考試科目	資料結構及演算法 81411	系所別	資訊科學系	考試時間	2月19日(星期日) 第一節

1. (10%) Design an algorithm of running time $O(n)$ for the following task.

   Input: $a_0, a_1, a_2, \ldots, a_n$, and x

   Output: the value of $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$

   Note:

   A. Each addition takes constant time.

   B. Each multiplication takes constant time.

2. (10%) Consider the following function f. How many lines, as a function of n (in $\Theta(n)$ form), does f(n) print? Write a recurrence and solve it. You may assume n is a power of 2.

```
void f(int n){
    if (n > 1){
        printf("still going\n");
        f(n / 2);
        f(n / 2);
    }
}
```

3. (10%) Design an algorithm of running time $O(|V|^2)$ for the following task.

   Input: An undirected graph $G = (V, E)$ in which each edge $e \in E$ is associated with a positive edge length $l(e)$; an edge $a \in E$.

   Output: The length of the shortest cycle containing edge a in G.

4. A server has n customers waiting to be served. The service time required by each customer is known in advance; it is t_i minutes for customer i. So if, for example, the customers are served in order of increasing i, then the ith customer has to wait $\sum_{j=1}^{i-1} t_j$ minutes. We wish to minimize the total waiting time

$$T = \sum_{i=1}^{n} (\text{time spent waiting by customer } i).$$

   A. (8%) Design an algorithm for computing the optimal order in which to process the customers.

   B. (2%) Prove your algorithm is correct.

考試科目	資料結構及演算法	系所別	資訊科學系	考試時間	2月19日(星期日) 第一節

5. (10%) Design an algorithm of running time $O(nt)$ for the following task.
   Input: A set S of n positive integers $S = \{a_1, a_2, \dots, a_n\}$; a positive integer t.
   Question: Does some subset of S add up to t? (You can use each a_i at most once.)
   Hint: Dynamic programming.

6. Recall the traveling salesman problem (TSP):
   Input: A list of cities; the distances between each pair of cities; a budget b.
   Output: A route of length less than or equal to b that visits each city exactly once and returns to the origin city, if such a route exists.

   The optimization version of this problem asks directly for the shortest route (TSP-OPT):
   Input: A list of cities; the distances between each pair of cities.
   Output: The shortest possible route that visits each city exactly once and returns to the origin city.

   A. (5%) Show that if TSP-OPT can be solved in polynomial time, then so can TSP.
   B. (5%) Show that if TSP can be solved in polynomial time, then so can TSP-OPT.

7. In this problem, you need to implement queues by linked lists.

```
struct ListNode{
    int data;
    struct ListNode* next;
};
struct Queue{
    struct ListNode* front;
    struct ListNode* end;
};
struct Queue* CreateQueue(){
    struct Queue* Q = (struct Queue*) malloc(sizeof(struct Queue));
    if (!Q)
        return NULL;
    Q->front = Q->end = NULL;
    return Q;
}
```

   A. (15%) Implement a function `void EnQueue(struct Queue* Q, int data)` by inserting an element `data` at the end of the linked list.
   B. (15%) Implement a function `int DeQueue(struct Queue* Q)` by outputting and deleting the first element of the linked list. You can assume that the queue is not empty.

考試科目	資料結構及演算法	系所別	資訊科學系	考試時間	2月19日(星期日) 第一節

8. (10%) Suppose we use left-child right-sibling binary tree data structure to represent general trees with the following tree node structure. Given the root of a tree, `root`, implement function `int nNode(struct TreeNode* root)` that outputs the number of nodes in the tree rooted at `root`.

Note:

A. You CANNOT use loops or jumps in your code.

B. Since `root` is a root of a tree, `root->rightSibling == NULL`.

```
struct TreeNode{
    int data;
    struct TreeNode* leftChild;
    struct TreeNode* rightSibling;
};
```

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	作業系統 31412	系所別	資訊科學系一般生	考試時間	2月19日(日)第二節

1. Please answer the following questions about processes and threads management:

   (a) (4%) When an interrupt or a system call transfers control to the operating system, a kernel stack area separate from the stack of the interrupted process is generally used. Why?

   (b) (6%) In the following Figure 1, three process states are shown. In theory, with three states, there could be six transitions, two out of each state. However, only four transitions are shown. Are there any circumstances in which either or both of the missing transitions might occur? If yes, how it occurs. If no, why cannot occur?

