst case. Let $T(n)$ be the worst-case time complexity of this variant of merge sort. Which of the following recurrence relations is true?

A. $T(n) = T(n/2) + \Theta(n)$.

B. $T(n) = 2T(n/2) + \Theta(n)$.

C. $T(n) = T(n/2) + \Theta(n^2)$.

D. $T(n) = 2T(n/2) + \Theta(n^2)$.

5) (5%) Consider again the above variant of merge sort. What is the worst-case time complexity?

A. $\Theta(n \log n)$.

B. $\Theta(n^2)$.

C. $\Theta(n^2 \log n)$.

D. $\Theta(n^3)$.

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6) (5%) Which of the following statements about binary search trees is false?
   A. Tree height is always $\Omega(\log n)$.
   B. Given the root of the tree, there is an $O(n)$-time algorithm that sorts all the data in the tree.
   C. Given the root of the tree and an integer k in $\{1,2,\ldots,n\}$, there is an $O(hk)$-time algorithm that outputs the kth smallest data in the tree, where h is the tree height.
   D. The smallest data is always stored in a leaf node.

7) (5%) Which of the following statements about binary heaps is false?
   A. It is a binary tree.
   B. Given the root of the tree, there is an $O(n)$-time algorithm that sorts all the data in the tree.
   C. Tree height is always $\Theta(\log n)$.
   D. It can be implemented using an array.

8) (5%) Which of the following data structures is best suited for range query (i.e., output all the data in the data structure that are between x and y, where x and y are input numbers)?
   A. Red black tree.
   B. Stack.
   C. Fibonacci heap.
   D. Hash table.

9) (5%) Which of the following statements about hash tables is true?
   A. The best-case time complexity of inserting a key-value pair is $\Theta(\log n)$.
   B. After a hash table is created, the hash function cannot be changed.
   C. The best-case time complexity of searching a key is $O(1)$.
   D. Let h be the hash function. If $x \neq y$, then $h(x) \neq h(y)$.

10) (5%) Which of the following problem is NP-hard?
   A. Find the largest cycle in a graph.
   B. Find the largest component in a graph.
   C. Find the minimum spanning tree in an edge-weighted graph.
   D. Find the maximum spanning tree in an edge-weighted graph
   (i.e., find the spanning tree that has the largest total edge weight).

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11) (5%) Which of the following statements is true?

A. If a problem is NP-hard, then no algorithm can solve the problem.

B. If a problem is NP-complete, then every algorithm needs $\Omega(2^n)$ time to solve the problem in the worst case, where n is the input size of the problem.

C. If there is a polynomial time algorithm that solves some NP-hard problem, then every NP problem has a polynomial time algorithm.

D. If a problem is NP-hard, then it is an NP problem.

12) (5%) Which of the following statements about graphs is false?

A. The sum of the degrees of all vertices is equal to twice the number of edges.

B. Let G be a connected graph, and let n be the number of vertices in G. Then G has at least $n-1$ edges.

C. If a graph is stored in an adjacency matrix, then it takes $O(1)$ time to check whether two given vertices are adjacent or not.

D. Let G be a graph stored in an adjacency matrix. Let d_{max} be the largest vertex degree in G. Then it takes $O(d_{max})$ time to compute the degree of any given vertex.

2. (20%) In the knapsack problem, the input contains n items $1,2,3,\ldots,n$, where item i has weight $w[i]$ and value $v[i]$. The input also contains a number W. All the weights, values, and W are positive integers. The output is a set $S \subseteq \{1,2,3,\ldots,n\}$ such that the total weight of S is at most W (i.e., $\sum_{i\in S} w[i] \leq W$) and the total value of S (i.e., $\sum_{i\in S} v[i]$) is maximized.

Consider the following subproblem:

$K(j,W')$: return the maximum achievable total value under the constraints that only items $1,2,3,\ldots,j$ can be chosen and the total weight of the chosen items is at most W'.

Thus, $K(n,W)$ returns the total value of the optimal solution of the original knapsack problem.

1) (10%) Design a dynamic programming algorithm (in pseudocode) for the knapsack problem based on the above subproblem. In your answer, please state the recursive formula for $K(j,W')$ clearly.

2) (5%) Please analyze the time complexity of your algorithm.

3) (5%) Is your algorithm a polynomial time algorithm? Please explain your answer.

3. (20%) Consider again the knapsack problem.

1) (10%) Design a greedy heuristic (in pseudocode) for the knapsack problem. Please explain why your heuristic can be considered to be greedy.

2) (5%) Please analyze the time complexity of your heuristic.

3) (5%) Does your heuristic always output the optimal solution? If so, please give a proof. If not, please give a toy example to show that your heuristic may not obtain the optimal solution.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。