

考試科目	統計學	4121 所別	金融 政策組	考試時間	3月18日 星期日	第一節
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國立政治大學圖書館

1. (13%) Suppose that a survey of MBA students reveals that among GMAT score above 650, 52% took a preparatory course, whereas among GMAT scores of less than 650, only 23% took a preparatory course. An applicant to an MBA program has determined that he needs a score of more than 650 to get into a certain MBA program, but he feels that his probability of getting that high a score is quite low—10%. He is considering taking a preparatory course that cost \$500. He is willing to do so only if his probability of achieving 650 or more doubles. What should he do?

2. (25%) If a dealer's profit, in units of \$5000, on a new car can be looked upon as a random variable X having the density function

$$f(x) = \begin{cases} 2(1-x), & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

- (1) Find the average profit per car. (4%)
- (2) Find the variance of X . (6%)
- (3) Demonstrate that Chebyshev's inequality holds for 2 standard deviations with the density function above. (10%)
- (4) What is the probability that the profit exceeds \$500? (5%)

3. (12%) Suppose that a researcher postulates a model of the type

$$Y_i = \beta_0 + \beta_1 x_{1i} + \epsilon_i, \quad i = 1, 2, \dots, n,$$

where $\epsilon_i \stackrel{iid}{\sim} (0, \sigma^2)$. However, in fact, an additional variable, say x_2 , also contributes linearly to the response. The true model is then given by

$$Y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i, \quad i = 1, 2, \dots, n.$$

Compute the expected value of the estimator

$$B = \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1) Y_i}{\sum_{i=1}^n (x_{1i} - \bar{x}_1)^2}$$

備	考	試	題	隨	卷	繳	交
命題委員：							
111 (簽章) 年 月 日							

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考試科目	統計學	所別	金融(國際金融) 財政金融組	考試時間	3月18日 星期日	第一節
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國立政治大學圖書館

4. (50%) NCCU Bank wishes to compare two companies (Great Appraisals and Excellence Real Estate) they use to appraise the value of residential homes. NCCU Bank selected a sample of 10 residential properties and scheduled both firms for an appraisal. The results, reported in NT\$ million, are

Home	Great	Excellence
1	13.5	12.8
2	11.0	10.5
3	13.1	11.9
4	14.2	14.0
5	10.5	9.8
6	13.0	12.3
7	13.1	12.7
8	11.0	11.5
9	12.5	12.2
10	14.9	14.5

At the .05 significant level, please answer the following questions.

- (1) If σ_1 and σ_2 denote the population standard deviations of the appraised values by Great Appraisals and Excellence Real Estate, respectively. Please test the hypotheses $H_0 : \sigma_1^2 = \sigma_2^2$ vs. $H_a : \sigma_1^2 \neq \sigma_2^2$. (10%)
- (2) Can we conclude there is a difference in the mean appraised values of the homes?
 - (a) Apply the approach of dependent (or paired) samples. (10%)
 - (b) Apply the approach of independent samples. (10%)
 - (c) Which method do you prefer? And why? (5%)
- (3) Please carry out a one-way analysis of variance (ANOVA) for these data by treating company as a factor. (15%)

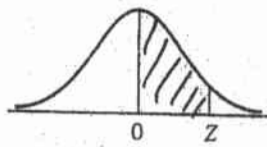
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考試科目	統計學	4121 所別	金兩考 總體金融 及策組	考試時間	3月18日 星期日	第一節
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Areas of the Standard Normal Distribution



The entries in this table are the probabilities that a standard normal random variable is between 0 and Z (the shaded area).

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986

國立政治大學圖書館

備 考 試 題 隨 卷 繳 交

命 題 委 員 : 113 (簽章) 年 月 日

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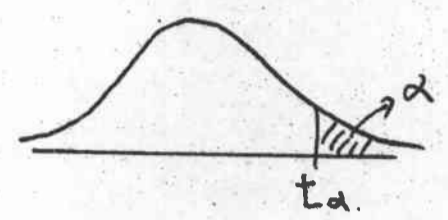
國立政治大學九十六學年度研究所(碩)博士班入學考試命題紙

