

考 試 科 目	資料結構及演算法	所 別	8141 資訊科學系	考 試 時 間	2 月 25 日(六) 第一節
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1. (20%) **True or False.** Please answer "T" or "F" for the following question. Each 2%

- (1) Both insert and delete operations of a stack take $O(1)$ time.
- (2) $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$.
- (3) Adjacency-list representation is suitable for dense graph.
- (4) The complexity of parent operation of each element in a binary heap is $O(1)$.
- (5) $\Theta(n!) + O(2^n) = O(2^n)$.
- (6) A complete binary tree having n nodes have $\log n$ levels
- (7) A greedy algorithm always leads to global optimal solution.
- (8) Merge sort is a "in place" sort algorithm.
- (9) Any problem in P is also in NP.
- (10) The class NP consists of those problems that are solvable in non-polynomial time.

2. (20%) **Single selection.** Each 2%

- (1) Which of the following sorting algorithm takes the least number of comparisons for sorting of the following sequence of data (2, 17, 66, 80, 98, 123, 170, 200)?
 (a) quick sort (b) insertion sort (c) merge sort (d) heap sort (e) selection sort
- (2) Solve the following prefix expression: / - * 2 5 * 1 2 - 13 11
 (a) 0 (b) 1 (c) 4 (d) 12
- (3) Which of the following formula is the time complexity in term of comparison operations for merge sort of n records?
 (a) $T(n) = T(n/2) + cn$
 (b) $T(n) = 2T(n/2) + cn^2$
 (c) $T(n) = T(n-1) + cn$
 (d) $T(n) = 2T(n/2) + cn$
- (4) The worst case time complexity of finding the maximum in a N-key AVL tree
 (a) $O(1)$ (b) $O(\log N)$ (c) $O(N)$ (d) $O(N \log N)$ (e) $O(N^2)$ (f) $O(N^2 \log N)$
- (5) The minimum edges need for a n-node connected undirected graph.
 (a) n (b) n^2 (c) $n-1$ (d) $\log n$

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(6) A cycle in a graph that visits every vertex us called as

- (a)Euler circuit (b)Hamiltonian circuit (c)Connected cycle (d)Hamiltonian cycle (e)Euler cycle

(7) Which of the following are max-heaps?

	1	2	3	4	5	6	7	8	9
(a)	1	3	2	7	9	8	14		
(b)	16	14	10	8	7	9	3	2	4
(c)	17	10	14	7	8	3	9	11	
(d)	18	8	9	14	5	6	3	1	

(8) The worst case time complexity of finding the minimum or maximum in a Binary Search Tree with N elements is

- (a) $O(1)$ (b) $O(\log N)$ (c) $O(N)$ (d) $O(N \log N)$ (e) $O(N^2)$

(9) What is the time complexity of $T(n) = 5T(n/2) + \Theta(n^3)$

- (a) $\Theta(n^2)$ (b) $\Theta(n^{\log 5})$ (c) $\Theta(n^3 \log n)$ (d) $\Theta(n^3)$

(10) The maximum height of the recursion tree for the recurrence $T(n)=T(n/3)+T(2n/3)+cn$ will be

- (a) $\log n$ (b) $\log_{3/2} n$ (c) $3 \log n$ (d) 2^n

說明: 3 ~ 8 題，請書寫必要的解題過程。僅書寫答案而缺乏必要的過程，亦無法獲得該題滿分。可使用中文或英文作答。

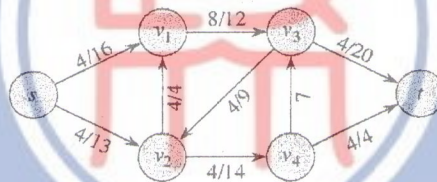
3. (10%) Procedure Q3 takes a n-element integer array $p[1..n]$ and an integer n as input. Let $T(n)$ denote the total number of calls made to $Q3(p, n)$. Please analyze the complexity of $T(n)$ in following pseudo code, and propose a better pseudo code

```

Q3(p, n)
1 if n == 0
2   return 0
3 q = -∞
4 for i = 1 to n
5   q = max( q, p[i]+Q3(p, n-i) )
6 return q
    
```