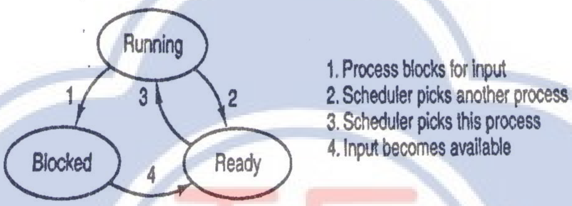


Figure 1: A process can be in running, blocked, or ready state. Transactions between these stats are as shown.

   (c) (5%) In a system with threads, is there one stack per thread or one stack per process when user-level threads are used? What about when kernel-level threads are used? Explain.

   (d) (5%) In Figure 2, the register set is listed as a per-thread rather than a per-process item. Why? After all, the machine has only one set of registers.

Per-process items	Per-thread items
Address space	Program counter
Global variables	Registers
Open files	Stack
Child processes	State
Pending alarms	
Signals and signal handlers	
Accounting information	

Figure 2: The first column lists some items shared by all threads in a process. The second one lists some items private to each thread.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	作業系統	系所別	資訊科學系一般生	考試時間	2月19日(日)第二節

2. Please answer the following questions about memory management:

   (a) (5%) It has been observed that the number of instructions executed between page faults is directly proportional to the number of page frames allocated to a program. If the available memory is doubled, the mean interval between page faults is also doubled. Suppose that a normal instruction take 1 μs (micro-second), but if a page fault occurs, it takes 2 ms (mini-second) to handle the fault. If a program takes 60 second to run, during which time it gets 15,000 page faults, how long would it take to run if twice as much memory were available?

   (b) (5%) Explain the difference between internal fragmentation and external fragmentation. Which one occurs in paging systems? Which one occurs in system using pure segmentation?

   (c) (5%) A computer has 32-bit virtual address uses a two-level page table. Virtual addresses are split into a 9-bit top-level page table field, and 11-bit second-level page table field, and an offset. How large are the pages and how many are there in the address space?

   (d) (5%) Use the page table of Figure 3, give the physical address corresponding to the virtual address of 8200.

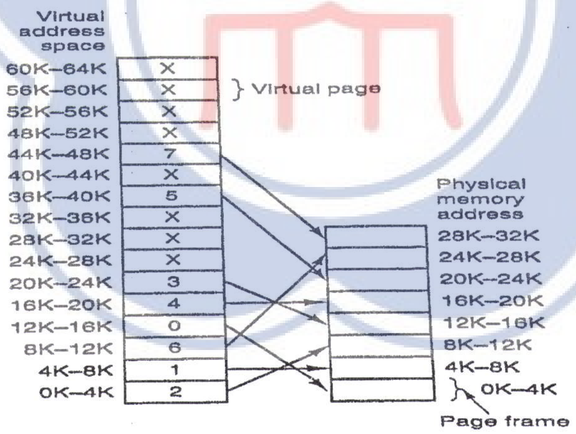


Figure 3: The relation between virtual addresses and physical memory addresses is given by the page table. Every page begins on a multiple of 4096 and ends 4095 addresses higher, so 4K-8K really means 4096-8191 and 8K to 12K means 8192-12287.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	作業系統	系所別	資訊科學系-一般生	考試時間	2月19日(日)第二節

3. Please answer the following questions about file systems:

   (a) (5%) Consider the i-node shown in Figure 4. If it contains 8 direct addresses of 4 bytes each and one indirect address. All disk blocks are 1024 bytes (1 KB), what is the largest possible file?

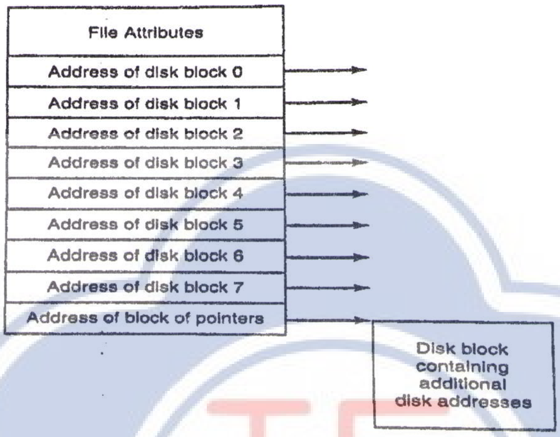


Figure 4: An example i-node.

   (b) (5%) The beginning of a free-space bitmap looks like this after the disk partition is first formatted: 1000 0000 0000 0000 (the first block is used by the root directory). The system always searches for free blocks starting at the lowest-numbered block, so after writing file A, which uses six blocks, the bitmap looks like this: 1111 1110 0000 0000. Show the bitmap after each of the following additional actions: (a) File B is written, using five blocks. (b) File A is delected.

