

考試科目	經濟學	系所別	商學院共同科	考試時間	3月6日(二)第二節
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Multiple Choice (1 point each)

Identify the letter of the choice that best completes the statement or answers the question.

- A consumer's utility function is $U(x, y, z) = 5y + \min\{x, 2z\}$. Now, at $p_x = 1, p_y = 5$, and $p_z = 2$, which of the following descriptions is correct as p_z marginally decreases.
 - The income effect increases the consumption of y .
 - There is no substitution effect.
 - There is no income effect.
 - The substitution effect increases the consumption of z .
- Suppose a consumer with a monotonic preference consumes only two goods. Then,
 - both goods need to be normal goods.
 - two goods are substitutes for each other.
 - if she has a downward sloping demand on one good, so as on another good.
 - None of the above is correct.
- Ann's preferences can be represented by the utility function $U(x, y) = (x - 3)^2 + (y - 4)^2$, then which of following is correct?
 - Her preference is convex.
 - Her preference is monotonic.
 - Commodity x is a "bad" for Ann.
 - If Ann has 10 dollars and $p_x = p_y = 1$, then Ann will consume $x = y = 5$.
- Consider a monopolistic competition market with a limited number of licenses that can be traded. In the long run, which of the following statements is incorrect?
 - All firms in the market earn zero profit.
 - Some firms have incentives to sell their licenses.
 - The potential entry firms will be indifferent between entering the market or not.
 - All firms produce at a quantity with decreasing average cost.
- Consider a market with demand, $Q_D = 200 - 5P$ and supply, $Q_S = 5P - 75$. The production process causes pollution for the environment as a negative external cost per unit is represented as $E_C = Q_S/5 + 10$. If the government plans to levy a Pigouvian ad valorem tax, which of the following is the correct tax rate?
 - 45%
 - 66%
 - 75%
 - 100%

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- 作答於試題上者，不予計分。
- 試題請隨卷繳交。

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6. What is the producer surplus of the monopoly when it conducts perfect price discrimination, given the demand function and marginal cost as $Q = 10 - P$ and $MC = 5$, respectively?

- A. 3.125
- B. 6.25
- C. 12.5
- D. 25

7. Among the following total cost functions of firms in a perfectly competitive market, which one has a meaningful shutdown point ($Q_{shutdown} > 0$)?

- A. $5Q^3 + 20Q^2 + 50Q + 100$
- B. $5Q^3 - 20Q^2 + 50Q + 100$
- C. $5Q^3 + 20Q^2 + 50Q - 400$
- D. $5Q^3 + 2Q^2 - 100Q + 100$

8. The long-run total cost function of a firm in a monopolistic competitive market is $TC(Q) = 7700Q - 100Q^2 + 1/3Q^3$. Which of the following represents a possible long-run equilibrium price and output?

- A. $(P, Q) = (100, 100)$
- B. $(P, Q) = (200, 150)$
- C. $(P, Q) = (500, 120)$
- D. $(P, Q) = (500, 180)$

9.

		Player 2		
		A	B	C
Player 1	A	(1,2)	(2,1)	(1,0)
	B	(0,5)	(1,2)	(7,4)
	C	(-1,1)	(3,X)	(2,2)

Which of the following values of X yields the above game exactly two pure strategy Nash equilibria?

- A. 0
- B. 1
- C. 1.5
- D. 3

10. Suppose a market has the following well-defined market supply curve: $Q = P + 3$. The market structure of this market could be:

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(i) Perfect competition.

(ii) Monopolistic competition.

(iii) Monopoly.

(iv) Oligopoly.

A. (i) only

B. (ii) and (iii)

C. (ii), (iii), and (iv)

D. (i), (ii), (iii), and (iv)

11. Suppose Iran has a comparative advantage over other countries in producing pistachios, but other countries have an absolute advantage over Iran in producing pistachios. If trade in pistachios is allowed, Iran

A. will import pistachios.

B. will export pistachios.

C. will have nothing to gain either from exporting or importing pistachios.

D. will either import or export pistachios, but it is not clear from the given information.

12. In the United States, the CPI was 100 in 1983 and is 250 in 2018, then \$100 in 2018 purchases the same amount of goods and services as

A. \$25 purchased in 1983.

B. \$40 purchased in 1983.

C. \$100 purchased in 1983.

D. \$250 purchased in 1983.

13. Seeing a looming credit crunch, banks choose to hold more excess reserves relative to their deposits. This action

A. does not change the money multiplier, but increase the money supply.

B. does not change the money multiplier, but decrease the money supply.

C. increases the money multiplier, and increase the money supply.

D. decreases the money multiplier, and decrease the money supply.

14. Suppose the inflation rate is high, but unemployment is at its natural rate. If the central bank pursues a contractionary monetary policy, which of the following results would be expected in the short run?

