國立政治大學　105　學年度碩士班招生考試試題

考試科目	資料結構及演算法 81411	所別	資訊科學系	考試時間	2月 28 日(日) 第一節

僅書寫答案而缺乏必要的解題過程，亦無法獲得該題分數。

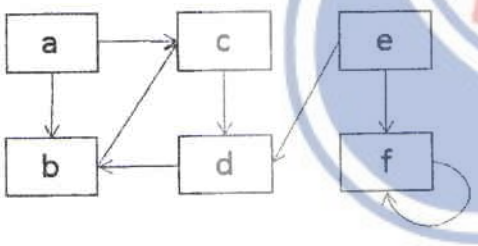
1. (10 points) Draw a diagram to precisely describe the definition of "$f(n) = \Theta(g(n))$", explain the detail.

2. (10 points) What are the **minimum** and **maximum** numbers of elements for a **min-heap** of height **h**? explain detail.

3. Time complexity
   (a) (5 points) Formulate the **time complexity** for the following procedure.
   (b) (5 points) Show a worst case example, and why?

```
Q3(A)
for i = 1 to A.length - 1
    for j = A.length downto i+1
        if A[j] <A[j-1]
            swap (A[j], A[j-1])
```

4. (10 points) Design a data structure with as small as possible storage to represent the following graph, explain detail of the data structure and compare to alternate data structure.



5. Binary search tree
   (a) (5 points) What is binary search tree property?
   (b) (5 points) Propose an $O(n \log n)$ algorithm to output a sorted array from a binary search tree.

6. Use master theorem to solve following equatioin:
   (a) (5 points)　$T(n) = 6T(n/3) + n^2$
   (b) (5 points)　$T(n) = 4T(n/4) + n \lg n$

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7. (10 points) Answer following questions after executes the code fragment.

(a) k =

```
int n = 4096;
int k = 0;
for (int i = 1; i < n; i = i*4)
    k = k + 10;
```

(b) f(5) =

```
int f(int x) {
    if (x==1 || x==0 ) return 2;
    else return f(x-1) + f(x-2);
}
```

8. Give the precise definition of problem class in following categories.
   (a) (2 points) class of P,
   (b) (2 points) class of NP
   (c) (2 points) polynomial-time reduction
   (d) (4 points) What will happen if P = NP? Draw a diagram and explain.

9. (10 points) Show how to implement a queue using two stacks. Analyze the running time of the queue operations.

10. (10 points) Write pseudo code for a recursive version of Binary Search. Analyze the worst-case time complexity.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	作業系統 (Operating System) 81412	所別	資訊科學系一般生	考試時間	2 月 28 日（日）第二節

1. a. (6%) What were the three main requirements considered while designing VMware Virtual Machine Monitor (or hypervisor)?
   b. (4%) Give an example of IaaS (Infrastructure as a Service), PaaS (Platform as a Service), and SaaS (Software as a Service).

2. a. (4%) A 255-GB disk has 65,536 cylinders with 255 sectors per track and 512 bytes per sector. How many platters and heads does this disk have?
   b. (4%) Assuming an average cylinder seek time of 11 *ms*, average rotational delay of 7 *ms* and reading rate of 100 MB/sec, calculate the average time in *ms* it will take to read 100 KB from one sector.

3. (8%) Consider a computer system that has cache memory, main memory (RAM) and disk, and an operating system that uses virtual memory. It takes 2 *ns* to access a word from the cache, 10 *ns* to access a word from the RAM, and 10 *ms* to access a word from the disk. If the cache hit rate is 95% and main memory hit rate (after a cache miss) is 99%, what is the average time in *ns* to access a word?

4. (8%) Suppose that a 10-MB file is stored on a disk on the same track (track 50) in consecutive sectors. The disk arm is currently situated over track number 100. How long will it take to retrieve this file from disk? Assume that it takes about 1 *ms* to move the arm from one cylinder to the next and about 5 *ms* for the sector where the beginning of the file is stored to rotate under the head. Also, assume that reading occurs at a rate of 100 MB/sec.

5. a. (4%) What are three requirements for a solution to the critical-section problem?
   b. (6%) Consider the following solution to the critical-section problem involving two processes P_0 and P_1. Assume that the variable turn is initialized to 0. Process $P_0's$ code is presented below.

```
/* Other code*/

While (turn !=0){} /* Do nothing and wait. */
Critical Section /* … */
turn=0;

/*Other code*/
```

For process P_1, replace 0 by 1 in above code. Determine if the solution meets ***all*** the required conditions for a correct critical-section problem solution.

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6. Measurements of a certain system have shown that the average process runs for a time T before blocking on I/O. A process switch requires a time S, which is effectively wasted (overhead). For round-robin scheduling with time quantum Q, give a formula for the CPU efficiency for each of the following:
   (a). (4%) $S < Q < T$
   (b). (4%) $Q=\infty$

7. (8%) You are given the following data about a virtual memory system:
   (a). The TLB (Translation Lookaside Buffer) can hold 1024 entries and can be accessed in 1 clock cycle (1 *ns*).
   (b). A page table entry can be found in 100 clock cycles or 100 *ns*.
   (c). The average page replacement time is 6 *ms*.
   If page references are handled by the TLB 99% of the time, and only 0.01% lead to a page fault, what is the effective address-translation time in clock cycle (or *ns*)?

