國立政治大學 107 學年度 碩士班 招生考試試題

考試科目	資料結構及演算法	系所別	資訊科學系	考試時間	2月3日(星期六)第一節

1. (10%) Design a polynomial-time algorithm for the following task.
   Input: A graph G.
   Output: "Yes", if G has a cycle;
   "No", if G does not have a cycle.
2. (10%) Compare arrays with linked lists.
   a. (5%) Give one advantage of linked lists over arrays.
   b. (5%) Give one advantage of arrays over linked lists.
3. (10%) Explain the reason that the worst-case running time of quick sort is $\theta(n^2)$, where n is the number of elements to be sorted.
4. (10%) Explain the reason that the worst-case running time of merge sort is $O(n \log n)$, where n is the number of elements to be sorted.
5. (10%) Often there are multiple shortest paths between two vertices of a graph. Design a polynomial-time algorithm for the following task:
   Input: An undirected graph G, where all edges have the same length, and vertices v and w in G.
   Output: The number of distinct shortest paths from v to w.
6. (10%) Design a polynomial-time algorithm to compute the maximum spanning tree, i.e., the spanning tree that has the largest total edge weight.
7. (15%) A contiguous subsequence of a list S is a subsequence made up of consecutive elements of S. For instance, if S is 5, 15, −30, 10, −5, 40, 10, then 15, −30, 10 is a contiguous subsequence but 5, 15, 40 is not. Design a polynomial-time algorithm for the following task:
   Input: A list S of numbers $a_1, a_2, \ldots, a_n$.
   Output: The contiguous subsequence of S of maximum sum.
   Note that a subsequence of length zero has sum zero.
   For the preceding example, the answer would be 10, −5, 40, 10, with a sum of 55.
8. (5%) What is a min heap? Give an example.
9. (10%) Answer the following questions about balanced binary search trees.
   a. (5%) What is a balanced binary search tree?
   b. (5%) Give one advantage of balanced binary search trees over binary search trees.
10. (10%) Let A be a problem that belongs to NP. For each of the following statements, explain whether or not the statement is true.
    a. (5%) There is no polynomial-time algorithm for A.
    b. (5%) If A can be solved in polynomial time, then P=NP.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	作業系統	系所別	資訊科學系	考試時間	2 月 3 日(六) 第 2 節

1. (16%) Virtualization and Operation Systems:

   (1) What is the "Trap-and-Emulate"?

   (2) What is the problem when we implement Trap-and-emulate in an old X86-based CPU?

   (3) How does binary translation solve this problem?

   (4) What is Light-weight Virtualization (e.g. Docker) ? What are the benefits of such technology and why?

2. (16%) Please define the following terms

   (1) Microkernel

   (2) Spin lock

   (3) Processor affinity

   (4) EDF Scheduling

3. (8%) Consider the classic bounded-buffer producer-consumer problem (in which the producer 'writes data into a share bounded buffer and the consumer reads data from the same buffer). Assume the size of the bounded buffer is n. Please enhance the following pseudo code by using binary and counting semaphores to ensure that :

   (1) Semaphores are properly initialized,

   (2) there is no buffer overflow and underflow, and

   (3) accesses to the buffer are mutual exclusive.

   Suppose we have an abstract data type **semaphore** that provides three operations: **wait () , signal(), and count()**, where operation **count()** is used for initializing a semaphore. For example, assuming *sem* is a variable of type **semaphore**, **count(5)** will initialize *sem* to the value of 5.

```
// Semaphore declaration and initialization code
…
// Producer
while (true) {
   …
   Add an item to buffer
   …
}
```

