國立政治大學 108 學年度 碩士暨碩士在職專班 招生考試試題

考試科目	機率	系所別	資訊科學系/智慧計算組	考試時間	2月18日(星期一) 第四節

1. (20%) 依據表一中的C語言程式作答。表一有兩個欄位，「**行號**」的欄位是為了出題溝通之用，不是程式的本身。「**程式**」欄位則是各行號的C語言程式指令；有一些行是蓄意留白的。

1.1. (3%)依照表一目前的程式，第15行可能會印出甚麼？說明你的答案。

1.2. (3%)依照表一目前的程式，第21行可能會印出甚麼？說明你的答案。

1.3. (3%)如果我們執行表一的程式很多次，第15行所列印的資料是否每一次都會一樣？如果有可能不一樣的話，會是怎樣的不一樣？舉例說明並且說明原因。

1.4. (3%)如果我們執行表一的程式很多次，第21行所列印的資料是否每一次都會一樣？如果有可能不一樣的話，會是怎樣的不一樣？舉例說明並且說明原因。

1.5. (4%)如果我們把第10行的1000000改成1000，對於上面第1.3和第1.4的答案會不會有影響？如果有影響的話是怎樣的影響？舉例說明並且說明原因。

1.6. (4%)如果我們把第19行的0.6改成0.8，對於上面第1.3和第1.4的答案會不會有影響？如果有影響的話是怎樣的影響？舉例說明並且說明原因。

行號	程式
01	`#include <time.h>`
02	`#include <stdlib.h>`
03	
04	`int crossover(double x);`
05	
06	`int main()`
07	`{`
08	`  srand(time(NULL));`
09	`  int i;`
10	`  int repeats=1000000;`
11	`  double sum=0;`
12	`  for (i=0; i<repeats; i++)`
13	`    sum += (double)rand()/(double)RAND_MAX;`
14	
15	`  printf("%6.4f\n",sum/(double)repeats);`
16	
17	`  sum = 0;`
18	`  for (i=0; i<repeats; i++)`
19	`    sum += crossover(0.6);`
20	
21	`  printf("%6.4f\n",sum/(double)repeats);`
22	
23	`  return 0;`
24	`}`
25	
26	`int crossover(double x)`
27	`{`
28	`  if ((double)rand()/(double)RAND_MAX<=x)`
29	`    return 1;`
30	`  else`
31	`    return 0;`
32	`}`

表一：第一題的C語言程式

2. (18%)資料分類(classification)是人工智慧工作的重要項目之一。在比較單純分類工作之中，程式的分類結果只會被劃分為分類正確和分類錯誤兩種。假設我們有某甲、某乙兩個分類器設計者。某甲和某乙測試他們的分類器時，分別使用了1000個和10000個測試案例，某甲和某乙的分類器分別把其中750和7500個案例分類正確(也就是個別有250和2500個分類錯誤)。假設你是競賽的裁判者，從confidence interval的角度來評論是某甲還是某乙的分類器的正確率何者較好？回答下列問題。

2.1. (6%)簡要說明confidence interval的定義和應用的意義。

2.2. (6%)假設我們已經知道這兩個分類器的正確率的variance，說明計算confidence interval時，牽涉到的相關變數和相關影響。

2.3. (6%)從第2.2題的立場出發，以confidence interval 的角度公平地評比某甲和某乙的分類器的品質，兩者有無差異？詳細說明你的判斷。

考試科目	機率	系所別	資訊科學系/智慧計算組	考試時間	2 月 18 日(星期一) 第四節

3. (20%) 我們以 CI(A,B,C)表示當給定 B 前提之下，A 和 C 是條件獨立(conditionally independent)。 這裡的 A、B 和 C，可以代表多個機率變數(random variables)。舉例來說 CI({X_1, X_2},{X_3, X_4},{X_5})表示 "{X_1, X_2} and {X_5} are conditionally independent given {X_3, X_4}"。在以下的題目中，我們定義並且採用以下的簡易寫法。

$$\Pr(x_1, x_2, \cdots, x_n) \equiv \Pr(X_1 = x_1, X_2 = x_2, \cdots, X_n = x_n)$$

在以上所列的簡易寫法之中，X_j 代表某一個機率變數($1 \le j \le n$)，x_j 則是 X_j 數個可能的狀態(state)中的某一個可能的狀態。

我們可以用 chain rule 的方式輕易展開 n 個機率變數的 joint probability distribution，如以下公式所述。

$$\Pr(x_1, x_2, \cdots, x_n) = \prod_{i=1}^{i=n} \Pr(x_i | x_1, x_2, \cdots, x_{i-1})$$