第四頁，共四頁

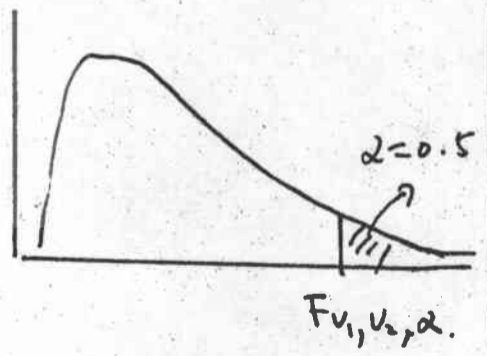
考試科目	統計學	所別	金(經)總(體)金(融)政策組	考試時間	3月18日 星期日 第一節
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國立政治大學圖書館

df	f.100	f.050	f.025	f.010	f.005	f.001
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.894
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450



v ₂	v ₁ α = .05								
	Numerator Degrees of Freedom								
	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28



備考 試題隨卷繳交

命題委員： 114 (簽章) 年 月 日

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考試科目	經濟學	421.402 所別	金融學系總體金融政策 組一般生 金融學系金融機構與市 場管理組一般生	考試時間	3月18日 星期日	第3節
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共四題，每題 25 分。

1. Suppose OPEC consists of two countries, whose production costs are

$$C_1 = q_1 + \frac{q_1^2}{2}$$

$$C_2 = 2q_2 + q_2^2$$

The demand for OPEC's oil is

$$Q = 10 - 2p$$

where $Q = q_1 + q_2$. To maximize OPEC's total profits, what should be the optimal quotas of q_1 and q_2 ? Will the two countries obey the quotas if they act non-cooperatively?

2. An entrepreneur has a project. This project requires a fixed investment I . The entrepreneur initially has cash $A < I$. To implement the project the entrepreneur must borrow $I - A$ from lenders.

If undertaken, the project either succeeds and yields verifiable income $R > 0$, or fails and yields no income. The project is subject to moral hazard. The entrepreneur can "behave" or "misbehave". Behaving yields probability $p = p_H$ of success and no private benefit to the entrepreneur, and misbehaving results in probability $p = p_L < p_H$ of success and private benefit $B > 0$.

Both the entrepreneur and the potential lenders are risk neutral. The riskless rate of return expected by lenders is taken to be 0. Lenders behave competitively in the sense that the loan makes zero profit. The entrepreneur is protected by limited liability, so her income cannot be negative. The loan contract specifies that in the case of success, R_b goes to the entrepreneur and R_l goes to the lenders ($R_b + R_l = R$). Both sides receive 0 in the case of failure.

- Write down the entrepreneur's incentive compatibility constraint for behaving.
- What is the highest expected income the lenders can receive without violating (a)?
- Combining your answer in (b) with the lenders' zero profit constraint, what is the threshold value of A that makes the lenders willing to lend to the entrepreneur?

國立政治大學圖書館

備 考 試 題 隨 卷 繳 交

命 題 委 員 : 115 (簽章) 96 年 3 月 5 日

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考試科目	經濟學	411、412 所別	金融學系總體金融政策 組一般生 金融學系金融機構與市 場管理組一般生	考試時間	3月18日 星期日	第3節
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國立政治大學圖書館

3. 請以 AD-AS 模型分析，所得稅上升對產出、利率、以及物價水準，分別在短期與長期的影響。假設在所得稅上升前，經濟體是達到充分就業下之自然產出水準
4. 請簡單回答以下問題：
 - a. 總合需求線與總合供給線的斜率為正或為負？原因為何？
 - b. 通貨膨脹的成本有哪些？什麼人可從通貨膨脹中獲利？
 - c. 根據一份 1989 年的報導，在當年美國花在研究治療 AIDS 的經費為 13 億美元，而花在心臟病的研發費用為 10 億美元。然而，在該年心臟病預期將奪走 77 萬 7 千人的生命，而 AIDS 則預期將導致 3 萬 5 千人的死亡。根據以上數據，從經濟學的角度是否美國花太多錢在 AIDS 上頭？



備考	試題隨卷繳交
命題委員：	116 (簽章) 96 年 3 月 5 日

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考試科目	財務管理	4122 所別 4123	金融系	考試時間	3月18日 星期日	第一節
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- I. True or false and briefly explain why (each 5 points, 40 points in total)
- (1). Investors prefer diversified companies because they are less risky.
 - (2). If stock were perfectly positively correlated, diversification would not reduce risk.
 - (3). A well-diversified portfolio with a beta of 2.0 is twice as risky as the market portfolio.
 - (4). A bond that is callable has a chance of being retired earlier than its stated term to maturity. Therefore, if the yield curve is upward sloping, an outstanding callable bond should have a lower yield to maturity than an otherwise identical noncallable bond.
 - (5). Suppose a firm is considering production of a new product whose projected sales include sales that will be taken away from another product the firm also produces. The lost sales on the existing product are a sunk cost and are not a relevant cost to the new product.
 - (6). Two firms have the same current ratio, 0.75, and the same amount of sales. However, Firm A has a higher inventory turnover ratio than Firm B. Therefore, we can conclude that the quick ratio of Firm A will be smaller than that of Firm B.
 - (7). Underlying the dividend irrelevance theory proposed by Miller and Modigliani is their argument that the value of the firm is determined only by its basic earning power and its business risk.
 - (8). The CAPM is built on expected conditions, although we are limited in most cases to use past data in applying it. Betas used in the CAPM, calculated using historical data, are always subject to changes in future volatility and this is a limitation on the use of the CAPM.

