考試科目	經濟學	系所別	商學院共同科	考試時間	2月3日(五)第二節

I、Multiple Choice (1 point each)

Identify the letter of the choice that best completes the statement or answers the question.

1. John has an income of $\$m$ per week. He consumes only two commodities, x and y. Let p_x be the price of x and p_y be the price of y. If he consumes more than $\bar{X}$ units of x per week, he can use coupons to buy the next Z units of x at a price of $p_x(1-s)$. If he buys more than $\bar{X}+Z$, he has to pay the full price p_x for additional units. His weekly income is greater than $p_x[\bar{X}+(1-s)Z]$. The maximum amount of x that he can buy per week is

A. $\bar{X}+\frac{mZ}{p_x}$.
B. $\frac{m+\bar{X}}{p_x}+Z$.
C. $\frac{m}{p_x}+sZ$.
D. $Z-\frac{(m+p_x)}{1-s}p_x$.

2. Professor Lin gives 3 midterm exams. Only the highest one counts. You are taking his course and have a 60 on your first exam. Let x_2 be your score on the second exam and x_3 be your score on the third exam. If you draw your indifference curves for scores on the second and third exams with x_2 represented by the horizontal axis and x_3 represented by the vertical axis, then your indifference curve through the point $(x_2, x_3) = (50, 70)$ is

A. ㄱ-shaped with a kink where $x_2 = x_3$.
B. three line segments, one vertical, one horizontal, and one running from $(70, 60)$ to $(60, 70)$.
C. a straight line, running from $(0, 70)$ to $(70, 0)$.
D. an L-shaped curve with its point at $(50, 70)$.

3. Consider the utility function to be $\min\{x, yz\}$. The price of x is \$1, the price of y is \$4, and the price of z is \$4. Henri's income is \$20. How many units of x does Henri demand?

A. 5
B. 20/9
C. 2
D. 3

4. Suppose that the production function is $f(x_1, x_2) = (x_1^a + x_2^a)^b + c$, where a, b, and c are positive constants. For what values of a, b, and c does the firm have constant returns to scale?

A. For any values of a if $b < 1$ and $c = 0$.
B. For any values of a and c if $ab < 1$.
C. For any values of a and c if $ab = 1$.
D. For any value of c if $a < 1$ and $b < 1$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	經濟學	系所別	商學院共同科	考試時間	2月3日(五)第二節

5. Roommate 1's utility function is $3X_1 + G$ and Roommate 2's utility function is X_2G, where G is their expenditures on the public goods they share in their apartment and where X_1 and X_2 are their respective private consumption expenditures. The total amount they have to spend on private goods and public goods is \$30,000. They agree on a Pareto optimal pattern of expenditures in which the amount that is spent on Roommates 1's private consumption is \$5,000. How much do they spend on public goods?

A. \$1,000
B. \$15,000
C. \$7,999
D. \$18,000

6. Consider a monopolistically competitive market in an economy moves from autarky to free trade. Which of the following statements is (are) correct?
(i) The domestic demand curve for a domestic firm will shift to the right.
(ii) The price elasticity of domestic demand that a domestic firm faces will increase in the absolute value.
(iii) The domestic firm's markup will decrease.

A. Only (i)
B. (i) and (ii)
C. (ii) and (iii)
D. (i) and (iii)

7. Which of the following curves is not affected by the existence of diminishing marginal product of input factors?

A. The average fixed cost curve.
B. The average variable cost curve.
C. The marginal cost curve.
D. The variable cost curve.

8. Suppose the shutdown point of a firm in a perfectly competitive market is that the market price is \$10. At the shutdown point, the average total cost of the firm is \$20. What is the average fixed cost?

A. 5
B. 10
C. 15
D. Need more information.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	經濟學	系所別	商學院共同科	考試時間	2月3日(五)第二節

9. The information in the table below shows the total demand for gasoline in a small urban market. Assume that each gasoline provider pays a fixed cost of $100,000 (per year) to operate in the market area and that the marginal cost of providing the gasoline to a household is zero.

Quantity	Quantity (per year)
0	120
3000	90
4000	80
5000	70
6000	60
7000	40
8000	30

Assume any agreement between two firms are not enforceable and two firms compete in quantity. What is the market price under the Nash equilibrium according to the table?

A. 80
B. 70
C. 60
D. 40

10. In a monopolistically competitive market, if the long run marginal cost curve intersects the long run marginal revenue cost curve and long run average cost curve when the marginal cost is $10 and $20 respectively, which of the following choices is a possible long run equilibrium market price?

A. 5
B. 10
C. 15
D. 21

11. How would a decrease in the frictional unemployment affect the long-run Phillips curve?

A. It would shift the long-run Phillips curve right.
B. It would shift the long-run Phillips curve left.
C. There would be an upward movement along a given long-run Phillips curve.
D. There would be a downward movement along a given long-run Philips curve.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	經濟學	系所別	商學院共同科	考試時間	2月3日(五)第二節

12. In the Unites States, the CPI was 80 in 1980 and is 300 today, then $100 today purchases the same amount of goods and services as
A. $26.67 purchased in 1980.
B. $33.33 purchased in 1980.
C. $40.00 purchased in 1980.
D. $80.00 purchased in 1980.

13. If total spending rises from one year to the next, then which of the following could not be true?
A. the economy is producing a smaller output of goods and services, and goods and services are selling at higher prices.
B. the economy is producing a larger output of goods and services, and goods and services are selling at lower prices.
C. the economy is producing a larger output of goods and services, and goods and services are selling at higher prices.
D. the economy is producing a smaller output of goods and services, and goods and services are selling at lower prices.