8. A computer has 32-bit virtual addresses and 4-KB pages. The program and data together fit in the lowest page (0-4095). The stack fits in the highest page.
   (a). (4%) How many entries are needed in the page table if traditional (one-level) paging is used?
   (b). (4%) How many page table entries are needed for two-level paging, with 10 bits in each part?

9. (8%)A computer has four page frames. The time of loading, time of last access, and the R and M bits for each page are as shown below (the times are in clock ticks):

Page	Loaded	Last ref.	R	M
0	126	280	1	0
1	230	265	0	1
2	140	270	0	0
3	110	285	1	1

   (a) Which page will NRU replace?
   (b) Which page will FIFO replace?
   (c) Which page will LRU replace?
   (d) Which page will enhanced second chance replace?

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10. (a). (4%) One way to use contiguous allocation of the disk and not suffer from holes is to compact the disk every time a file is removed. Since all files are contiguous, copying a file requires a seek and rotational delay to read the file, followed by the transfer a full speed. Writing the file back requires the same work. Assuming a seek time of 5 *ms*, a rotational delay of 4 *ms*, a transfer rate of 8 MB/sec, and average file size of 8 KB, how long (in *ms*) does it take to read a file into memory and then write it back to the disk at a new location?

    (b). (4%) Using these numbers, how long (in second) would it take to compact half of a 16-GB disk?

11. Suppose that a system uses DMA for data transfer from disk controller to main memory. Further assume that it takes t_1 nsec on average to acquire the bus and t_2 nsec to transfer one word over the bus ($t_1 \gg t_2$). After the CPU has programmed the DMA controller, how long will it take to transfer 1000 words from the disk controller to main memory, if

    (a). (4%) word-at-a-time mode is used.

    (b). (4%) burst mode is used.

    Assume that commanding the disk controller requires acquiring the bus to send one word and acknowledging a transfer also requires acquiring the bus to send one word.

12. (8%) A system has four processes and five allocatable resources. The current allocation and maximum needs are as follows:

	Allocated	*Maximum*	*Available*
Process A	1 0 2 1 1	1 1 2 1 3	0 0 ***x*** 1 1
Process B	2 0 1 1 0	2 2 2 1 0	
Process C	1 1 0 1 0	2 1 3 1 0	
Process D	1 1 1 1 0	1 1 2 2 1	

What is the smallest value of ***x*** for which this is a safe state?

備註	一、作答於試題上者，不予計分 二、試題請隨卷繳交。

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考試科目	計算機數學 8143	所別	資訊科學系	考試時間	2 月 28 日（日）第三節

I. Discrete Mathematics (60%)

1. (10%) Consider that $2^n > n^3$ for $n \geq K$, where K is a positive integer. What is the smallest value of K?
2. (10%) If we want to find the sum of 11 numbers by using the hypothetical computer that has an instruction which computes the sum of three numbers, how many times the addition instruction will always be executed? [Hint: Any sequence of execution of this instruction to obtain the result is a regular ternary tree with nine leaves.]
3. (10%) Consider the big-O notation and the following two functions, g and h. What is gh?

   $g = n + O\left(\frac{1}{n}\right)$, $h = n^{1/2} + O\left(\frac{1}{n^{1/2}}\right)$

4. (10%) Simplify the following proposition.

   $(a \wedge b \wedge c) \vee (\sim a \wedge b \wedge c) \vee (a \wedge b \wedge \sim c) \vee (\sim a \wedge b \wedge \sim c) \vee (a \wedge \sim b \wedge \sim c)$

5. (10%) Consider the following recursive function. What is $f(n)$, or how is $f(n)$ defined directly?

   $f(0) = 2,\ f(1) = 6, f(n) = 4f(n-1) - 4f(n-2)$ for $n > 1$

6. (10%) Consider a connected graph that can be described by the following adjacency matrix A and degree matrix D. What is the number of spanning trees of the graph?

   $$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

II. Linear Algebra(40%)

7. (10%) Please find the inverse of each of the following matrices.

   (1) $A = \begin{bmatrix} 2 & 3 \\ 5 & 8 \end{bmatrix}$　　(2) $A = \begin{bmatrix} 2 & 4 \\ 5 & 10 \end{bmatrix}$

8. (10%) Suppose that $A = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ and $R = [1 \quad 3 \quad 2]$. We construct a new matrix B by adding R to the third row of A. We construct a new matrix M by replacing the third row of A by R. Please find the determinants of A, B, and M.

9. (10%) Suppose that $A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 2 & -1 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix}$. Please find the determinantal rank of A.

10. (10%) Suppose that $A = \begin{bmatrix} 5 & -1 & 2 \\ 0 & 2 & 0 \\ 8 & 4 & 5 \end{bmatrix}$. Please find the eigenvalues of A and the corresponding eigenvectors.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。