

考試科目	統計學	所別	金融	考試時間	3月16日 星期日	第 / 節
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1. (20%) A fast-food restaurant finds that its daily profits have a normal distribution with mean US\$140 and standard deviation US\$80.
  - (1) (6%) Find the probability that the restaurant loses money on a given day.
  - (2) (5%) Find the probability that the restaurant makes money for the next seven days in a row.
  - (3) (5%) What is the expected number of days that the restaurant makes money for the next seven days in a row?
  - (4) (4%) What assumptions must you make for this calculation to be valid for parts (2) and (3)?
  
2. (30%) For a large supermarket chain, a women's group claimed that female employees were passed over for management training in favor of their male colleagues. The company denied this claim, saying that they picked the employees from the eligible pool at random to receive this training. The large pool of more than 1000 eligible employees who can be tapped for management training is 40% female and 60% male. Since this program began, 28 of the 40 employees chosen for management training were male and 12 were female.
  - (1) (3%) In a significant test, the random sampling assumption is the claim of the company. Let  $p$  be the probability of selecting a male for any given selection. State the null and alternative hypotheses for a test based on  $p$  to investigate the strength of evidence to support the women's claim.
  - (2) (8%) Conduct a statistical test for part (1) using a 0.05 significance level.
  - (3) (5%) Find and interpret the  $P$ -value for this test.
  - (4) (6%) Construct a 95% confidence interval for  $p$ .
  - (5) (8%) What is the probability of committing a type II error when  $p = 0.4$ ?
  
3. (20%) A tax assessor wants to estimate the mean property tax bill for all homeowners in a small town. A survey ten years ago got a sample mean and standard deviation of \$1400 and \$1000, respectively.
  - (1) (6%) How many tax records should the tax processor randomly sample in order for a 95% confidence interval for the mean to have a margin error equal to \$100?

備考	試題隨卷繳交
命題委員：	(簽章) 年 月 日

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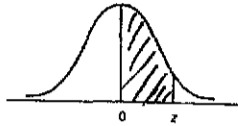
考試科目	統計學	所別	金融 42	考試時間	3月16日 第一節
<p>(2) (7%) In reality, suppose that they'd now got a standard deviation equal to \$1500. Using the sample size you derived in part (1), please find the margin of error. Explain whether and why this margin of error would be less \$100, equal to \$100, or more than \$100.</p> <p>(3) (7%) Refer to part (2). What is the probability that the sample mean falls within \$100 of the population mean? Explain whether and why this probability would be less 0.95, equal to 0.95, or more than 0.95.</p> <p>4. (30%) The following estimated regression equation was developed after 20 fast-food outlets were surveyed,</p> $\hat{y} = 10.1 - 4.2x_1 + 6.8x_2 + 15.3x_3,$ <p>where</p> <p><math>x_1</math> = number of competitors within one mile,  <math>x_2</math> = population within one mile (1000s),  <math>x_3</math> = <math>\begin{cases} 1 &amp; \text{if drive-up window present} \\ 0 &amp; \text{otherwise} \end{cases}</math>,  <math>y</math> = sales (US\$1000s).</p> <p>The following results are also obtained</p> $SS_{Reg} = 250 \text{ and } SS_{Total} = 310.$ <p>(1) (10%) Please complete the ANOVA table for the regression analysis and interpret the resulting table. Make a decision for the model using a 0.05 significance level.</p> <p>(2) (5%) Compute the value of <math>R^2</math> and interpret it.</p> <p>(3) (5%) Compute the standard error of the regression model and interpret its meaning.</p> <p>(4) (3%) What is the expected amount of sales attributable to the drive-up window?</p> <p>(5) (3%) Predict sales for a store with two competitors, a population of 8000 within one mile, and no drive-up window.</p> <p>(6) (4%) What assumptions must you make for carrying out a linear regression analysis?</p>					
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國立政治大學九十七學年度研究所<sup>412</sup>博士班入學考試命題紙 第三頁，共四頁

考試科目	統計學	所別	金融所	考試時間	3月16日 星期日 第1節
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Normal Probabilities



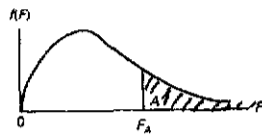
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

備考	試題隨卷繳交
命題委員：	(簽章) 年 月 日

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Critical Values of F: A = .05