現在假設已知 CI({X_3},{X_1},{X_2})、CI({X_4},{X_1, X_2},{X_3})、CI({X_4},{X_1},{X_2, X_3})、CI({X_3,X_4},{X_1, X_2},{X_5})。證明下列等式。

$$\Pr(x_1, x_2, \cdots, x_5) = \Pr(x_1)\Pr(x_2|x_1)\Pr(x_3|x_1)\Pr(x_4|x_1)\Pr(x_5|x_1, x_2)$$

4. (20%) The probability distribution of the Poisson random variable X, representing the number of outcomes occurring in a given time interval or specific region, is

$$\Pr(x; \mu) = \frac{e^{-\mu}\mu^x}{x!}, x = 0, 1, 2, \ldots,$$

where μ is the average number of outcomes occurring in the given time interval or specified region and e=2.71828... (Euler's number)

證明：the mean and variance of X are both μ

5. (12%) 假設在沒有任何額外資訊的前提之下，某人 P 得到某一病症 D 的機率是 0.2，沒有得到 D 的機率是 0.8。再假設有一檢測，當 P 罹患 D 時，在該檢測呈現陽性的機率為 0.9，呈現陰性的機率為 0.1；又若 P 沒有罹患 D 時，在該檢測呈現陽性的機率為 0.3，呈現陰性的機率為 0.7。在某一次健康檢查時，P 在該檢測中呈現陽性，從機率計算的角度，P 得到 D 的機率比較高還是沒有得到 D 的機率比較高，詳細說明你的答案？

6. (10%) 以 X 代表一個機率變數，且 X 代表的數值不小於零 (X 小於零的機率為零)，假設 X 有兩個可能的機率分布，分別以 f 和 g 代表這兩個機率分布的 probability density function，並且分別以 F 和 G 代表這兩個機率分布的 cumulative distribution function。假設對於 X 任何的值 x，$F(x) \ge G(x)$ 。假設我們必須採用 F 或者 G，選擇之後得到的評價分數就是 X 的期望值，我們應該選擇何者才能提高所得的評價？詳細說明你的選擇的理由。

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	計算機數學	系所別	資訊科學系	考試時間	2月18日(一) 第三節

*作答務必註明題號，請工整書寫必要的解題過程，僅答案而缺乏過程，無法獲得該題滿分。

I. 離散數學 (60%)

1. (6%) Use **resolution** to show that the following compound proposition is not satisfiable.

$$(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$$

2. (6%) Find the transitive closures of the relation, {(1,2), (2,1), (2,3), (3,4), (4,1)}, on {1,2,3,4}.

3. (7%) Please prove $x \in (A - B) \cup (A - C) \Leftrightarrow x \in A - (B \cap C)$ step by step.

4. (7%) What is the big-O time complexity estimation of the following procedure regarding of n?

```
Procedure HANOL(n, A, B, C)
    IF(n <= 1)
        PRINT("Move top disk from " A " to " C)
    ELSE
        HANOL(n-1, A, C, B)
        HANOL(1, A, B, C)
        HANOL(n/2, B, A, C)
```

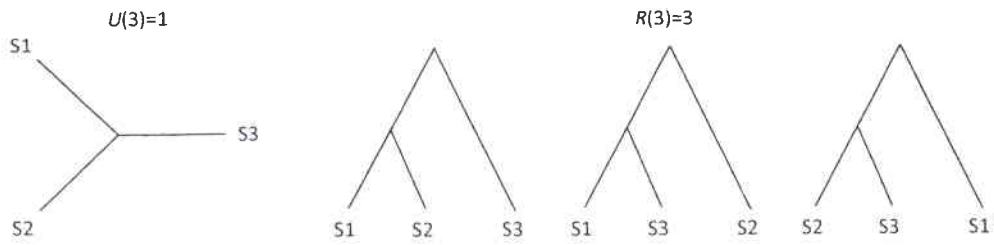
5. Consider a complete undirected graph, G, with vertex set $\{v_1, v_2, v_3, v_4, v_5\}$. Entry w_{ij} in the matrix W below is the weight of the edge $\{v_i, v_j\}$ (i.e., w_{12}=1, w_{13}=2).

$$W = \begin{pmatrix} 0 & 5 & 2 & 3 & 1 \\ 5 & 0 & 1 & 17 & 9 \\ 2 & 1 & 0 & 7 & 19 \\ 3 & 17 & 7 & 0 & 14 \\ 1 & 9 & 19 & 14 & 0 \end{pmatrix}$$

   (a) (5%) What is the *minimum spanning tree* by **Prim's** algorithm with the initial vertex, v_1? Please represented the tree in Adjacency matrix form.

