考試科目	統計學	所別	金融所 總體金融政策組	考試時間	3 月 15 日 星期日 第一節

1. (20%) Prove or calculate the probability :

   a) (5%) If $P(A|C) > P(B|C)$ and $P(A|\overline{C}) > P(B|\overline{C})$ then $P(A) > P(B)$.

   b) (5%) If Y is a binomial random variable based on n trials with success probability p, calculate the conditional probability $P(Y > 1 | Y \geq 1)$.

   c) (10%) In a quality control plan for some production process, the worker samples $n = 10$ items per day and counts the number of defectives Y. Let p be the probability of observing a defective and p varies from day to day having a uniform distribution on the interval $[0,1/2]$. Find the expected value $E(Y)$ and variance $V(Y)$.

2. (20%) Calculate the following :

   a) (10%) Suppose the random variable Y has a distribution function given as

$$F(y) = \begin{cases} 0, & y < 0 \\ \left(\frac{y}{\theta}\right)^{\alpha}, & 0 \leq y \leq \theta \\ 1, & y > \theta \end{cases}$$

   where $\alpha, \theta > 0$. For fixed values of α and θ, find a transformation $G(U)$ such that $G(U)$ has a distribution function F when U is a uniform distribution on $[0,1]$ and prove your answer.

   b) (10%) The opening prices per share S_1 and S_2 of two similar stocks are independent variables, each with a density function given by

$$f(s) = \begin{cases} \left(\frac{1}{2}\right)\exp\left(\left(-\frac{1}{2}\right)(s-4)\right), & s \geq 4, \\ 0, & \textit{elsewhere.} \end{cases}$$

   On a given morning an investor will buy share of whichever stock is less expensive. Find the expected cost that the investor will pay.

3. (20%) Let $X_1, X_2, \cdots X_{50}$ be i.i.d. random sample from a Poisson distribution. The following table summarizes the outcome of the random sample :

x	0	1	2	3	4	5
frequency	5	13	10	11	6	5

   Find the maximum likelihood estimate of $P(X = 0)$.

備考	試題隨卷繳交
命題委員：	（簽章）

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考試科目	統計學	所別	金融所 總體金融政策組	考試時間	3 月 15 日 星期日 第一節

4. (15%) A tennis racket manufactory produces two kinds of rackets named R05 and R09, where R09 is the latest product. To test whether R09 is better than R05, ten players are randomly selected to test the improvement which is based on the success rate of the first serve. The number of the first serves that are good in 30 trials for each player and for each kind of rackets is recorded in the following table.

Player	1	2	3	4	5	6	7	8	9	10
R05	16	20	15	14	12	16	17	18	18	16
R09	14	18	16	14	14	13	21	20	21	22

Assuming the number of first serves that are good is normally distributed, then what would be the conclusion if the significance level is α.

5. (10%) It is common in economics or finance literature to assume that the rate of return on an asset is normally distributed. How would you test whether the assumption of normality is valid? (You may state as many methods as you can).

6. (15%) Let $X=[X_1,X_2,X_3]'$ be a 3-dimensional random vector distributed as $N_3(0,\Sigma)$, where

$$\Sigma=\begin{pmatrix}3 & 2 & 1\\ 2 & 2 & 1\\ 1 & 1 & 3\end{pmatrix}$$

a) (7%) Find the eigen values and the associated eigen vectors of Σ.

b) (8%) What is the distribution of $(X_1-2X_2+X_3)^2$?

備考	試題隨卷繳交
命題委員：	（簽章）

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考試科目	經濟學	所別	金融所 4/21 4/22	考試時間	3月15日 星期日 第3節

1. 何謂「財政赤字的貨幣化」？財政赤字貨幣化的現象，是在央行「釘住利率水準」還是「釘住貨幣數量」的政策下較容易發生？(13分)

2. 假設金融資產的風險性(riskiness)大幅上升，這會如何影響總合需求線（AD curve)？ (12分)

3. IS線的斜率如何影響貨幣政策與財政政策的有效性？根據以上的答案，不景氣的時候是貨幣政策還是財政政策比較有效？(13分)

4. On October 8, 2008, the Fed announced it would begin to pay interest on depository institutions' required and excess reserve balances. What's the implication of this policy on the federal funds rate? (12分)

5. Consider two ice cream firms, firm 1 and firm 2. The demand for firm i ($i = 1,2$) is $q_i = a + b\left(A_i + A_j\right)^{1/2}$, where A_i and A_j are the advertising expenditures from the two firms. Firm i's profit function is $\pi_i = p_i\left(a + b\left(A_i + A_j\right)^{1/2}\right) - A_i$.

   a. Does firm 2's advertising expenditure affect firm 1's market share? (10分)
   b. Assume that $p_1 > p_2$. What are the Nash equilibrium advertising expenditures? (10分)