DENOMINATOR DEGREES OF FREEDOM $\nu_2$	$\nu_1$	NUMERATOR DEGREES OF FREEDOM								
		1	2	3	4	5	6	7	8	9
1		161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2		18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3		10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4		7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5		6.61	5.79	5.41	5.19	5.03	4.95	4.88	4.82	4.77
6		5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7		5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8		5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9		5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10		4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11		4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12		4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13		4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14		4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15		4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16		4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17		4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18		4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19		4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20		4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21		4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22		4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23		4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24		4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25		4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26		4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27		4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28		4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29		4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30		4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40		4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60		4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120		3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞		3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

備考	試題隨卷繳交
命題委員：	(簽章) 年 月 日

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國立政治大學九十七學年度研究所<sup>博</sup>士班入學考試命題紙

第 1 頁，共 2 頁

考試科目	經濟學	所別	金高 4121 金高 4122	考試時間	3月16日 星期日	第3節
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1. A consumer has a utility function given by  $u(w) = \ln w$ , where  $w$  denotes income. He is offered the opportunity to bet on the flip of a coin that has a probability  $\pi$  of coming up heads. If he bets  $\$x$ , he will have  $w+x$  if head comes up and  $w-x$  if tails comes up.
- (a) Solve for the optimal  $x$  that maximizes the person's expected utility. (10 points)
- (b) Will the person accept a sufficiently small bet if he is offered favorable odds? (10 points)

2. There is one nondurable consumption good, of which  $q_1$  is consumed in period 1 and  $q_2$  in period 2. The corresponding prices are  $p_1$  and  $p_2$ , and incomes (paid at the beginning of each period) are  $y_1$  and  $y_2$ . Let  $A_0$  be the value of assets at the end of period 0 and  $r_1$  and  $r_2$  be the interest rates in the two periods, where interests are paid at the beginning of each period.
- (a) Find a person's lifetime budget constraint. (5 points)
- (b) If the utility function is given by
- $$u = \beta_1 \ln(q_1 - \bar{q}_1) + \beta_2 \ln(q_2 - \bar{q}_2),$$
- where  $\bar{q}_1$  and  $\bar{q}_2$  are two known constants, derive the Marshallian (ordinary) demands for  $q_1$  and  $q_2$ . (10 points)

3. Suppose each firm in a competitive industry has the long-run total cost function
- $$LTC = 60y - 20y^2 + 2y^3,$$
- and that the demand curve for the industry is
- $$Y^d = 610 - P.$$
- (a) In long-run equilibrium, what are the market price, industry output, the number of firms, and the profit of each firm? (12 points)
- (b) In perfectly competitive industries, economic theory predicts there cannot be relatively large incomes earned in the long run. True, false, or uncertain. Explain your answer. (8 points)

備 考 試 題 隨 卷 繳 交

命 題 委 員 :

(簽章) 97年 3 月 3 日

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國立政治大學 九十七學年度 碩士班暨碩士在職專班招生考試 命題紙

第 2 頁，共 2 頁

考試科目	經濟學	所別	金融 4121 4122	考試時間	3 月 16 日 星期日	第 3 節
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4. (a) What is the connection between the Phillips curve and the aggregate supply curve? Explain why the modern theories of the two curves are consistent. (10 points)

(b) No one is really certain about the level of the natural rate of unemployment in the economy at any moment. Suppose that the fiscal and monetary authorities underestimated the natural rate and pursued policies that would be appropriate for combating cyclical unemployment. What would happen in the short and long run? (10 points)

5. You are given the following IS-LM model for a closed economy, where the consumption function  $c(y, r)$ , the investment function  $i(y, r)$ , and the function of demand for money  $l(y, r)$  are all assumed to be dependent upon real output  $y$  and the interest rate  $r$ , and the government expenditure is denoted by  $g$ , an exogenous variable. In addition,  $M/P$  is the real money balances, where  $M$  signifies the supply of money and  $P$  the price level.

(1)  $y = c(y, r) + i(y, r) + g$

(2)  $\frac{M}{P} = l(y, r)$ .