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4. (10%) What is an optimal Huffman code tree for the following set of frequencies, a:33 b:4 c:5 d:12 e:6 f:8 g:13 . The result of each pass must be listed.
5. (10%) $A=(a_{ij})$ and $B=(b_{ij})$ are square $n \times n$ matrices, write a recursive pseudo code to compute their multiplication $C=(c_{ij})$, and analyze the complexity of the pseudo code.
6. (10%) Describe an algorithm that, given n integers in the range 0 to k , preprocess the input and then answers any query about how many integers fall into a range $[a \cdots b]$ in $O(1)$ time, where $a, b \in \{0, 1, \dots, k\}$. The preprocess algorithm should be linear time complexity.
7. (10%) Give the definition of residual network and augmenting path in a flow network, and draw the residual network and augmenting path of the following flow network G and flow. (note: flow/capacity)



8. (10%) The Fibonacci numbers are defined by recurrence: $F_0=0, F_1=1, F_i=F_{i-1}+F_{i-2}$ for $i \geq 2$. Write an $O(n)$ -time dynamic-programming algorithm to compute the n th Fibonacci number.

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第一題[48%]: 單一選擇題，每小題答對得六分，未答得零分，答錯倒扣兩分。

1.1 Which of the following statements is true regarding the Interprocess communication?

- (A) Shared memory is typically faster than message passing.
 (B) Message passing is typically faster than shared memory.
 (C) Message passing is most useful for exchanging large amounts of data.
 (D) Shared memory is far more common in operating systems than message passing.

1.2 Which one of the followings is true of multilevel queue scheduling?

- (A) Process can move between queues.
 (B) Each queue has its own scheduling algorithm.
 (C) A queue cannot have absolute priority over lower-priority queues.
 (D) It is the most general CPU-scheduling algorithm.

1.3 Suppose that the following processes arrive for execution at the times indicated. Each process will run for the amount of time listed.

Process	Arrival Time	Burst Time
P1	0.0	8
P2	0.4	4
P3	1.0	1

Which one of the followings is correct?

- (A) The average turnaround time with the FCFS scheduling algorithm is 11.
 (B) The average turnaround time with the non-preemptive SJF scheduling algorithm is 9.53
 (C) The average turnaround time with the preemptive SJF (shortest remaining time first) scheduling algorithm is 4.33
 (D) None of the above.

1.4 When using semaphores, a process invokes the *wait()* operation before accessing its critical section, followed by the *signal()* operation upon completion of its critical section. Consider reversing the order of these two operations – first calling *signal()*, then calling *wait()*. What would be a possible outcome of this?

- (A) Starvation is possible
 (B) Several processes could be active in their critical sections at the same time.
 (C) Mutual exclusion is still assured.
 (D) Deadlock is possible.

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1.5 Consider a system with 5 processes P_0 through P_4 and 4 resource types A, B, C, and D. Suppose that the system has the following resource allocation status:

	Allocation				Max				Available			
	A	B	C	D	A	B	C	D	A	B	C	D
P_0	0	0	1	2	0	0	1	2	1	5	2	0
P_1	1	0	0	0	1	7	5	0				
P_2	1	3	5	4	2	3	5	6				
P_3	0	6	3	2	0	6	5	2				
P_4	0	0	1	4	0	6	5	6				

Using the Banker's algorithm, which one of the following statements is incorrect?

- (A) The matrix *Need* is
- | | A | B | C | D |
|-------|---|---|---|---|
| P_0 | 0 | 0 | 0 | 0 |
| P_1 | 0 | 7 | 5 | 0 |
| P_2 | 1 | 0 | 0 | 2 |
| P_3 | 0 | 0 | 2 | 0 |
| P_4 | 0 | 6 | 4 | 2 |
- (B) The system is in a safe state.
- (C) If a request from process P_1 arrives for (0,4,2,0), the request can be granted immediately.
- (D) None of the above.

1.6 Which one of the following statements is correct?

- (A) Consider a logical address space of 64 pages of 1024 words each, mapped onto a physical memory of 32 frames, there are 15 bits in the logical address.
- (B) Consider a logical address space of 64 pages of 1024 words each, mapped onto a physical memory of 32 frames, there are 16 bits in the physical address.
- (C) Consider a computer system with a 32-bit logical address and 4-KB page size. The system supports up to 512MB of physical memory. Using a conventional single-level page table, there are 2^{19} (2 to the power of 19) entries in the page table.
- (D) Consider a computer system with a 32-bit logical address and 4-KB page size. The system supports up to 512MB of physical memory. Using an inverted page table, there are 2^{17} (2 to the power of 17) entries in the page table.

