考試科目	資料結構及演算法	所別	資訊科學系	考試時間	2 月 26 日(六) 第 一 節

1. (20%) True/False. Please answer "T" or "F" for the following question. 答對每題 2%，答錯倒扣 1%

(1) $2^{n+1} = O(2^n)$

(2) nlogn = $O(n^2)$

(3) The lower bound of worst case time complexity of sort algorithm is Ω(nlogn).

(4) f(n) = Ω(g(n)) and g(n) = Ω(h(n)) imply f(n) = Ω(h(n)).

(5) Heap is useful in the Kruskal's algorithm for minimum spanning tree problem.

(6) Given a graph with n vertex, the minimum spanning tree of this graph contains exactly n edges.

(7) Binary search can perform on an arbitrary input array.

(8) If (u, v) is a minimum-weight edge in graph G, then (u, v) must belong to some minimum spanning tree of G.

(9) Merge sort needs extra O(N) space for sorting N data.

(10) Every deterministic algorithm have a correspondent non-deterministic version algorithm.

2. (20%) Single selection. 答對每題 2%，答錯倒扣 1%

(1) Which of the following input is the worst case of quick sort?

(a) 95, 78, 66, 6, 16

(b) 6, 16, 66, 78, 95

(c) 66, 16, 78, 95, 6

(d) 78, 66, 16, 6, 95

(2) Solve the following postfix express. 6 1 2 + 3 * 3 - /

(a) 0 (b) 1 (c) 3 (d) 12

(3) Which of the following formula is the worst case time complexity in term of comparison operations for quick sort of n records?

(a) T(n) = T(n/2) + cn

(b) T(n) = 2T(n/2) + cn

(c) T(n) = T(n-1) + cn

(d) T(n) = 2T(n/2) + n^2

(4) Which one is true?

(a) The best-case time complexity of quick sort is the same as that of bubble sort algorithm

(b) The worst-case time complexity of quick sort is the same as the radix sort algorithm.

(c) The average-case time complexity of quick sort is the same as the merge sort algorithm.

(d) Both quick sort and merge sort are stable.

備註	試題隨卷繳交

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(5) Which of the following problem is NOT NP-complete?

(a) 3 Coloring problem

(b) Hamilton cycle problem

(c) Elur tour

(d) Clique problem

(6) Suppose the postorder sequence is DCEBGFA and inorder sequence is DCBEAFG for the same binary tree. What is the preorder traversal?

(a) DCEBFAG

(b) ABCDEFG

(c) CDEBGFA

(d) CABGEFD

(7) Which of the following data structure is the least likely to be used in Dijkstra's shortest path algorithm

(a) heap (b) queue (c) stack (d) hashing

(8) The best time complexity of finding the maximum in a N-key AVL tree

(a) O(1) (b) O(logN) (c) O(N) (d) O(NlogN) (e) $O(N^2)$ (f) $O(N^2 \log N)$

(9) What is the time complexity of $T(n) = 7T(n/2) + \Theta(n^2)$

(a) $\Theta(n^2)$ (b) $\Theta(n^{\lg 7})$ (c) $\Theta(n^2 \log n)$ (d) $\Theta(n^2)$

(10) Consider a binary tree with height H. The maximum number of nodes will be

(a) $\log H$ (b) $2H$ (c) $2\log H+1$ (d) $2^H - 1$

說明: 3 ~ 8 題，請書寫必要的解題過程。僅書寫答案而缺乏必要的過程，亦無法獲得該題滿分。可使用中文或英文作答。

3. (10%) Use O notation to solve the two recurrences.

(a) T(n) = T(n-1) + n.

(b) T(n) = T(n/3) + T(2n/3) + n.

4. (10%) Use mathematical induction to show that when n is an exact power of 2, the solution of the recurrence

$$T(n) = \begin{cases} 2 & if \quad n = 2 \\ 2T(n/2) + n & if \quad n = 2^k, for\ k > 1 \end{cases}$$

is T(n) = nlgn.