6. Consider an exchange economy with three consumers and two goods. Utility functions for the three consumers are $u_1(x_1^1, x_1^2) = \log x_1^1 + \log x_1^2$, $u_2(x_2^1, x_2^2) = \log x_2^1 + 2\log x_2^2$, and $u_3(x_3^1, x_3^2) = 2\log x_3^1 + \log x_3^2$. Endowments are $w_1 = (1,1)$ (meaning consumer 1's endowments are 1 unit of good 1 and 1 unit of good 2), $w_2 = (2,1)$ and $w_3 = (3,1)$. Let the price of good 1 be 1 and the price of good 2 be p. Find each consumer's demand functions. Find equilibrium for p. (30分)

備考	試題隨卷繳交
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考試科目	財務管理	所別	金融 4/22 4/23	考試時間	3月15日 星期日 第一節

I. Explain the following terms briefly (each 4 points, total 24 points).

(1) Anomalies

(2) Venture capital

(3) Winner's curse

(4) Miller and Modigliani's proposition on dividend irrelevance

(5) Arbitrage

(6) Trade-off theory of capital structure

II. Computational problems (each 6 points, total 30 points).

(1) If you wish to provide $200,000 for your newborn's college education, how much should you invest now if the interest rate is 8% compounded annually?

(2) Bond B has a 5 year maturity with a 12% coupon. Suppose that the market discount rate is 13%. What's the duration for Bond B?

(3) A company has just paid a dividend of US $2 per share. Suppose the company has undertaken new management. It is now expected that the rate of growth in dividends will be 15 percent per year for 3 years, and then 10 percent per year thereafter. What is the stock price if the required return remains at 12 percent?

(4) Suppose an investor, who has square root utility and initial wealth of $100,000, is deciding whether or not to buy car insurance. The outcomes and associated probabilities, based on the investor's driving record are:

Event	Loss	Probability
No Accident	0	$p_u = 95\%$
Accident	-50,000	$p_d = 5\%$

How much should he/she pay for car insurance?

(5) Suppose *Assets* 1 and 2 have expected returns and standard deviations as follows:

Asset	Expected Return	Standars Deviation
1	20%	20%
2	10%	16%

Also assume that the returns of the two securities are perfectly negatively correlated $(\rho = -1)$. What is the composition of the minimum variance portfolio?

III. Short answer questions (each 6 points, total 18 points).

(1) Briefly explain why, in a competitive securities market, successive price changes are

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考試科目	財務管理	所別	金融系 4/22 4/23	考試時間	3月15日 星期日 第1節

random.

(2) Briefly explain the information content of share repurchase.

(3) Given the following data: Cost of debt = r_D = 6%; Cost of equity = r_E = 12.1%; Marginal tax rate = 35%; and the firm has 50% debt and 50% equity. Calculate the after-tax weighted average coat of capital (WACC).

IV. Questions (total 28 points).

(1) Assume that on November 1, the spot rate of the British pound was $1.58 and the price on a December futures contract was $1.59. Assume that the pound depreciated during November so that by November 30 it was worth $1.51.

a. What do you think happened to the futures price over the month of November? Why? (6 points)

b. If you had known that this would occur, would you have purchased or sold a December futures contract in pounds on November 1? Explain. (6 points)

(2) Assume that the United States invests heavily in government and corporate securities of Country K. In addition, residents of Country K invest heavily in the United States. Approximately $10 billion worth of investment transactions occur between these two countries each year. The total dollar value of trade transactions per year is about $8 million. This information is expected to also hold in the future.

Because your firm exports goods to Country K, your job as international cash manager requires you to forecast the value of Country K's currency (the "krank") with respect to the dollar. Explain how each of the following conditions will affect the value of the krank, holding other things equal. Then, aggregate all of these impacts to develop an overall forecast of the krank's movement against the dollar.

a. U.S. inflation has suddenly increased substantially, while Country K's inflation remains low. (4 points)

b. U.S. interest rates have increased substantially, while Country K's interest rates remain low. Investors of both countries are attracted to high interest rates. (4 points)

c. The U.S. income level increased substantially, while Country K's income level has remained unchanged. (4 points)

d. The U.S. is expected to impose a small tariff on goods imported from Country K. (4 points)

備考	試題隨卷繳交

命題委員：（簽章）

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國立政治大學九十八 學年度研究所 碩 士班入學考試命題紙

考試科目	數理統計學	所別	金融系	考試時間	3 月 15 日 星期日 第三節

1. (10%) Prove or Disprove

(1)(5%) If $P(A)>P(B)>0$, then $P(A|C)>P(B|C)$, where $P(C)>0$.

(2)(5%) If $P(A)=\alpha>0$ and $P(B)=\beta>0$, then $P(A|B)\geq\dfrac{\alpha+\beta-1}{\beta}$.