A. The short-run Phillips curve would shift to the left.

B. The short-run Phillips curve would shift to the right.

C. The economy would move up and to the left along a given short-run Phillips curve.

D. The economy would move down and to the right along a given short-run Phillips curve.

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15. Which of the following would transfer wealth from old to young?
- Increases in the budget deficit
 - Increases in housing prices
 - More generous education subsidies
 - Indexation of Social Security benefits to inflation
16. Which of the following results in higher inflation and higher unemployment in the short run?
- The Fed institutes an interest rate cut.
 - The Fed raises interest rates.
 - The price of oil increases.
 - All of the above.
17. Other things being equal, as the price level of a country increases,
- its real value of money falls and its currency appreciates.
 - its real value of money rises and its currency appreciates.
 - its real value of money falls and its currency depreciates.
 - its real value of money rises and its currency depreciates.
18. Which of the following shifts both the long-run aggregate supply curve and the short-run aggregate supply curve?
- a change in the price level
 - a change in the money wage rate
 - an advance in technology
 - All of the above.
19. If the price level is constant and the slope of the aggregate expenditure curve in the Keynesian model is 0.75, a decrease in investment of \$100 leads to a(n) _____ in real GDP of _____.
- decrease; \$75.
 - increase; \$75.
 - increase; \$400.
 - decrease; \$400.
20. Suppose the economy is experiencing an unanticipated event. Real borrowing costs rise for debtors due to the occurrence of this event. What kind of event might it be and what kind of action could the Fed take to help the debtors?
- An unexpected fall in price; increase the money supply.
 - An unexpected fall in price; decrease the money supply.
 - An unexpected rise in price; decrease the money supply.
 - An unexpected rise in price; increase the money supply.

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Problems and Short-essay Questions

1. David consumes two goods: pencils (x_1) and erasers (x_2). The price of a pencil is P_1 while the price for an eraser is P_2 . David has m dollars. David's utility function is

$$u(x_1, x_2) = \min\{6x_1 + x_2, x_1 + 2x_2\}.$$

- (7 points) Calculate the marginal rate of substitution ($\frac{dx_2}{dx_1}$) for David.
- (7 points) What are David's demand functions in terms of P_1 , P_2 , and m .
- (6 points) Suppose $P_1 = 2$, $P_2 = 4$, and $m = 40$. Moreover, if one buys 10 pencils or more, he will get 5 erasers for free. What is David's optimal choice now?

2. A perfectly competitive market in a small country has 50 identical firms. Suppose the market demand function is $P = 130 - Q$ and the supply curve of a firm is $P = -50 + 50q$, where P is the market price, Q is the market quantity supplied, and q is a firm's quantity supplied. Please answer the following questions in detail:

- (5 points) What is the market supply curve?
- (5 points) What is the market equilibrium under autarky?
- (10 points) Suppose the world price of the market product is 30, and the small country is currently under free trade. What level of import tariff should the government impose to make consumer surplus equal to 4512.5?

3. Below are some data from the land of milk and honey.

Year	Price of Milk	Quantity of Milk	Price of Honey	Quantity of Honey
2020	7	10	2	15
2021	5	17	5	3
2022	4	30	6	6

- (6 points) Compute the percentage change in nominal GDP between 2020 and 2022.
- (8 points) Suppose the government adopts the fixed-based approach to measure the real GDP by setting 2020 as the base year. What is the cumulative real GDP growth rate between 2020 and 2022?
- (6 points) Suppose the government adopts the chain-linked approach to measure the real GDP. What is the cumulative real GDP growth rate between 2020 and 2022?

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4. Suppose the nominal exchange rate between the dollar and the euro is 2 euros per dollar. Answer the following questions.

a. (5 points) The price of a book in Europe is 20 euros. If the purchasing power parity holds, how much will the same book cost in the United States?

b. (10 points) If the price level in the U.S. is 120 and the price level in Europe is 150, what is the real exchange rate in terms of European goods per U.S. good? By how much is the Euro undervalued or overvalued than predicted by the purchasing-power parity?

c. (5 points) If the nominal exchange rate between the dollar and the euro becomes 1.5 euros per dollar, what will happen to U.S. net capital outflow?