備註	試題隨卷繳交

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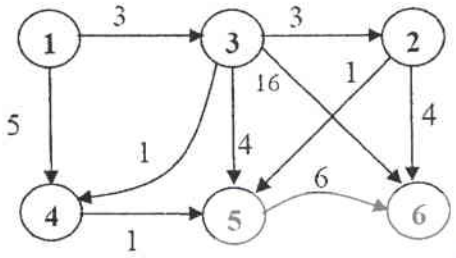
5. (10%) (a) Solve the longest path step by step from vertex 1 to vertex 6 for the graph in the following figure.
(b) Use Prim's algorithm step by step to find the minimum spanning tree for the below graph.(Assume all the edges are undirected)



6. (10%) Give the following keys 62, 105, 67, 22, 117, 92, please write down the result using the hash function h(X) = (X mod 10), with
(1) open addressing hash table using linear probing (F(i) = i)
(2) open addressing hash table with second hash function $h_2 = 7 - (X \mod 7)$. i.e. $F(i) = i * h_2(X)$

7.(10%) A example of convex/concave polygon shown in the below figure. Please describe the properties of a convex polygon, and write a pseudo to determine if a give polygon is convex or concave.

Convex polygon　　Concave polygon

8. (10%) Consider a binary tree as a graph, define the distance between two nodes as the number of branches traversed from first node to second node. Please write a pseudo code to find the maximum distance in the graph, and explain the key idea of your code.

備註	試題隨卷繳交

考試科目	作業系統	所別	資訊科學系	考試時間	2月 26日(六) 第 2 節

第一題[48%]：單一選擇題，每小題答對給六分，未答得零分，答錯倒扣兩分。

1.1 Which statement of the followings is wrong?

(A) A process may switch to the Ready state when an I/O event is completed.

(B) The short-term scheduler selects a process from the processes in memory that are ready to execute and allocates the CPU to that process.

(C) In a bounded temporary queue for communicating processes, the sender must always block until the recipient receives the message.

(D) The degree of multiprogramming refers to the number of processes in memory.

1.2 The following program uses the Pthreads API. What would be the output from the program at LINE C and LINE P, respectively?

```
#include <pthread.h>
#include <stdio.h>
int value = 0;
void *runner(void *param); /*the thread*/
int main(int argc, char *argv[])
{
int pid;
pthread_t tid;
pthread_attr_t attr;
   pid = fork();
   if(pid == 0) { /*child process*/
      pthread_attr_init(&attr);
      pthread_create(&tid, &attr, runner, NULL);
      pthread_join(tid, NULL);
      printf("CHILD: value = %d", value); /*LINE C*/
   }
   else if (pid > 0) { /*parent process*/
      wait(NULL);
      printf("PARENT: value = %d", value); /*LINE P*/
   }
}

void *runner(void *param)
{
   value = 5;
   pthread_exit(0);
}
```

(A) 5, 0

(B) 0, 5

(C) 5, 5

(D) 0, 0

備註	試題隨卷繳交	請注意：背面還有試題。

考試科目	作業系統	所別	資訊科學系	考試時間	2月 26日(六) 第 2 節

1.3 All five processes arrive at time 0, in the order given, with the length of the CPU burst given in milliseconds:

Process	Burst Time
P_1	10
P_2	29
P_3	3
P_4	7
P_5	12

Which statement is correct?

(A) With FCFS scheduling algorithm, the average waiting time is 29 milliseconds.

(B) With nonpreemptive SJF scheduling, the average waiting time is 12 milliseconds.

(C) With RR (quantum = 10 milliseconds) scheduling, the average waiting time is 23 milliseconds.

(D) All of the aboves are correct.

1.4 Consider a system consisting of three processes, P_0, P_1, and P_2, each accessing three semaphores, A, B, C, set to the value 1:

```
P0              P1              P2
wait(A)         wait(C)         wait(A)
wait(B)         wait(B)         ...
wait(C)         wait(A)         signal(A)
...             ...             wait(B)
signal(C)       signal(A)       ...
signal(B)       signal(B)       signal(B)
signal(A)       signal(C)       wait(C)
                                ...
                                signal(C)
```

The system

(A) always has deadlocks.