(a) Check that the slopes of the IS and LM curves have the expected signs. (10 points)

(b) Derive mathematically the effects on equilibrium output and interest rate of an increase in the supply of money by open market operations. (10 points)

(c) Under what conditions will money be neutral, i.e., money supply changes have no effect on equilibrium output? Explain the economic implications. (5 points)

備	考試題隨卷繳交
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考試科目	財務管理	所別	金融	考試時間	3月16日 星期日	第一節
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I. Explain the following terms briefly (each 4 points, total 28 points).

- (1) Abnormal return
- (2) Consol
- (3) CMOs
- (4) Hedge ratio (Delta)
- (5) LBO
- (6) Liquidity premium
- (7) Weak-form efficient market

II. Computational problems (each 6 points, total 30 points)

(1) The Chinese economy is predicted to average double digit inflation over the next two years of 12% per annum. The inflation forecast for the US is 2.5% per annum. If the current exchange rate is US\$0.13/RMB, what will be the exchange rate two years from now (US\$/RMB)?

(2) Consider a bond with a face value of \$1,000, a coupon rate of 0%, a yield to maturity of 9%, and ten years to maturity. What is the duration of this bond?

(3) The following table gives the available projects for a firm:

Project	A	B	C	D	E	F	G
Initial investment	90	20	60	50	150	40	20
NPV	140	70	65	-10	30	32	10

If the firm has a limit of 210 million to invest, what is the maximum NPV the company can obtain?

(4) The market value of NCCU Corporation's common stock is \$20 million, and the market value of its risk-free debt is \$5 million. The beta of the company's common stock is 1.25, and the market risk premium is 8%. If the Treasury bill rate is 5%, what is the company's cost of capital? (Assume that there are no taxes.)

(5) Suppose NCCU's stock price is currently \$20. In the next six months it will either fall to \$10 or rise to \$40. What is the current value of a six-month call option with an exercise price of \$12? (Assume that the six-month risk-free interest rate is 5%).

備 考 試 題 隨 卷 繳 交

命 題 委 員 :

(簽章) 97年3月4日

考試科目	財務管理	所別	金融 4122 4123	考試時間	3月16日 星期日	第1節
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III. Answer the following questions **briefly** (each 8 points, total 24 points).

- (1) Considering two different financial systems – one is market-based and the other is bank-based. Please explain which financial system emphasize “transparency” more, and why?
- (2) Company Z-Speed has issued a 5-year coupon bond and company A-tom has issued a zero-coupon bond with the same maturity. Explain which company's debt is more difficult to value, and why?
- (3) An option analyst argue that “the writer of the call and the buyer of the put both hope that the stock price will decline. Therefore the two positions are identical.” Is this analyst correct? Please explain with diagrams.

IV. Questions (each 9 points, total 18 points)

- (1) Consider the following two tickets: ticket A pays \$100 if Mr. Hsieh is elected as president, ticket B pays \$100 if Mr. Ma is elected. Could the two tickets sell for less than the present value of \$100? Could they sell for more? What are the implications for MM's proposition I?
- (2) In Zombie Republic, the top personal income tax rate is 40%. This rate applies to interest and dividends. Capital gains are tax free, providing that realized gains do not exceed an annual allowance of \$10,000.  
All the individual stockholders of Super Group are in the top Zombie's tax bracket, but manage their portfolios so that realized capital gains never exceed their annual allowances. Suppose that the Group's effective corporate tax rate is 30%. How does the **sum** of corporate and personal taxes change if the Group:
  - (a) Operates at a higher or lower debt ratio?
  - (b) Increases or reduces cash dividend payout, holding capital investment and debt constant?
  - (c) Given your answers to (a) and (b), how would you advise the Group about debt and dividend policy?

備 考 試 題 隨 卷 繳 交

命 題 委 員 :

(簽章) 97年 3月 4日

考試科目	數理統計	所別	金融(財工)	考試時間	3月16日 星期日	第三節
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- Consider a random sample  $X_1, X_2, \dots, X_n$  from a discrete distribution with pdf
 
$$f(x; \theta) = \frac{(\theta+1)^x}{(\theta+2)^{x+1}} \text{ if } x = 0, 1, \dots \text{ where } \theta > 0. \text{ Find a UMP test of}$$

$$H_0: \theta = \theta_0 \text{ against } H_a: \theta > \theta_0. \text{ (10\%)}$$
- Let  $X \sim Poi(\lambda)$ , and let  $\theta = P(X \leq 1)$ .
  - Find an unbiased estimator of  $\theta$ . (6%)
  - Find a sufficient statistic for  $\theta$ . (5%)
  - Find a UMVUE of  $\theta$ . (7%)
- If  $X_1, X_2, \dots, X_n$  is a random sample from a distribution with p.d.f.
 