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```
//Consumer
while (true) {
   …
   Read and remove an item from buffer
   …
}
```

4. (10%) Please "illustrate" and compare the layered structure of Android and iOS platforms with respect to kernel, services and library, and user interfaces.

5. (10%) System protection and security

   - Buffer-overflow attacks can be avoided by adopting a better programming methodology or by using special hardware support. Briefly explain how each of these solutions prevents buffer-overflow.
   - What is the purpose of using a "salt" along with the user-provided password? Where should the "salt" be stored, and how should it be used?

6. (15%) Consider a computer system with a 32-bit virtual address space where paging is used. Assuming the page size is 4K bytes and the memory is byte-addressable, please answer the following questions:

   - How many pages can a process have at most? Suppose the maximum physical memory size is 32 GB. What is the number of bits for physical addresses? What is the maximum number of frames for the system?
   - Let the memory access time and TLB access time be 100 ns and 20ns, respectively. If we want an effective memory access time of less than 140ns, what is the minimal TLB hit ratio that needs to be achieved?
   - Suppose the virtual memory of the system adopts demand paging. Assume the effective memory access time of the computer system without any page fault is loons, and the service time for a page fault is 15ms. If the page fault rate is 0.0000004, what is the effective access time under demand paging?

7. (10%) A blockchain can be seen as a distributed file system which consists of continuously growing list of records (blocks). The blocks are linked and secured using cryptography. Each block typically contains a hash pointer as a link to a previous block, a timestamp and transaction data. The data stored in a blockchain are inherently resistant to modification of the data. A blockchain network is typically peer-to-peer and therefore is de-centralized. Any new data must be verified by peers before they can be added in to the file system. According to the above descriptions of blockchain, answer the following questions.

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(1) From the view point of File System, which "consistency semantics" should the blockchain uses and why?

(2) Assuming that you are going to design a new blockchain. Sketch an approach to store the data (in a distributed way) and make these data being consistent at the same time.

8. (15%) File systems and I/O: Consider DISK scheduling algorithms (FCFS, SSTF, SCAN, C-SCAN, and LOOK),

(1) Which algorithm is best for heavily loaded system? Why?

(2) Which algorithm is truly fail, no starvation? Why?

(3) Which algorithm is best for DISK scheduling when reading data in SSD (Solid-State Disk)? Why?

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考試科目	計算機數學	系所別	資訊科學系	考試時間	2月 3日(六) 第三節

I. 離散數學 (60%)，作答務必註明題號

單選題，每題 2%，五題共 10%

1. Which of following statements is **NOT** tautology?
   (a) $(p \vee \neg p)$ (b)$(p \wedge q) \rightarrow (p \vee q)$ (c) $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$ (d) $((\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)) \rightarrow q$
2. Which of following pair statements is logically equivalent?
   (a) $\exists x P(x) \wedge \exists x Q(x)$, $\exists x(P(x) \wedge Q(x))$ (b) $\forall x P(x) \vee \forall x Q(x)$, $\forall x(P(x) \vee Q(x))$
   (c) $\forall x\big(P(x) \rightarrow Q(x)\big)$, $\forall x P(x) \rightarrow \forall x Q(x)$ (d) $\exists x\big(P(x) \vee Q(x)\big)$, $\exists x P(x) \vee \exists x Q(x)$
3. Which of following functions is **NOT** a bijection (one-to-one correspondence) from **R** to **R**?
   (a) $f(x) = -5x + 1$ (b) $f(x) = x^3$ (c) $f(x) = (x^4 + 2)/(x^2 + 2)$ (d) $f(x) = x^{117} + 2$
4. What will be the solution of the recurrence relation, $a_n = 3a_{n-1}$, and its initial condition $a_o = 3$?