   (c) (10%) One way to use contiguous allocation of the disk and not suffer from holes is to compact the disk every time a file is removed. Since all files are contiguous, copying a file requires a seek and rotational delay to read the file, followed by the transfer at full speed. Writing the file back requires the same work. Assuming a seek of 5 *ms* (mini-second), a rotational rate of 15,000 rpm, a transfer rate of 4 MB/sec, and an average file size of 4 KB, how long does it take to read a file into main memory and then write it back to the disk at a new location? Using these numbers, how long would it take to compact half of a 8-GB disk?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	作業系統	系所別	資訊科學系一般生	考試時間	2月19日(日)第二節

4. Please answer the following questions about virtualization and the cloud:

   (a) (5%) Why do you think virtualization took so long to become popular? After all, the key paper was written in 1974 and IBM mainframes had the necessary hardware and software through the 1970s and beyond.

   (b) (5%) Does it make sense to paravirtualization an operating system if the source code is available? What if it is not? Explain.

   (c) (5%) Why is virtual machine migration important? Under what circumstances might it be useful?

   (d) (5%) Migration virtual machines may be easier than migrating processes, but migration can still be difficult. What problems can arise when migrating a virtual machine?

5. Please answer the following questions about protection and security:

   (a) (10%) Suppose that a system has 1000 objects and 100 domains at some time. 1% of the objects are accessible (some combination of r, w and x) in all domains, 10% are accessible in two domains, and remaining 89% are accessible in only one domain. Suppose one unit of space is required to store an access right (some combination of r, w, x), object ID, or a domain ID. How much space is needed to store the full protection matrix, protection matrix as ACL, and protection matrix as capability list?

   (b) (5%) Secret-key cryptography is more efficient than public-key cryptography, but requires the sender and receiver to agree on a key in advance. Suppose that the sender and receiver have never met, but there exists a trusted third party that shares a secret key with the sender and also shares a (different) secret key the receiver. How can the sender and receiver establish a new shared secret key under these circumstances?

   (c) (5%) Suppose that two strangers A and B want to communicate with each other using secret-key cryptography, but do not share a key. Suppose both of them trust a third party C whose public key is well known. How can the two strangers establish a new shared secret key under these circumstances?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	計算機數學 31413	系所別	資訊科學系	考試時間	2月19日(日) 第三節

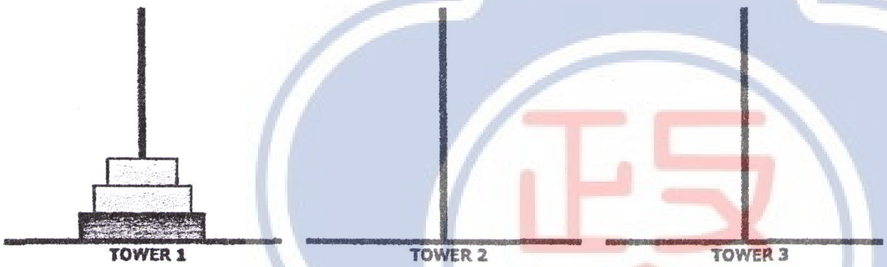
I.　離散數學: 四大題(1-4)，每大題有三小題，共十二小題，每小題 5%，共 60%

II.　線性代數: 四大題(5-8)，每題 10%，共 40%

請書寫必要的解題過程，僅書寫答案而缺乏必要的過程，無法獲得該題滿分。可使用中文或英文作答，力求書寫工整，如字跡潦草，無法閱讀，將影響評分。

1. (15%) The Tower of Hanoi consists of three towers, and a number of disks of different sizes which can slide onto any tower. The puzzle starts with the disks in a neat stack in ascending order of size on one tower, the smallest at the top. The objective of the puzzle is to move the entire stack to another tower, obeying the following simple rules:

- Only one disk can be moved at a time.
- Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack i.e. a disk can only be moved if it is the uppermost disk on a stack.
- No disk may be placed on top of a smaller disk.



(a). (5%)$f(n)$ denotes the minimum number of moves needed for n disks for three towers. What is $f(n)$ in *recurrence relation* form (*boundary condition* also needed)?

(b). (5%)What is the numeric function (or the solution of the recurrence relation) in big-O notation?

(c). (5%)If there are four towers instead of three towers (the *Reve's puzzle*), $g(n)$ denotes the minimum number of moves needed for n disks for four towers. What is $g(n)$ in *recurrence relation* form [HINT: $g(n)$ should contain $f(n)$]?

2. (15%) The greatest common divisor (gcd) of two or more integers, when at least one of them is not zero, is the largest positive integer that is a divisor of both numbers.

(a). (5%)Suppose that x and y are integers, that $x = 3^{111} \mod 143$ and $y = 209^{263} \mod 53$. Please compute x, y and $gcd(x, y)$.