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1.7 Consider a two-level page table scheme. A 32 bit logical address is divided into a 12 bits of outer page number and 12 bits of inner page number and 8 bits of page offset. If we trace a particular process and record the following address sequence with hexadecimal representation:

0x07EF9812, 0x07EF9823, 0x07EF98DE, 0x00000012, 0x00000045, 0x00000089, 0x01AABB99, 0x01AABB54, 0x01AABB77, 0x02989876, 0x00000078, 0x03DFDF89, 0x03DFDF77, 0x00000098, 0x048890FF, 0x048890EE, 0x02989874, 0x03DFDFDE, 0x00000099, 0x03DFDF78, 0x03DFDF24, 0x02989822, 0x02989845, 0x01AABB88, 0x01AABB70, 0x02989899, 0x02989845, 0x00000011, 0x01AABB23, 0x07EF9888, 0x00000099, 0x01AABB34, 0x01AABB87.

Which of the following statements is incorrect?

- (A) The number of page faults for the LRU replacement algorithms, with four page frames is 8.
- (B) The number of page faults for the optimal replacement algorithms, with three page frames is 9.
- (C) If we set working-set window to $\Delta = 10$ memory references, the working set size at the 10th memory references is 5.
- (D) The number of page faults for the FIFO replacement algorithms, with five page frames is 9.

1.8 Consider a disk queue holding requests to the following cylinders in the listed order: 116, 22, 3, 11, 75, 185, 100, 87. Using the SSTF scheduling algorithm, what is the order that the requests are serviced, assuming the disk head is at cylinder 88 and moving upward through the cylinders?

- (A) 116-22-3-11-75-185-100-87
- (B) 100-116-185-87-75-22-11-3
- (C) 87-75-100-116-185-22-11-3
- (D) 100-116-185-3-11-22-75-87

第二題 [12%]

Consider a multiprocessor system and a multithreaded program written using the many-to-many threading model. Let the number of user-level threads in the program be more than the number of processors in the system. Discuss the performance implications of the following scenarios.

- (a) The number of kernel threads allocated to the program is less than the number of processors.
- (b) The number of kernel threads allocated to the program is equal to the number of processors.
- (c) The number of kernel threads allocated to the program is greater than the number of processors but less than the number of user-level threads.

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第三題 [12%]

The following algorithm is proposed for trying to solve the critical-section problem for only 2 processes, P0 and P1. Shared variables flag[2] of Boolean data type are initially set to false; i.e., flag[0]=flag[1]=false. If variable flag[i] is set to true, then process Pi is ready to enter its critical section. The algorithm for Process Pi is as follows: (where i = 0 or 1, and $j = (i+1) \% 2$)

```
do {
    flag[i] = true;
    while (flag[j]);
        critical section
    flag[i] = false;
    remainder section
}while(1);
```

Does the above algorithm satisfy all three requirements (mutual exclusion, progress, bounded waiting)? Justify your answer.

第四題 [16%]

Consider a demand-paging system with the following time-measured utilizations:

CPU utilization	20%
Paging disk	97.7%
Other I/O devices	5%

For each of the following, say whether it will (or is likely to) improve CPU utilization. Explain your answers.

- (a) Install a bigger paging disk.
- (b) Increase the degree of multiprogramming.
- (c) Install more main memory.
- (d) Increase the page size.

第五題 [12%]

Consider a file system on a disk that has both logical and physical block size of 512 bytes. Assume that the information about each file is already in memory. For each of the three allocation strategies (contiguous, linked, and indexed),

How is the logical-to-physical address mapping accomplished in this system? (For the indexed allocation, assume that a file is always less than 512 blocks long.)

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I 離散數學部分(60%)

(1) 是非題 (10%)

- 1.1. $f(n) = O(n^2)$ implies $f(n) = O(n^3)$.
- 1.2. If A, B and C are sets of strings, then $(A \cap B) C = AB \cap AC$.
- 1.3. The logical expressions $\sim(p \rightarrow q)$ and $\sim p \rightarrow \sim q$ are logically equivalent.
- 1.4. Every complete lattice is bounded.
- 1.5. There are regular languages which are not context-free.
- 1.6. If G is a simple graph with 10 vertices and 35 edges, it must be connected.
- 1.7. There exists a Euler circuit in a complete simple graph K_6 with 6 vertices.
- 1.8. A binary tree of height $n \geq 0$ has no more than 2^n leaves.
- 1.9. A tree has more vertices than edges.
- 1.10. If G is a digraph with 20 vertices and has a path of length $k \geq 20$, then it must have also a path of length 133.