   (a) $a_n = 3^n$ (b) $a_n = 3^n + 2$ (c) $a_n = 3^{n+1}$ (d) $a_n = 3^{n+1} + 2$
5. What will be the number of $(99^2 \bmod 32)^3 \bmod 15$?
   (a) 9 (b) 3 (c) 1 (d) 0

請工整書寫必要的解題過程，僅答案而缺乏過程，無法獲得該題滿分。

6. (5%) How many solutions are there to the inequality $x_1 + x_2 + x_3 \leq 17$, where x_1, x_2, x_3 are nonnegative integers such that $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$?
7. (5%) Find $p(Y|X)$ if $p(X|Y) = 1/3$, $p(X|\bar{Y}) = 1/4$, and $p(Y) = 2/3$ by using Bayes' theorem where X and Y are events from a sample space S.
8. (5%) Find the solution to the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 5$, $a_1 = -9$, and $a_2 = 15$, that is, rewrite a_n by n.
9. (5%) Let R be the relation, {(1,1), (1,2), (2,2), (2,5), (3,3), (3,5), (4,4), (4,1), (5,5), (5,1)}, on the set {1, 2, 3, 4, 5}. Find the directed graph of the relation R^*, where $R^* = \bigcup_{n=1}^{\infty} R^n$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	計算機數學	系所別	資訊科學系	考試時間	2月 3日(六) 第三節

10. (5%) The complete m-partite graph $K_{n_1,n_2,\ldots,n_m}$ has vertices partitioned into m subsets of n_1, n_2,…, n_m elements each, and vertices are adjacent if and only if they are in different subsets in the partition. What is graph notation, $K_{n_1,n_2,\ldots,n_m}$, of the following figure (2%)? What is its **intendance number**, where the intendance number is the size of a maximum independent vertex set and a set of vertices is *independent* if no two vertices in the set are adjacent (1%)? What is the intendance number of $K_{n_1,n_2,\ldots,n_m}$ (2%)?

11. (5%) Find a deterministic finite-state automaton that can recognize $\{1^n | n = 2, 3, 4, \ldots\}$.

12. (10%) Use Fermat's little theorem and Chinese remainder theorem to find $5^{2003} \bmod 1001$?

13. (10%) Show that the length of the shortest path between vertices x and y in a connected simple graph, G, equals the level number of y in the breath-first spanning tree of G with root x. [HINT: by Mathematical Induction]

II. 線性代數 (40%)，作答務必註明題號

1. (10%) Determine h and k such that the solution set of the system (i) is empty, (ii) contains a unique solution, and (iii) contains infinitely many solutions.

(a)
$$\begin{aligned} x_1 + 3x_2 &= k \\ 4x_1 + hx_2 &= 8 \end{aligned}$$

(b)
$$\begin{aligned} -2x_1 + hx_2 &= 1 \\ 6x_1 + kx_2 &= -2 \end{aligned}$$

2. (10%) Assume that all the following matrices are $n \times n$. Determine each of the matrices whether it is invertible or not invertible.

(a)
$$\begin{bmatrix} 3 & 0 & -3 \\ 2 & 0 & 4 \\ -4 & 0 & 7 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 4 & 0 & -3 & -7 \\ -6 & 9 & 9 & 9 \\ 7 & -5 & 10 & 19 \\ -1 & 2 & 4 & -1 \end{bmatrix}$$

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3. (10%) Mark each statement **True** or **False**. Please justify each answer. If true, cite appropriate facts of theorems. If false, explain why or give a counterexample that shows why the statement is not true.

(a) The nonpivot columns of a matrix are always linearly dependent.

(b) Row operations on a matrix A can change the linear dependence relations among the rows of A.

(c) Row operations on a matrix can change the null space.

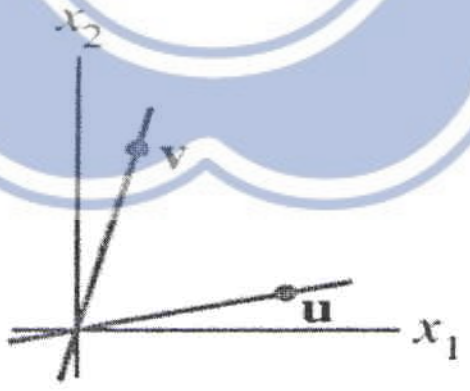
(d) If an $m \times n$ matrix A is row equivalent to an echelon matrix U and if U has k nonzero rows, then the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$ is $m - k$.

(e) If matrices A and B have the same reduced echelon form, then Row A = Row B.

4. (10%) Let **u** and **v** be the vectors shown in the following figure, and suppose **u** and **v** are eigenvectors of a 2×2 matrix A that correspond to eigenvalues 2 and 3, respectively.

Let $T : R^2 \rightarrow R^2$ be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$ for each **x** in R^2, and let

$\mathbf{w} = \mathbf{u} + \mathbf{v}$. Make a copy of the figure, and on the same coordinate system, carefully plot the vectors $T(\mathbf{u})$, $T(\mathbf{v})$ and $T(\mathbf{w})$.



備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。