2. (15%)

Monte Carlo simulation has become one popular method in pricing derivatives. We often assume that a stochastic process follows a geometric Brownian motion. It is known that a Brownian motion is a special setting of a normal distribution. The following is one essential tool to generate correlated normal distributions.

(1)(10%) The Cholesky decomposition of a real symmetric matrix A is to decompose A as the product of two matrixes, i.e. $A=LL^T$, where L is a lower triangular matrix with strictly positive diagonal entries and L^T denotes the transpose of L. In addition, suppose that

$$A=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix}=LL^T.$$

Find the entries of L in terms of a_{ij}'s.

(2)(5%) A random vector X follows a multivariate normal distribution with mean vector μ and variance-covariance matrix Σ, where

$$\mu=\begin{bmatrix}0\\0\\0\end{bmatrix}\quad\text{and}\quad\Sigma=\begin{pmatrix}1&2&2\\2&5&5\\2&5&14\end{pmatrix}$$

Let $(Z_1,Z_2,Z_3)^T=(0.53,1.83,-2.25)^T$ be an i.i.d. random sample from $N(0,1)$. Use this random sample and Cholesky decomposition to determine the sample of random vector X.

3. (10%)

Suppose that $\varnothing$ is convex on an open interval I and has a second order derivative. State and Prove the theorem of Jensen's inequality.

4. (10%)

Suppose that X,Y are continuous random variables with joint probability density function $f(x,y)$.

Show that the distribution of $Z=\dfrac{Y}{X}$ is

$$f_Z(u)=\int_{-\infty}^{\infty}|x|f(x,u-x)\,dx.$$

備考	試題隨卷繳交
命題委員：	（簽章）

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考試科目	數理統計學	所別	金融系	考試時間	3 月 15 日 星期日 第三節

5. (10%)
Given that $N=n$, the conditional distribution of Y is $\chi^2(2n)$. The distribution of the number, N, is Poisson (λ). Calculate $E(Y)$ and $Var(Y)$.

6. (15%)
Suppose that $X_1, X_2, ..., X_n$ are i.i.d. with the probability density function given by

$$f(x;\theta)=\begin{cases}\frac{2x}{\theta^2}, 0\le x\le\theta,\\ 0 \quad , elsewhere.\end{cases}$$

(1)(10%) Find the maximum likelihood estimator $\hat{\theta}$ for θ.
(2)(5%) Find the maximum likelihood estimator for the median of the distribution.

7. (20%)
Suppose that $X_1, X_2, ..., X_n$ are i.i.d. normally distributed with mean μ and variance σ^2. Here, assume that σ^2 is a fixed positive number. Let Y be the median of the random sample $X_1, X_2, ..., X_n$ and let n be an odd number.

(1)(10%) Find the minimum variance unbiased estimator of μ among all unbiased estimators of μ.
(2)(5%) Compute $E(Y|\overline{X}=\overline{x})$.
(3)(5%) Is the variance of Y larger than the variance of $\overline{X}$? Explain.

8. (10%)
Let the sample space $\Omega=[0,1]$ with uniform distribution on this unit interval. Futher, let the random variables X and Y are defined as

$$X(\omega)=2\omega^2,\ Y(\omega)=\begin{cases}1 & if\ \omega\in[0,1/3],\\ 2 & if\ \omega\in(1/3,2/3],\\ 0 & if\ \omega\in(2/3,1].\end{cases}$$

Calculate the conditional expectation $E(X|Y)$.

備考	試題隨卷繳交
命題委員：	（簽章）

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考試科目	微積分	所別	金融學系(4123)	考試時間	3月15日 星期日 第3節

Instructions: Answer All Questions

1. [20 points]

   Show that, by introducing the change of variables: $x = r\cos\theta$, $y = r\sin\theta$, the following partial differential equations

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

   results in:

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} = 0$$

2. [20 points]

   Find the equations for the (a) tangent line, (b) normal plane to the curve $x = \varphi - \cos\varphi$, $y = 3 + \sin 2\varphi$, $z = 1 + \cos 3\varphi$ at the point where $\varphi = \frac{\pi}{2}$.

3. [30 points]
   Evaluate the following integrals:

   (a) (15 points)

$$\int_0^1 \frac{x}{(x^2+1)(x+1)^2}dx$$

   (b) (15 points)

$$\int_0^{\frac{\pi}{2}} \cos\theta\,(\sin\theta\cos\theta)^4\,d\theta$$

4. [30 points]

   (a) (15 points) Prove that, for non-exact first order differential equations of the form:

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

   , a function $\xi(x,y)$ is an integrating factor if and only if the following condition is satisfied:

$$M\xi_y - N\xi_x = -\xi(M_y - N_x)$$

   (b) (15 points) Hence solve the following differential equation:

$$(x^4y^2 - y) + (x^2y^4 - x)\frac{dy}{dx} = 0$$

備考	試題隨卷繳交
命題委員：	（簽章）

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