$$f(x; \theta) = \theta^2 x e^{-\theta x}, \quad 0 < x < \infty, \text{ zero elsewhere, where } 0 < \theta < \infty;$$
  - Find a complete sufficient statistic for  $\theta$ . (5%)
  - Show that  $X_1 / \sum_i X_i$  and  $\sum_i X_i$  are independent. (8%)
  - What is the distribution of  $X_1 / \sum_i X_i$ ? (12%)
- Consider a simple linear regression model,  $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ , where
 
$$E(\varepsilon_i) = \mu \neq 0, \quad Var(\varepsilon_i) = \sigma^2, \quad \beta_0 \text{ and } \beta_1 \text{ are unknown parameters.}$$
  - If we use the traditional least squares estimator of  $\beta_0$  and  $\beta_1$  ( $\hat{\beta}_0, \hat{\beta}_1$ ), will the estimators still have the unbiasedness? (12%)
  - $Cov(\hat{\beta}_1, \bar{y}) = ?$  (6%)
- Let  $Y_i \sim LOGN(\mu_i, \sigma_i^2) \quad i = 1, \dots, n$  be independent. Find the distribution of:
  - $\prod_{i=1}^n Y_i$ . (4%)
  - $Y_1/Y_2$ . (5%)
  - Find  $E\left[\prod_{i=1}^n Y_i\right]$ . (5%)
- Let  $X$  be a Negative Binomial random variable with parameter  $k=3$  and  $p=0.3$ .
  - Compute the moment generating function of  $X$ . (8%)
  - What is the limiting distribution of  $X$  as  $k \rightarrow \infty, p \rightarrow 1$  and  $k(1-p) = \lambda$  (constant). (7%)

考試科目	微積分	所別	金融系 4123 財務工程與金融創新組	考試時間	3月16日 星期日 第3節
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Instructions: Answer All Questions.

1. [25 Points]

(a) (10 Points) Let

$$f(x, y, z) = \frac{z \cdot \sin y}{x}$$

where  $x = 3r^2 + 2s$ ,  $y = 4r - 2s^3$ ,  $z = 2r^2 - 3s^2$ .

Find

$$\frac{\partial f}{\partial r}$$

and

$$\frac{\partial f}{\partial s}$$

(b) (15 Points) If

$$f(\lambda x, \lambda y) = \lambda^n f(x, y)$$

for some constant  $n$ , and all real values of  $\lambda$ .

Show that

$$nf(x, y) = x \left( \frac{\partial f}{\partial x} \right) + y \left( \frac{\partial f}{\partial y} \right)$$

2. [25 Points] Evaluate the following integrals:

(a) (10 Points)

$$\int \cos(x - \ln x) \left( \frac{x-1}{x} \right) dx$$

(b) (15 Points)

$$\int_0^3 \int_{x^2}^9 x^3 \exp(y^3) dy dx$$

3. [25 Points] Solve the following differential equations:

(a) (10 Points)

$$\frac{dy}{dx} = e^{y-x} \cdot \sec y \cdot (1+x^2)$$

with initial condition:  $y(0) = 0$ .

(b) (15 Points)

$$\frac{d^2 y}{dx^2} - 10 \frac{dy}{dx} + 21y = 3 \sin x$$

4. [25 Points]

Given that

$$f_n(x) = \frac{nx}{e^{nx^2}}$$

is a sequence of functions, where  $x \in [0, 1]$ , and  $n = 1, 2, 3, \dots$

(a) (10 Points) Evaluate

$$\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx$$

and

$$\int_0^1 \lim_{n \rightarrow \infty} f_n(x) dx$$

And explain why

$$\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx \neq \int_0^1 \lim_{n \rightarrow \infty} f_n(x) dx$$

(b) (15 Points) Let

$$f_n(x) = \frac{\sin nx}{n^3}$$

where  $n = 1, 2, 3, \dots$

Show that

$$\frac{1}{2} \int_0^\pi \left[ \sum_{n=1}^{\infty} f_n(x) \right] dx - \sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = 0$$