II. Explain the following term briefly (each 4 points, 20 points in total)

- | | |
|--------------------------------|-----------------------------|
| (1) value-additivity principle | (2) principal-agent problem |
| (3) LIBOR | (4) value-at-risk |
| (5) APT | |

III. (Computation problem, 16 points)

- (1) A merchant pays €100,000 for a shipment of red wine from France and is certain that it can be resold at the end of one year for €150,000. If the interest rate is 10%, what is the NPV of this project? (4 points)
- (2) You believe that ABC Company will be worth \$100 per share one year from now. How much are you willing to pay for one share today if the riskless rate is 2%, the expected return on the market is 8%, and the company's beta is 2? (4 points)

備 考 試 題 隨 卷 繳 交

命 題 委 員 : 117 (簽章) 96 年 3 月 6 日

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考試科目	財務管理	4122 所別 4123	金融系	考試時間	3月18日 星期日	第一節
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國立政治大學圖書館

(3) Two assets with the following expected return (μ) and risk (σ):

$$\text{Asset 1 } (\mu, \sigma) = (10\%, 10\%)$$

$$\text{Asset 2 } (\mu, \sigma) = (20\%, 20\%)$$

If Asset 1 and 2 are perfectly negatively correlated, please find the portfolio with the minimum risk, and what is the expected return of this portfolio? (8 points)

IV. (Computation problem, 24 points in total)

The ROC Company (ROC) currently has \$200,000 market value (and book value) of perpetual debt outstanding carrying a coupon rate of 6 percent. Its earnings before interest and taxes (EBIT) are \$100,000, and it is a zero-growth company. ROC's current cost of equity is 8.8 percent, and its tax rate is 40 percent. The firm has 10,000 shares of common stock outstanding selling at a price per share of \$60.00.

- (1). What is ROC's current total market value and weighted average cost of capital? (8 points)
- (2). The firm is considering moving to a capital structure that is comprised of 40 percent debt and 60 percent equity, based on market values. The new funds would be used to replace the old debt and to repurchase stock. It is estimated that the increase in riskiness resulting from the leverage increase would cause the required rate of return on debt to rise to 7 percent, while the required rate of return on equity would increase to 10 percent. If this plan were carried out, what would be ROC's new WACC and total value? (8 points)
- (3). Now assume that ROC is considering changing from its original capital structure to a new capital structure with 50 percent debt and 50 percent equity. If it makes this change, its resulting market value would be \$820,000. What would be its new stock price per share? (4 points)
- (4). Now assume that ROC is considering changing from its original capital structure to a new capital structure that results in a stock price of \$64 per share. The resulting capital structure would have a \$336,000 total market value of equity and \$504,000 market value of debt. How many shares would ROC repurchase in the recapitization? (4 points)

備 考 試 題 隨 卷 繳 交

命題委員： 118 (簽章) 96 年 3 月 6 日

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考試科目	微積分	4123 所別	金融學系 財務工程金融創新組	考試時期	3月18日 星期日	13:20~15:00
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1. Assume the existence and continuity of all derivatives under consideration. The equations $u = f(x, y)$, $x = X(t)$, $y = Y(t)$ define u as a function of t , say $u = F(t)$.

(a) Show that

$$F'(t) = \frac{\partial f}{\partial x} X'(t) + \frac{\partial f}{\partial y} Y'(t)$$

where $\partial f/\partial x$ and $\partial f/\partial y$ are to be evaluated at $[X(t), Y(t)]$.

(b) Compute $F'(t)$ in terms of t for the following special case:

$$f(x, y) = \ln[(1 + e^{x^2}) / (1 + e^{y^2})], \quad X(t) = e^t, \quad Y(t) = e^{-t}$$

2. If k is a positive constant and $g(x, t) = \frac{1}{2} \frac{x}{\sqrt{kt}}$, let $f(x, t) = \int_0^{g(x,t)} e^{-u^2} du$.