(B) has a race condition

(C) is always in safe state.

(D) None of the aboves.

備註	試題隨卷繳交

考試科目	作業系統	所別	資訊科學系	考試時間	2月 26 日(六) 第 2 節

1.5 Consider a system with 5 processes P_0 through P_4 and 3 resource types A, B, C. Suppose that the system has the following resource allocation status:

	Allocation			Max			Available		
	A	B	C	A	B	C	A	B	C
P_0	0	1	0	7	5	3	3	3	2
P_1	2	0	0	3	2	2			
P_2	3	0	2	9	0	2			
P_3	2	1	1	2	2	2			
P_4	0	0	2	4	3	3			

Which request can be granted according to the Banker's algorithm?

(A) Request for (4, 3, 1) by P_4
(B) Request for (1, 2, 2) by P_0
(C) Request for (1, 0, 2) by P_1
(D) None of the aboves

1.6 Consider a 32-bit address for a two-level paging system with an 8 KB page size. The outer page table has 1024 entries. How many bits are used to represent the second-level page table?

(A) 10
(B) 8
(C) 12
(D) 9

1.7 Suppose that we have the following page accesses: 1 2 3 4 2 3 4 1 2 1 1 3 1 4 and that there are three frames within our system. Using the true LRU replacement algorithm, what is the number of page faults for the given reference string?

(A) 14
(B) 13
(C) 8
(D) 10

1.8 Consider a disk queue holding requests to the following cylinders in the listed order: 116, 22, 3, 11, 75, 185, 100, 87. Using the C-SCAN scheduling algorithm, what is the order that the requests are serviced, assuming the disk head is at cylinder 88 and moving upward through the cylinders?

(A) 116-22-3-11-75-185-100-87
(B) 100-116-185-87-75-22-11-3
(C) 87-75-100-116-185-22-11-3
(D) 100-116-185-3-11-22-75-87

第二題[10%]

(a) What is the major difference between multi-process and multi-threaded programming?

(b) What are two differences between user-level threads and kernel-level threads? Under what circumstances is one type better than the other?

備註	試題隨卷繳交	請注意：背面還有試題。

考試科目	作業系統	所別	資訊科學系	考試時間	2月 26日(六) 第 2 節

第三題[12%]

The first known correct software solution to the critical-section problem for two processes was developed by Dekker. The two processes, P_0 and P_1, share the following variables:

```
boolean flag[2];   /*initially FALSE*/
int turn;          /*initially 0 or 1*/
```

The structure of process P_i in Dekker's algorithm:

Note: P_i (i == 0 or 1), the other process is P_j (j == 1 or 0)

```
do {
   flag[i] = TRUE;
   while(flag[j]) {
      if(turn == j) {    // part (b): this line removed
         flag[i] = FALSE;
         while(turn == j)
            ; // do nothing
         flag[i] = TRUE;
      }                  // part (b): this line removed
   }

   // critical section

   turn = j;
   flag[i] = FALSE;

   // remainder section
}while(TRUE);
```

(a) Prove that the algorithm satisfies all three requirements (mutual exclusion, progress, bounded waiting) for the critical-section problem.

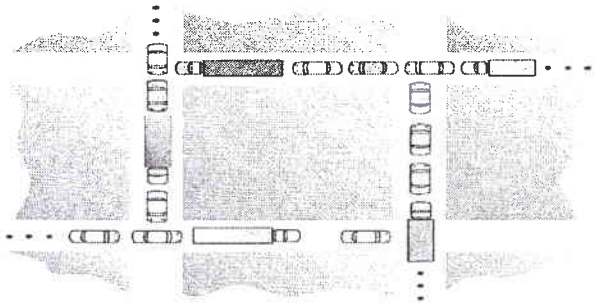
(b) If we removed the checking `if(turn == j)`, then what requirement(s) will not be satisfied?

