國立政治大學 111 學年度 碩士暨碩士在職專班 招生考試試題

考試科目	資料結構及演算法	系所別	資訊科學系	考試時間	2 月 9 日(三) 第二節

1. (10%) Give the tight asymptotic bound of the following recurrence: $T(n) = \frac{4}{9}T\left(\frac{3n}{2}\right) + n$

2. (25%) Design *a singly linked list* that supports the following operations in C or pseudo code.
   - 2.1 Initializing an empty linked list
   - 2.2 Inserting a node of value, x, before the first element of the linked list
   - 2.3 Inserting a node of value, x, before the i-th element of the linked list
   - 2.4 Delete the i-th element of the linked list
   - 2.5 Reverse the linked list in the most memory-efficient way.

3. (15%) A *stack* can be implemented by a linked list. Design the following stack operations in C or pseudo code mainly based on the operations of Question 2.
   - 3.1 Inserting (or 'push') an element at the top of the stack
   - 3.2 Removing (or 'pop') an element at the top of the stack
   - 3.3 Getting the value of the element at the top of the stack

4. (10%) Design Depth First Search algorithm, *DFS-search*(G, s), traversing a graph G from a source vertex s using the above stack operations.

5. (10%) Design *insert sort* which sorts the following playing cards according to its number while the suit of card does not matter (不管花色). [HINT: insert sort here is not *stable*.]
   - Input: club11, spade 10, club10, heart 7
   - Output: heart 7, club10, spade 10, club11

6. (10%) Use Kruskal's algorithm to find the minimum spanning tree of the graph, G, with vertices $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and the adjust matrix A.

$$A = \begin{pmatrix} 0 & 10 & 16 & 18 & 13 & 8 \\ 10 & 0 & 14 & 17 & 15 & 9 \\ 16 & 14 & 0 & 9 & 10 & 12 \\ 18 & 17 & 9 & 0 & 9 & 19 \\ 13 & 15 & 10 & 9 & 0 & 11 \\ 8 & 9 & 12 & 19 & 11 & 0 \end{pmatrix}$$, where $A[i, j]$ is the weight of the edge between v_i and v_j

7. (20%) Design an algorithm for the following task and prove its correctness.

   Input: a graph $G(V, E)$

   Task: find the compact set C of G, C is a compact set if $\min\{E(v_i, v_k) | v_i \in C, v_k \in V \backslash C)\} > \max\{E(v_i, v_j) | v_i, v_j \in C\}$ (the smallest external distance of C is still larger than the longest internal distance in C, i.e., the C of Question 6 is $\{v_1, v_6\}, \{v_1, v_2, v_6\}, \{v_3, v_4, v_5\}$)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

國立政治大學 111 學年度 碩士暨碩士在職專班 招生考試試題

考試科目	作業系統	系所別	資訊科學系/資訊科學與工程組、智慧計算組	考試時間	2 月 9 日(三) 第 3 節

1. (40%) Select the best answer

(1) Which of the following statements is false? (a) Bare-Metal VM is more efficient than Hosted VM (b) CP/CMS is the first OS that supports virtualization (c) VMM and hypervisor are synonyms (d) most VMMs run on all hardware platform.

(2) OS does not maintain one copy of ___ for each process? (a) TLB (b) Page Table (c) PTBR (d) PCB

(3) _____ is used to approximate the Working Set? (a) WSS (b) PFF (c) LFU (d) none of the above

(4) What is the advantage of using a solid state disk over a magnetic disk? (a) Fast (b) Cheap (c) Larger (d) more secure

(5) Given: a. the kernel starts systemd; b. the boot firmware loads and runs the boot loader; c. the kernel mounts the root file system; d. the boot loader finds the kernel image on disk, loads into memory, and starts it; e. the kernel initializes the devices and its drivers; The correct order of the booting sequence is: (a) bdeac (b) bdeca (c) abdec (d) abdce

(6) Which of the following is an incorrect transition of process states? (a) running→waiting (b) waiting→running (c) ready→running (d) waiting→ready

(7) Which of the following statements is false? (a) TLB is the cache of page table (b) hierarchical paging can reduce the size of page table (c) the frame table without ASID has to refresh the TLB after each context-switch (d) TLB is part of MMU

(8) Which of the following statements is true? (a) The machine running a blockchain node (miner) is I/O bound (b) the process runs on a soft-affinity system may migrate among CPUs (c) Linux CFS places the runnable tasks in a red-black tree structure (d) the vmstat command is used to count the number of demand paging occurrence of a process

(9) Which of the following statements is false about Docker and Container? (a) the Docker image is built based on dockerfile (b) by default, the network of a container is separated from the network of the host (c) the Docker images are always platform-independent (d) the Docker Hub stores published docker images

(10) Which of the following statements is false? (a) Each process is associated with a PCB (b) In Linux, PCB is usually located at /proc (c) the pointers to the opened files are stored in PCB (d) PCB information such as parent, child, and register values are usually kept in a structure called mm_struct.