(a) Show that $\frac{\partial f}{\partial x} = e^{-g^2} \frac{\partial g}{\partial x}$ and $\frac{\partial f}{\partial t} = e^{-g^2} \frac{\partial g}{\partial t}$.

(b) Show that f satisfies the partial differential equation $k \frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial t}$.

3. Consider the second-order differential equation $(1 - x^2)y'' = -2y$.

Assume there exist a solution, say $y = f(x)$, which may be represented by a power-series

expansion in some neighborhood of the origin, say $y = \sum_{n=0}^{\infty} a_n x^n$.

(a) Show that $a_{n+2} = \frac{n-2}{n+2} a_n$.

(b) Show that the series for y can be written as follows:

$$y = a_0(1 - x^2) - a_1 \sum_{n=0}^{\infty} \frac{1}{(2n+1)(2n-1)} x^{2n+1}$$

4. Given the fact that $\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$, show that

$$\int_0^{\infty} f(x) e^{-\lambda x^2} dx \approx \frac{\sqrt{\pi} f(0)}{2\sqrt{\lambda}} + \frac{f'(0)}{2\lambda} + \frac{\sqrt{\pi} f''(0)}{8\lambda^{3/2}} + \frac{f'''(0)}{12\lambda^2} + \dots$$

where λ is large and positive.

備 考 試 題 隨 卷 繳 交

命 題 老 師 :

121 (簽章) 96年3月5日

國立政治大學圖書館

考試科目	數理統計學	所別	金融 ^{財務工程} 金融創新組	考試時間	3月18日 星期日	第3節
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國立政治大學圖書館

1. (10%) Let X have the cdf $F(x)$ which is a mixture of the continuous and discrete types :

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{2}, & 0 \leq x < 1 \\ 1, & 1 \leq x \end{cases}$$

Compute the mean and the variance of this distribution.

2. (15%)
 (a) What is Jensen's Inequality?
 (b) What is the inequality between $E(-\log X)$ and $-\log E(X)$?
 (c) Let the random variable X such that

$$\text{Prob}[X = a_i] = \frac{1}{n}, \quad i = 1, 2, \dots, n,$$

where a_i are positive numbers. Using (b) to show that the geometric mean $(a_1 a_2 \dots a_n)^{\frac{1}{n}}$ is less than or equal to the arithmetic mean $\frac{1}{n} \sum_{i=1}^n a_i$.

3. (10%) Let the joint pdf of X_1 and X_2 be

$$f_{X_1, X_2}(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)}, & 0 < x_i < \infty, \quad i = 1, 2, \\ 0, & \text{otherwise} \end{cases}$$

Find the pdf of $X_1 + X_2$

4. (15%) Let X and Y have the joint pdf

$$f(x, y) = \begin{cases} 6y, & 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional mean $E[Y | X]$ and variance of the conditional mean

$$V(E(Y | X)).$$

備 考 試 題 隨 卷 繳 交

命 題 委 員 : 119 (簽章) 2007 年 3 月 2 日

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考試科目	數理統計學	423 所別	金融 ^{財務工程組} 金融創新組	考試時間	3月18日 星期日	第3節
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國立政治大學圖書館

5. (10%) Let X and Y have a trinomial distribution with parameters n, p_1 and p_2 , their joint pdf is

$$p(x, y) = \frac{n!}{x!y!(n-x-y)!} p_1^x p_2^y (1-p_1-p_2)^{n-x-y}$$

Find the covariance $COV(X, Y)$.

6. (10%) Let $X = [X_1, X_2, X_3]'$ be a multivariate normal random vector distributed as $N(0, \Sigma)$, where

$$\Sigma = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$

Find the distribution of $X_1 - 2X_2 + X_3 + 5$

7. (15%) Let X be a continuous-type random variable with a pdf $f(x)$ which is positive and continuous on the support (a, b) , $-\infty \leq a < b \leq \infty$. Suppose $Y_1 < Y_2 < Y_3$ are the order statistics of the random sample.
- (a) Derive the joint pdf of Y_1, Y_2 , and Y_3 .
- (b) What is the pdf of Y_2 ?
- (c) Compute $P(Y_2 \geq m)$, where m is the median of the distribution of X .

8. (15%) Let $X_1 \dots X_n$ be a random sample from a Poisson distribution with parameter θ .

- (a) Compute the expectation $E(X_1 + 2X_2)$.
- (b) Compute the conditional expectation $E(X_1 + 2X_2 \mid \sum_{i=1}^n X_i)$.
- (c) Explain the role of sufficient statistic and relevant theorems.

備 考 試 題 隨 卷 繳 交

命題委員： 120 (簽章) 2007年3月2日

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