第四題[10%]

Consider the traffic deadlock depicted in the following figure.

(a) Show that the four necessary conditions for deadlock hold in this example.

(b) State a simple rule for avoiding deadlock in this system.



備註	試題隨卷繳交

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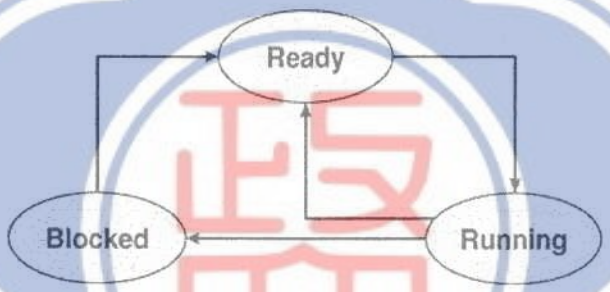
第五題[10%]

A simplified view of thread states is **Ready**, **Running**, and **Blocked**, where a thread is either ready and waiting to be scheduled, is running on the processor, or is blocked (i.e. is waiting for I/O). This is illustrated in the following figure. Assuming a thread is in the Running state, answer the following questions: (Be sure to explain your answer)

(a) Will the thread change state if it incurs a page fault? If so, to what new state?

(b) Will the thread change state if it generates a TLB miss that is resolved in the page table? If so, to what new state?

(c) Will the thread change state if an address reference is resolved in the page table? If so, to what new state?



第六題[10%]

Consider a file currently consisting of 100 blocks. Assume that the file control block (and the index block, in the case of indexed allocation) is already in memory. Calculate how many disk I/O operations are required for contiguous, linked, and indexed (single-level) allocation strategies, if, for one block, the following conditions hold. In the contiguous-allocation case, assume that there is no room to grow at the beginning but there is room to grow at the end. Also assume that the block information to be added is stored in memory.

(a) The block is added at the beginning.

(b) The block is removed from the end.

備註	試題隨卷繳交

考試科目	計算機數學	所別	資訊科學系碩士班	考試時間	2 月 26 日(六) 第 3 節

I 離散數學部分(60%)

(1). 選擇題 (20%)(單選不倒扣) :

1.1. Which of the following properties need not hold for a relation to be a partial order?

(a) transitivity (b) reflexivity (c) anti-symmetry (d) symmetry

1.2. How many equivalence relations are there on the set {1,2,3} ?

(a) 5 (b) 6 (c) 8 (d) 9

1.3. A simple graph is said to be regular if all of vertices have the same degree. Then which of the following graphs is not regular? Suppose n is an integer > 1.

(a) C_n (a cycle graph with n vertices) (b) K_n (a complete graph with n vertices)

(c) Q_n (A n-dimensional hypercube) (d) $K_{n,n}$ (a complete bipartite graph with n vertices in both partitions)

1.4. Which of the following relations is well-founded?

(a) $(Z, \leq)$ (b) $(Q, \leq)$ (c) $(R, \leq)$ (d) $(N, \leq)$

1.5. How many total orders are there on the set S = {1,2,4,8,3,9,72} which are compatible with the divisibility relation on S?

(a) 6 (b) 8 (c) 10 (d) 12

1.6. Let G =(V, T, S,P) be a context free grammar where N = {S} is the set of non-terminal symbols, T={0,1} is the set of terminal symbols, S is the start symbol and the set of productions P = {S → 1, S →0SS, S→S0S, S→SS0}. Then which of the following strings is derivable from the grammar?

(a) 01010 (b) 10010 (c) 11100 (d) 00011

1.7. Which of the following notations best describes the order of the function $(n^5+3n^3+2) / (7n^2 + 2n+100)$?

(a) $O(n)$ (b) $\Theta(n^2)$ (c) $\Theta(n^4)$ (d) $O(n^5)$

1.8. Which of the following propositions is not a tautology?

(a) $p\rightarrow(q\rightarrow p)$ (b) $(p\vee q)\wedge(\sim p \vee r)\rightarrow(q \vee r)$ (c) $(\sim p\rightarrow\sim q)\rightarrow(q\rightarrow p)$ (d) $(p\rightarrow q)\rightarrow(\sim p\rightarrow\sim q)$

1.9. If a planar graph G has 12 vertices, each of degree 3. Let e and r be the number of edges and regions, respectively, of G. Then what is the value of e + r ?