(b). (5%)You can use the *Euclidean algorithm* to compute *gcd*. Please write the pseudo-code of the Euclidean algorithm by recursive function.

(c). (5%)What is the worst-case complexity of your code? [HINT: The pair of Fibonacci numbers will result in the worst case, *gcd*(Fibonacci(n), Fibonacci(n-1))]

3. (15%)A chordal graph is one in which all cycles of four or more vertices have a chord, which is an edge that is not part of the cycle but connects two vertices of the cycle. Equivalently, every induced cycle in the graph should have exactly three vertices.

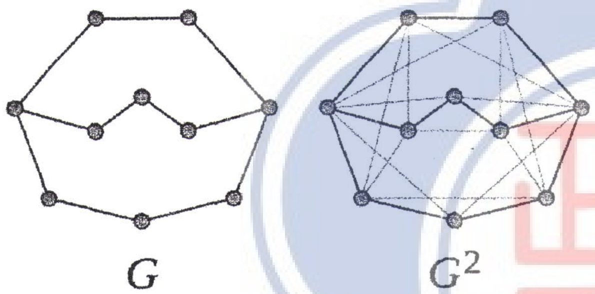
備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考 試 科 目	計算機數學	系 所 別	資訊科學系	考 試 時 間	2 月 19 日(日) 第三節

(a). (5%) We represent the following three graphs, *i*, *ii* and *iii*, by their adjacency matrix forms. Which graph is not a chordal graph? How do you make them as chordal graph by adding edges into their corresponding adjacency matrices?

$$\text{i. } \begin{bmatrix} 0&1&0&0&1&0\\ 1&0&1&0&0&1\\ 0&1&0&1&0&0\\ 0&0&1&0&1&0\\ 1&0&0&1&0&1\\ 0&1&0&0&1&0 \end{bmatrix} \quad \text{ii. } \begin{bmatrix} 0&1&1&1&1&1\\ 1&0&1&1&1&1\\ 1&1&0&1&1&1\\ 1&1&1&0&1&1\\ 1&1&1&1&0&1\\ 1&1&1&1&1&0 \end{bmatrix} \quad \text{iii. } \begin{bmatrix} 0&1&0&0&1&0\\ 1&0&1&0&1&0\\ 0&1&0&0&0&1\\ 0&0&0&0&1&1\\ 1&1&0&1&0&0\\ 0&0&1&1&0&0 \end{bmatrix}$$

(b). (5%) The square of an undirected graph *G*, G^2, is another graph that has the same set of vertices, but in which two vertices are adjacent when their distance in *G* is at most 2. i.e.,



Given the adjacency matrix of *G*, *A*, what would be the adjacency matrix of G^2? Why? [HINT: you can consider *A* as a relation matrix.]

(c). (5%) Please prove the square of *tree* is a chordal graph, where a tree is an undirected graph in which any two vertices are connected by exactly one path. In other words, any acyclic connected graph is a tree.

4. (15%)A *phrase structure grammar* consists of four items:

i. A set of *terminals T*.

ii. A set of *nonterminals N*.

iii. A set of *productions P*.

iv. Among all the nonterminals in *N*, there is a special nonterminal that is referred to as the *starting symbol*.

(a). (5%)What is the phrase structure grammar for the language *L*?

$$L = \{apple^i pen^j \mid i, j \geq 1, i > j\}, \text{ where } i \text{ and } j \text{ are integers}$$

(b). (5%)If $i+j = 10$, what is number of strings generated by *L*? [HINT: generating function]

(c). (5%)Please prove *L* is type-2 grammar. In other words, it cannot be specified by type-3 grammar. [HINT: Type-3 language is and only if is a finite state language].

考試科目	計算機數學	系所別	資訊科學系	考試時間	2月19日(日) 第三節

5. (10%) Find $Proj_v u$, the projection of *u* along *v* where *u=(3,-1,-2,4)* and *v=(1,1,3,6)*.

6. (10%) Consider the basis $u_1 = (1,0,1), u_2 = (0,1,1)$, and $u_3 = (1,0,0)$ for R^3. Form an orthonormal basis v_1, v_2, v_3.

7. (10%)Let the linear transformation $T: R^3 \to R^4$ be given by

$$T[(x,y,z)] = (x - 2y + z, 3x - 5y + 5z, 5x - 9y + 7z, 0)$$

Find rank *(T)* and *N(T)*.

8. (10%) Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 3 & -1 \end{bmatrix}$.

(a). (4%) Find the eigenvectors of A.

(b). (6%) Find the orthogonal matrix C that diagonalizes A.