(2) 填充題(20%;每格 2%) :

- 2.1. If we randomly construct a simple graph G with 5 vertices. Let Pr be the probability of G having more than 5 edges. If $Pr = q/p$ where p and q are non-negative integers and $\gcd(p, q) = 1$. Then $p+q = \underline{\hspace{2cm}}$.
- 2.2. There are $\underline{\hspace{2cm}}$ different Boolean functions $f(x_1, \dots, x_n)$ of degree $n > 0$, among which there are $\underline{\hspace{2cm}}$ different Boolean functions satisfying the condition that $f(x_1, \dots, x_n) = f(y_1, \dots, y_n)$ if $x_1 + \dots + x_n = y_1 + \dots + y_n$.
- 2.3. The rooted Fibonacci tree T_n are defined recursively in the following way: T_1 and T_2 are both rooted tree consisting of a single vertex, and for $n \geq 3$, the tree T_n is constructed from a root with T_{n-1} as its left subtree and T_{n-2} as its right subtree. Then T_{10} has $\underline{\hspace{2cm}}$ vertices and its height is $\underline{\hspace{2cm}}$.
- 2.4. Let f be an increasing function satisfies the divide-and-conquer relation $f(n) = 5 f(n/2) + 2n^2$ and the initial condition $f(1) = 1$. Then $f(n)$ has the asymptotic order $\Theta(\underline{\hspace{2cm}})$.
- 2.5 The equation $x + y + z < 20$ has $\underline{\hspace{2cm}}$ non-negative solutions.
- 2.6. There are $\underline{\hspace{2cm}}$ ways to assign 5 different jobs to 4 different employees if every employee is assigned at least one jobs.
- 2.7. Let M_R given below be the matrix representation of a relation R . Find the matrix $M_{R \cdot R}$ representing $R \cdot R$ and the matrix M_{R^+} representing the transitive closure R^+ of R .

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad M_{R \cdot R} = \underline{\hspace{2cm}}, \quad \text{and } M_{R^+} = \underline{\hspace{2cm}}.$$

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I 離散數學部分(續前頁)

計算與證明: (30%)

(3). [10%] Solve the following recurrence relation:

$$a_n = 2 a_{n-1} - 2 a_{n-2}$$

with the initial conditions that $a_0 = 1$ and $a_1 = 2$. Note your solution expression may contain complex numbers.

(4). [10%] Prove by induction that for all integer $n \geq 0$, the number $f_n =$

$$\frac{5+\sqrt{5}}{10} \left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-\sqrt{5}}{10} \left(\frac{1-\sqrt{5}}{2}\right)^n$$

is a non-negative integer.

(5) [10%] A poset (S, \leq) is said to be well-founded if there is no infinite sequence of elements x_1, x_2, x_3, \dots such that $x_1 > x_2 > x_3 > \dots$, where $x_i > x_{i+1}$ means $x_{i+1} \leq x_i$ but $x_i \neq x_{i+1}$. An equivalent definition of well-foundedness is called the minimal condition which the poset (S, \leq) is said to satisfy if every nonempty subset of S has a minimal element. Show that both definitions are indeed equivalent by completing the proof of the following propositions:

(a) if (S, \leq) is not well-founded, then it does not satisfy the minimal condition. [4%]

(b) if (S, \leq) does not satisfy the minimal condition, then it is not well-founded. [6%]

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II. 線性代數(40%)

(10%) 1. Let

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & k \\ 1 & k & 3 \end{bmatrix}$$

Find the values of k for which $\det A = 0$ and hence, or otherwise, determine the value of k for which the following system has more than one solution:

$$x + y - z = 1$$

$$2x + 3y + kz = 3$$

$$x + ky + 3z = 2.$$

Solve the system for this value of k and determine the solution for which $x^2 + y^2 + z^2$ has least value.

(10%) 2. Let

$$U = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 \\ 0 & 0 & 5 & 1 & 2 \\ 0 & 0 & 3 & 4 & 1 \end{bmatrix} \text{ and } V = \begin{bmatrix} 3 & -2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & -4 & 1 \end{bmatrix}$$

(a) Find UV using block multiplication (b) Are U and V block diagonal matrices? (c) Is UV block diagonal?

(10%) 3. Determine whether or not W is a subspace of R^3 where W consists of all vectors (a, b, c) in R^3 such that:

(a) $a = 3b$ (b) $a \leq b \leq c$ (c) $ab = 0$

(d) $a + b + c = 0$ (e) $b = a^2$ (f) $a = 2b = 3c$

(10%) 4. Suppose $v = (1, 3, 5, 7)$. Find the projection of v onto W or, in other words, find $w \in W$ that minimizes $\|v - w\|$, where W is the subspace of R^4 spanned by:

(a) $u_1 = (1, 1, 1, 1)$ and $u_2 = (1, -3, 4, -2)$

(b) $v_1 = (1, 1, 1, 1)$ and $v_2 = (1, 2, 3, 2)$