備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。
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考試科目	財務管理	系所別	金融學系	考試時間	2月6日(二)第三節
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Part I: Short answer questions (45 points).

- (15 points) How is duration linked to convexity in bond portfolio management, and what attracts investors to convexity?
- (20 points) What constitutes a private equity fund, and could you explain its operational mechanics and functions?
- (10 points) Define agency problems and suggest at least one method to alleviate potential issues related to agency problems.

Part II: Calculations. Calculation process is shown for full credit (55 points).

- (5 points) A firm has a stock price of \$50 per share and a P/E ratio of 70. If you buy the stock at this P/E and earnings fail to grow at all, how long should you expect it to take to just recover the cost of your investment?
- (5 points) The tax burden of the firm is 0.7, the interest burden is 0.5, the profit margin is 0.3, the asset turnover is 1.8, and the leverage ratio is 1.3. What is the ROE of the firm?
- (10 points) You consider buying a share of stock at a price of \$30. The stock is expected to pay a dividend of \$2 next year, and your advisory service tells you that you can expect to sell the stock in 1 year for \$35. The stock's beta is 1.8, r_f is 5%, and $E[r_m] = 12\%$. What is the stock's abnormal return?
- (10 points) A pension fund has an average duration of its liabilities equal to 20 years. The fund is looking at 8-year maturity zero-coupon bonds and 5% yield perpetuities to immunize its interest rate risk. How much of its portfolio should it allocate to the zero-coupon bonds to immunize if there are no other assets funding the plan?
- (10 points) You manage a hedge fund with \$200 million in assets. Your fee structure provides for a 1% annual management fee with a 20% incentive on returns over a 15% benchmark. If the fund value, before fees, is \$270 million at the end of the year, what is the net return to the investors?
- (15 points) An investor buys \$20,000 worth of a stock priced at \$25 per share using 70% initial margin. The broker charges 9% on the margin loan and requires a 35% maintenance margin. The stock pays a \$.80-per-share dividend in 1 year, and then the stock is sold at \$30 per share. What was the investor's rate of return?

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1. (30%) A nationwide sample survey shows that 40% of adults are committed to eating nutritious food when eating away from home. You decide to conduct a sample survey in your own area. You contact a simple random sample of 10 households by telephone.

(1) Five of the 10 respondents say they are concerned about nutrition. Is this reason to believe that the percent in your area is higher than the national 40%? (10%)

If you plan to contact a simple random sample of 600 households by telephone rather than just 10. Assume that the population proportion of adults concerned about nutritious food, $p=0.4$, is true.

(2) What is the probability that X lies between 220 and 230? (10%)

(3) What is the probability that 45% or more of respondents say they are concerned about nutrition? (10%)

2. (30%) The average cost per square foot for the office rental space in the central business district is NTD2358 according to a survey. A large real estate company wants to confirm this figure. The rent of per square foot paid by 100 offices in the central business district were investigated by the company. Suppose the sample average is NTD2288 per square foot, with a standard deviation of NTD511.

(1) Conduct a hypothesis test using 0.05 significance level to determine whether the cost per square foot reported by the survey should be rejected. (8%)

(2) If the decision in part (1) is to fail to reject and if the actual average cost per square foot is NTD2230, what is the probability of committing a Type II error? (12%)

(3) The probability of committing a Type II error is set at 0.1 in part (2). How large a sample is necessary to meet requirements? (10%)

3. (28%) A company wants to understand the impact of employee job satisfaction (score, Y , the higher the score, the higher the satisfaction) and their working experience (years,

X_1), salary (NTD\$1000, X_2) and gender (0 means female, 1 means male, X_3) relationship. The company's HR manager collected information from 25 employees and obtained the following estimation of the regression model

	Estimate	Standard error
Intercept	14.44	8.19
X_1	8.60	3.96
X_2	1.36	0.69
X_3	-5.95	3.30

$$SST=722.5, s=4.7$$

Here SST is the total sum of squares and s is the estimate of the standard deviation of the regression error.

(1) With a significance level of 0.05, test the significance of each regression coefficient in details and clearly

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H_0, H_1 , test statsitc, etc. (12%)

(2) Calculate the adjusted coefficient of determination. (8%)

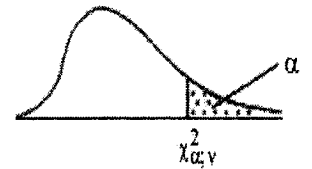
(3) Give the definition of the mean squared error (MSE), describe its menaing, and calculate MSE value. (8%)

4. (12%) A safety director samples at random from the file of minor accidents and classified them according to the time the accident took place

Time	Number of Accidents	Time	Number of Accidents
8 up to 9 A.M.	6	1 up to 2 P.M.	5
9 up to 10 A.M.	8	2 up to 3 P.M.	13
10 up to 11 A.M.	18	3 up to 4 P.M.	17
11 up to 12 P.M.	9	4 up to 5 P.M.	9

At the 0.02 significance level, can we conclude whether the accidents are evenly distributed throughout the day?