(a) 24 (b) 26 (c) 30 (d) 36

1.10 Let m and n be respectively the maximum and minimum number of nodes of all fully binary trees of height 8. The what is the value of m + n ?

(a) 528 (b) 511 (c) 513 (d) 530

備註	試題隨卷繳交

請注意：背面還有試題。

考試科目	計算機數學	所別	資訊科學系碩士班	考試時間	2 月 26 日(六) 第 3 節

I 離散數學部分(續前頁)

計算與證明: (40%)

(2). [15%] Consider the following linear non-homogeneous recurrence relation:

$a_n = 4\ a_{n-1} - 4\ a_{n-2} + 3x2^n$

(a) What is the general solution of its associated homogeneous recurrence relation $a_n = 4\ a_{n-1} - 4\ a_{n-2}$?

(b) Find a particular solution for the above non-homogeneous recurrence relation.

(c) Combine the above results to find a solution of the non-homogeneous relation satisfying the initial condition: a0=1 and a1= 7.

(3). [15%] Consider the following recursive procedure for computing the Ackermann's function A(m,n):

A(m,n) = if (m=0) then return 2n
else if(n = 0) then return 0
else if(n=1) then return 2
else return A(m-1, A(m, n-1)).

(a) Prove by induction that A(m,2) returns 4 if m is an integer ≥ 1.

(b) How many times will A(-,-) be called if we invoke A(1,n) with an integer n ≥ 1? Just give your answer and do not count the initial A(1,n) invocation.

(c) Explain briefly why A(m,n) will always terminate(run to end) if we call it by passing two non-negative integers.

(4). [10%] A set S is said to be *infinite* if there exists a sequence $x_0, x_1, x_2, \ldots$ of elements of S such that $x_i \neq x_j$ if $i \neq j$, or more formally if there exists a 1-1 mapping f:N→S from the set of non-negative integers N to S. Show that a given set S is infinite if and only if there is a 1-1 mapping g : S→S such that g(S) (i.e., the range of g) is a proper subset of S.

備註	試題隨卷繳交

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II. 線性代數部分(40%)

(5). [15%]

Determine whether or not each of the following matrices is *symmetric*, that is, $A^T = A$, or *skew-symmetric*, i.e., $A^T = -A$:

$$(a)\quad A = \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix}, \qquad (b)\quad B = \begin{bmatrix} 0 & 4 & -3 \\ -4 & 0 & 5 \\ 3 & -5 & 0 \end{bmatrix}, \qquad (c)\quad C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(6). [10%]

Let $u_1 = (1, 2, 4)$, $u_2 = (2, -3, 1)$, $u_3 = (2, 1, -1)$ in $\mathbf{R}^3$. Show that u_1, u_2, u_3 are orthogonal, and write v as a linear combination of u_1, u_2, u_3, where: (*a*) $v = (7, 16, 6)$, (*b*) $v = (3, 5, 2)$.

(7). [15%]

Find the dimension and a basis of the solution space W of each homogeneous system:

$$\begin{aligned} x + 2y + 2z - s + 3t &= 0 \\ x + 2y + 3z + s + t &= 0, \\ 3x + 6y + 8z + s + 5t &= 0 \end{aligned}$$
(*a*)

$$\begin{aligned} x + 2y + z - 2t &= 0 \\ 2x + 4y + 4z - 3t &= 0 \\ 3x + 6y + 7z - 4t &= 0 \end{aligned}$$
(*b*)

$$\begin{aligned} x + y + 2z &= 0 \\ 2x + 3y + 3z &= 0 \\ x + 3y + 5z &= 0 \end{aligned}$$
(*c*)

備註	試題隨卷繳交