   (b) (5%) What is the minimum spanning tree T in G such that the vertex v_1 is a leaf node in the tree T? Please represented the tree in Adjacency matrix form. [*Hint*: T is different from the answer (a)]

6. An unrooted binary tree is an unrooted tree in which each vertex has either one or three neighbors. A rooted binary tree has a root node and every node has at most two children. Let $U(n)$ and $R(n)$ be the number of unrooted and rooted binary trees with n labeled leaves and unlabeled internal nodes. The following show n=3.



考試科目	計算機數學	系所別	資訊科學系	考試時間	2月18日(一) 第三節

(a) (5%) What is the relation between $U(n)$ and $R(n)$?

(b) (5%) Please solve the $R(n)$ in term of n (find an explicit formula for $R(n)$).

7. RSA encrypts an integer X into another integer Y by $Y = X^e \bmod n$ with a particular key(n, e).

(a) (7%) If $n = p \times q$ (p, q primes), gcd(X, p)=gcd(X, q)=1, and $de \equiv 1(\text{mod}(p-1)(q-1))$, please prove $Y^d \equiv X(\text{mod } pq)$.

(b) (7%) Given $Y = 981$ and a key (n=2537, e=13), Please decrypt Y, what is the original integer X? [*Hint*: the fast modular exponentiation]

II. 線性代數 (40%)

8. Given two matrices M and N, please calculate

$$M=\begin{bmatrix}0 & 1 & 0 & 0\\ 3 & -2 & 2 & 1\\ 0 & 15 & 0 & 1\\ 5 & 5 & 5 & 5\end{bmatrix}, \quad N=\begin{bmatrix}3 & 9 & -1 & -1\\ 0 & 1 & 0 & 16\\ 0 & 0 & 2 & 3\\ 0 & 0 & 0 & 2\end{bmatrix}$$

(a) (2%) det(M)

(b) (3%) det(MN)

(c) (5%) det(adj(M)), where adj(M) satisfies $M \cdot \text{adj}(M) = \det(M) \cdot I$

9. (10%) Let the linear transform $T: E^3 \to E^2$ be defined by $T[(x, y, z)] = (x + 2y + z, -x + 3y + z)$ Please calculate the **kernel** and **nullity** of T (ket T and $\eta(T)$).

10. (10%) Give three integers (x, y and z), try to maximize $z = 25x + 60y$ subject to the following constraints

$$-x + 2y \le 5$$
$$4x + y \le 25$$
$$x + y \ge 7$$
$$x \ge 0, y \ge 0$$

11. Organize data as a matrix X (m x n), where m is the number of measurement types and n is the number of samples. The goal of Principle Component Analysis is to find an orthonormal matrix P for $Y = PX$, where the rows of P are considered as the principal components of X.

(a) (2%) Please prove $C_Y = PC_XP^T$, where C_x and C_y is $\frac{1}{n}XX^T$ and $\frac{1}{n}YY^T$.

(b) (3%) We select the matrix P to be a matrix where each row p_i is an eigenvector of C_X, where D is the diagonal matrix of eigenvector decomposition. Please prove $C_Y = D$, a diagonal matrix.

(c) (5%) Given the following C_X, what is C_Y?

$$C_x = \begin{bmatrix}3 & 0\\ 0 & 4\end{bmatrix}$$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

國立政治大學 108 學年度 碩士暨碩士在職專班 招生考試試題

考試科目	作業系統	系所別	資訊科學系	考試時間	2月 18日(一) 第2節

1. Single choice (45%)

(1) Which of the following statement is false? (a) Microservice is a method to structure the operating system by removing all nonessential components from the kernel and implementing them as system and user-level programs. (b) Microkernel aims to provide minimal process and memory management. (c) Spin lock means the lock that while a process is in its critical section, any other process that tries to enter its critical section must loop continuously in the call to acquire (d) In EDF scheduling, the earlier the deadline, the higher the priority.

(2) To starting up a computer system, the computer locates (a) an OS in ROM and loads it onto RAM (b) an OS in memory and loads it onto storage (c) an OS in storage and loads it into memory (d) a boot manager in ROM and loads it onto RAM.

(3) Which of the following mechanism is used to implement virtual memory? (a) paging (b) segmentation (c) segmentation with paging (d) all of the above.

(4) Which of the following is a multiple-user version of multitasking (a) multiprogramming (b) multiprocessing (c) time-sharing (d) multi-user tasking

(5) Which of the following refers to the concurrent execution of many programs on a multi-user system? (a) multiprogramming (b) multiprocessing (c) time-sharing (d) multi-user tasking

(6) Which of the following process scheduling strategy can cause starvation? (a) FCFS (b) Shortest Job First (c) Round-Robin (d) none of the above.