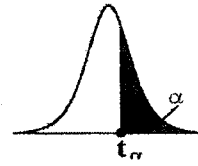
Table of the Chi-square Distribution



$\alpha =$	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001	$=\alpha$
V = 1	0.0000393	0.000157	0.000628	0.000982	0.00393	0.0158	0.0642	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827	V = 1
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815	2
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268	3
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465	4
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517	5
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457	6
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322	7
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125	8
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877	9
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588	10
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264	11
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909	12
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528	13
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123	14
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697	15

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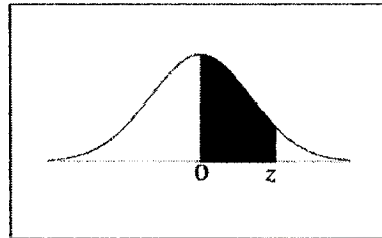
Percentage Points of the t Distribution; $t_{v, \alpha}$
 $P(T > t_{v, \alpha}) = \alpha$



v	α													
	0.40	0.30	0.20	0.15	0.10	0.05	0.025	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960	2.054	2.170	2.326	2.432	2.576	2.807	3.291

考 試 科 目	統計學 A	系 所 別	金融學系	考 試 時 間	2 月 6 日(二) 第四節
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Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

備

註

- 一、作答於試題上者，不予計分。
- 二、試題請隨卷繳交。

考試科目	統計學 B	系所別	金融學系財務工程與金融科技組	考試時間	2月6日(二)第三節
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1. (30 points) Consider the following simple linear regression model,

$$y_i = \alpha + \beta x_i + e_i, \quad i = 1, \dots, n,$$

where $e_i \stackrel{iid}{\sim} N(0, \sigma^2)$ and x_i is non-random.

- (10 points) Derive the ordinary least square (OLS) estimators, $\hat{\alpha}$ and $\hat{\beta}$.
 - (10 points) Derive the variances of $\hat{\alpha}$ and $\hat{\beta}$.
 - (10 points) Derive the variances of the fitted value \hat{y}_i and the residual \hat{e}_i .
2. (40 points) Suppose we estimate the following multiple linear regression model with OLS method,

$$y_i = \beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki} + \varepsilon_i, \quad i = 1, \dots, n,$$

where $\varepsilon_i \stackrel{iid}{\sim} N(0, \sigma^2)$. Let $R^2 = 1 - \frac{\text{residual sum of squares}}{\text{total sum of squares}}$ be the usual coefficient of determination which measures the goodness of fit.

- (10 points) Construct a statistic for testing $H_0: \beta_1 = \dots = \beta_k = 0$ in terms of R^2 , and please specify its distribution.
 - (10 points) Prove that if H_0 is true, the distribution of R^2 is $\text{Beta}\left(\frac{k}{2}, \frac{n-k-1}{2}\right)$.
Hint: If $z \sim F(d_1, d_2)$, then $\frac{d_1 z / d_2}{1 + d_1 z / d_2} \sim \text{Beta}\left(\frac{d_1}{2}, \frac{d_2}{2}\right)$.
 - (10 points) Let $k = 1$. Show that if H_0 is true, then R^2 converges to zero as the sample size n approaches infinity.
 - (10 points) Let $k = n - 1$. Show that if H_0 is true, then $\mathbb{P}(R^2 = 1) = 1$.
3. (30 points) Let $\{x_1, \dots, x_n\}$ be i.i.d. random variable with probability density function

$$f(x; \theta) = \frac{\theta}{(\theta + x)^2},$$

where $x \in [0, \infty]$ and $\theta \in [0, \infty]$.

- (10 points) Verify that $f(x; \theta)$ satisfies the definition of probability density function.
- (10 points) Evaluate the mean $\mathbb{E}[x]$.
- (10 points) Prove that the maximum likelihood estimator for θ exists.

Hint: You can use intermediate value theorem to show that the root for the first order condition exists.

備

註

- 作答於試題上者，不予計分。
- 試題請隨卷繳交。