考試科目	作業系統	系所別	資訊科學系/資訊科學與工程組、智慧計算組	考試時間	2 月 9 日(三) 第 3 節

2. (25%) Please explain the following terms
   - Copy on write (for process creation)
   - Reentrant Lock
   - Orphan
   - Windows ALPC
   - Safe state (for deadlock)
3. (10%) Please briefly explain the commonality of the following technologies in terms of the core design concept: Git, Union FS, Blockchain, Event Sourcing
4. (10%) What is DMA? How does it help the I/O tasks?
5. (5%) Consider the following configuration. Logical address = 36 bits; page size = 8K bytes; How many page table entries should the OS allocate for the page tables of a 116M bytes process under the one-level paging mechanism?
6. (10%) Please sketch and explain the active_list/inactive_list mechanism used by Linux that implements LRU to select a victim for page replacement

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國立政治大學 111 學年度 碩士暨碩士在職專班 招生考試試題

考試科目	計算機數學	系所別	資訊科學系	考試時間	2月 9日(三) 第4節

I. 離散數學：60% (第 1~9 題)

II. 線性代數：40% (第 10~12 題)

請書寫必要的解題過程，僅提供答案而無必要過程，將無法獲得該題滿分。可使用中文或英文作答，力求書寫工整，如字跡潦草，無法閱讀，將影響評分。

1. (4%) Which of the following pairs of values (n, k) (if any) make the following statement true. If 50 students are distributed into n classrooms, there must be a classroom with at least k students?
   (a) (3, 15), (b) (70, 3), (c) (15, 3), (d) (7, 8), (e) (51, 2)
2. (5%) Which of the following asymptotic notation(s) is/are correct for the equation $3N^3\log(N) + 4N\log(N)$?
   (a) $\Theta(N^2 log(N))$, (b) $O(N^3)$, (c) $\Omega(N^2\log(N))$, and (d) $O(N^3\log(N^2))$.
3. (5%) If A and B are disjoint events, and $P(A) = 0.4$ and $P(B) = 0.2$, what is $P(B|A)$?.
4. (6%) Please use a recursion tree to determine a good asymptotic upper bound on the recurrence
   $$T(n) = 3T(\lfloor \frac{n}{2} \rfloor) + n$$
5. (4%) Please prove that $n(n+1)(n+2)$ is divisible by 3 using mathematical induction.
6. (10%) Suppose that the two continuous random variables X and Z are statistically independent. Please prove that the mean and variance of their sum satisfy (please use definition of the expected value and variance):
   (a) (5%) E[X + Z] = E[X] + E[Z],
   (b) (5%) Var[X + Z] = Var[X] + Var[Z].
7. (6%) Please prove $222|(2^a - 2^b)$ for some positive integers, a and b.
8. (10%) Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$, for $n \geq 2, a_0 = 1, a_1 = 0$.
9. (10%) Find the set of all solutions x to the system of two congruences below:
   $$7x \equiv 14 (\text{mod } 6) \text{ and } 5x \equiv 3 (\text{mod } 6).$$
10. (5%) True or false.
    (a) If a rectangle matrix A is invertible, A could have a zero singular value.
    (b) If a matrix B is similar to A, then B has the same eigenvectors as A.
    (c) Any symmetric matrix is similar to a diagonal matrix.
    (d) Any matrix which is similar to a diagonal matrix is symmetric.
    (e) Every positive definite matrix is nonsingular.
11. (15%) Suppose a rectangle matrix A has full rank.
    (a) (4%) Please derive the best square solution $\tilde{x}$ to $Ax = b$ step by step.
    (b) (6%) Which fundamental subspace associated with A is the projection vector p in, where $p = A\tilde{x}$? Which

考試科目	計算機數學	系所別	資訊科學系	考試時間	2 月 9 日(三) 第 4 節

fundamental subspace is $p-b$ in then?

(c) (5%) Find the projection matrix P onto the column space of $A=\begin{bmatrix}1 & 0\\ 3 & 0\\ 0 & -1\\ 0 & -3\end{bmatrix}$.

12. (20%) Given a recurrence relation: $(M^TM+4I)x_{n+1}=(M^TM-4I)x_n$, where $M\in R^{5\times 3}$ (a real 5×3 matrix) and n is a positive integer.
   (a) (5%)Let $x_n=A^nx_0$. Please find out A (independent of any x_i, $i\in N\cup\{0\}$)
   (b) (5%) Let λ be an eigenvalue of M^TM, corresponding to the eigenvector λ. Please find an eigen pair of A. (an eigen pair means an eigenvector and its associated eigenvalue)
   (c) (5%) Given all the information above, choose all the possible behaviors of x_n for a large n from (1) decaying to zero, (2) oscillating but not growing or decaying in length, (3) going to a nonzero constant vector, and (4) going to a nonzero constant vector. Explain your answers by discussing $A's$ eigenvalue.
   (d) (5%) If $x_1=0$ for a nonzero x_0, please find one singular value of M.

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