(7) Which of the following is an incorrect transition of process state? (a) running→ waiting (b) waiting→ running (c) waiting →ready (d) ready →running

(8) Which of the following mechanism has the worst fault tolerance capability? (a) RAID 1 (b) RAID 0 (c) RAID 5 (d) RAID 0+1

(9) Which of the following is true for a hard real-time system but not true for a soft real-time system? (a) each task has a deadline (b) tasks are scheduled according to its deadline (c) missing a deadline results in a failure of the whole system (d) It is mostly used in time-critical applications

(10) When designing the virtual memory sub-system, which of the following factor is not typically used to determine the page size? (a) the size of page table (b) internal fragmentation (c) CPU speed (d) I/O time

(11) Which of the following strategy is preemptive? (a) FCFS (b) SJF (c) Earliest deadline first (d) FCLS

(12) Given that physical memory is 256KB and is partitioned into 8 page frames. If logical memory is 5MB, what is the number of pages needed in virtual memory? (a) 640 (b) 320 (c)160 (d)80

(13) Which of the following is not a condition of deadlock? (a) mutual exclusion (b) hold and wait (c) preemption (d) circular wait

(14) In a paging system with three-level page tables, suppose that the hit rate is 90% and it takes 20 ns to search the TLB and 200 ns to access memory. What is the effective memory-access time? (a) 280ns (b) 260ns (c) 300 ns (d) 240ns

(15) Which of the following mechanism does not implement access matrix? (a) Role Based Access Control (b)

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Global Table (c) Capability Lists for Domain (d) Lock-Key Scheme

2. (10%) Consider the following configuration. Virtual address = 32 bits, page size = 4K bytes, and a page table entry occupies 4 bytes. How many pages should the OS allocates for the page tables of a 12Mbytes process under the following paging mechanisms? Assuming that the number of entries is the same.
   (1) One-level paging
   (2) Two-level paging

3. (20%) Please define the following terms
   (1) System call
   (2) Thrashing
   (3) Context switch
   (4) TCB (trusted computer base)
   (5) Hypervisor

4. (10%) What is man-in-the-middle attack? Please draw a diagram and explain.

5. (15%) Point out and correct the problems when using the following two binary semaphores with S1 =1 and S2=0 to implement a counting semaphore.

wait operation:	**signal operation:**

wait(S1):
C--;
if(C<0) {
signal (S1);
wait(S2);
}else
signal(S1);
	wait(S1);
	C++;
	If(C<=0)
	signal(S2);
	signal(S1);

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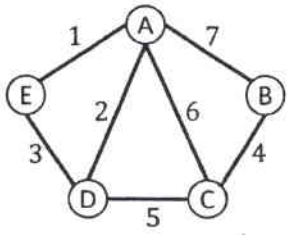
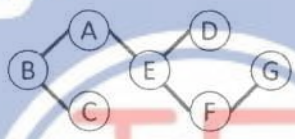
國立政治大學 108 學年度 碩士暨碩士在職專班 招生考試試題

考試科目	資料結構及演算法	系所別	資訊科學系/資訊科學與工程組、智慧計算組	考試時間	2 月 18 日(一) 第 一 節

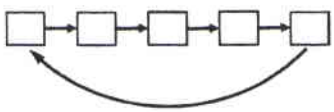
1. (10%) Use Kruskal's algorithm to find the minimum spanning tree of the following graph.



2. (10%) Give two advantages of hash tables over balanced binary search trees.
3. (15%) A vertex cover of a graph $G = (V, E)$ is a subset of vertices $S \subseteq V$ that includes at least one endpoint of every edge in E. Use dynamic programming to design a linear time algorithm that outputs the smallest vertex cover in a tree. For example, in the following tree, {A, C, D, F} is a vertex cover, and {C, E, F} is not a vertex cover. The smallest vertex cover in the following tree is {B, E, G}.



4. (10%) Design an $O(n)$ algorithm for the following task.
   Input: A list of n positive integers $a_1, a_2, \ldots, a_n$, where each a_i satisfies $1 \leq a_i \leq n$.
   Task: Sort $a_1, a_2, \ldots, a_n$ in non-decreasing order.
   For example, if the input contains 4 integers 3, 1, 1, and 3, then the output should be 1, 1, 3, 3.
5. (10%) Give the tight asymptotic bound of the following recurrence: $T(n) = 2T(n/2) + n$.
6. (10%) Give an application of the heap data structure.
7. (10%) Give a balanced binary search tree containing the following data: 1, 2, 3, 4, 5, 6, 7.
8. (10%) Give a binary heap containing the following data: 1, 2, 3, 4, 5, 6, 7.
9. (15%) Design an algorithm for the following task.
   Input: a linked list.
   Task: determine whether the linked list contains a cycle.
   For example, the following linked list contains a cycle.



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