14. You put money into an account that earns a 3 percent real interest rate. The inflation rate is 2 percent, and the tax rate on your interest income is 20 percent. What is your after-tax real rate of interest?
A. 2.0 percent.
B. 2.4 percent.
C. 3.0 percent.
D. 3.6 percent.

15. Time inconsistency will cause the
A. short-run Phillips curve to be higher than otherwise.
B. short-run Phillips curve to be lower the otherwise.
C. long-run Phillips curve to be farther to the right than otherwise.
D. long-run Phillips curve to be farther left than otherwise.

16. Which of the following policies should be used to close an inflationary GDP gap?
A. A decrease in government purchases.
B. An increases in taxes.
C. A decrease in money supply.
D. All of the above.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	經濟學	系所別	商學院共同科	考試時間	2月3日(五)第二節

17. If a country has a current account balance of -$120 and a capital and financial account balance of $80, there will be _____ in official reserves of _____ .

A. an increase; $40.
B. an increase; $200
C. a decrease; $40
D. a decrease; $200

18. When the central bank in a country decreases the money supply, its

A. price level rises and its currency appreciates relative to other currencies in the world.
B. price level falls and its currency appreciates relative to other currencies in the world.
C. price level falls and its currency depreciates relative to other currencies in the world.
D. price level rises and its currency depreciates relative to other currencies in the world.

19. The Ricardo-Barro effect states that government deficits

A. increase the equilibrium real interest rate, crowding out investment.
B. decrease private saving, the equilibrium real interest rate and investment.
C. increase private saving and have no effect on the equilibrium real interest rate and investment.
D. decrease the equilibrium real interest rate and increase investment.

20. Suppose potential GDP exceeds real GDP in a short-run macroeconomic equilibrium. If aggregate demand does not change, then the

A. short-run aggregate supply curve will shift rightward as the money wage rate decreases.
B. short-run aggregate supply curve will shift leftward as the money wage rate increases.
C. long-run aggregate supply curve will shift leftward as the money wage rate increases.
D. long-run aggregate supply curve will shift leftward as the money wage rate decreases.

Ⅱ. Problems and Short-essay Questions

1. Consider the following utility function.

$$u(x_1, x_2) = \begin{cases} 3x_1 + x_2 & \text{if } x_1 > 2x_2, \\ \frac{7}{5}x_1 + \frac{21}{5}x_2 & \text{otherwise.} \end{cases}$$

A. (7 points) Calculate the marginal rate of substitution.

B. (6 points) (9,1) and (c,6) sit on the same indifference curve. What is the value of c?

C. (7 points) Let m stand for the consumer's income. Please calculate the demand function as a function of p_1, p_2, and m.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	經濟學	系所別	商學院共同科	考試時間	2月3日(五)第二節

2. Suppose Firm A is the only seller of Product A in a closed economy. The firm faces the following demand, marginal revenue, and marginal cost curves.

$$\text{Demand: } P = 70 - Q$$
$$\text{Marginal Revenue: } MR = 70 - 2Q$$
$$\text{Marginal Cost: } MC = 10 + Q$$
$$\text{Total cost: } TC = 210 + 10Q + 0.5Q^2$$

A. (4 points) Please calculate the efficient loss due to the monopoly.

Suppose the government of the economy decides to open the market to the world. The world price of Product A is \$30.

B. (4 points) How many units of Product A will the economy export or import in the short run?

C. (4 points) How many units of Product A will the economy export or import in the long run?

D. (8 points) In the long run, if the government wants to maintain that both the domestic producer and foreign importers coexist in the domestic market, the government should tax or subsidize the importers? What is the minimum value of the per-unit tax or the per-unit subsidy to achieve this goal?

3. Consider an economy where the representative consumer has a utility function $U = CF$ over clothing C and food F, and has an income of \$40.

A. (8 points) Suppose in year one (the base year), the prices of clothing and food are $p_C^1 = 2$ and $p_F^1 = 2$, respectively. What is the consumer's optimal consumption bundle? How much utility does the consumer receive from this bundle?

B. (6 points) Suppose in year two, the prices of clothing and food become $p_C^2 = 2.5$ and $p_F^2 = 10$, and the consumer's income increases in proportion to the consumer price index (CPI). What is the consumer's optimal consumption bundle?

C. (6 points) What is the minimum income in year two that enables the consumer to achieve the same level of utility as in year one? How much does the CPI overstate actual inflation due to the *substitution bias*?

4. In an economy, autonomous consumption expenditure is \$100 billion, investment is \$300 billion, and government expenditure is \$150 billion. The marginal propensity to consume is 0.90 and net taxes are \$150 billion. Exports are \$450 billion and imports are \$550 billion. Assume that net taxes and imports are autonomous and the price level is fixed.

A. (5 points) What is the value of consumption multiplier?

B. (10 points) Calculate the equilibrium aggregate expenditure.

C. (5 points) If government expenditure increases by \$200 million, what is the change in the economy's equilibrium real GDP?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	財務管理	系所別	金融學系/金融管理組	考試時間	2 月 3 日(五) 第 三節

I. Explain the following term briefly (24 points, 4 points for each)

1. maturity premium
2. opportunity cost of capital
3. countercyclical
4. specific risks
5. value stocks
6. trade-off theory

II. Computational Questions (30 points, 5 points for each)

1. The market portfolio has an expected return of 18% and the risk-free rate is 6%. An investor borrows $100 at the risk-free rate and invests this and a further $100 of his own in the market portfolio. What is his expected return?

2. A stock is expected to pay a year-end dividend of $8 and then to sell at a price of $109. The risk-free interest rate is 4%, the expected market return is 12% and the stock has a beta of 0.8. What is the stock price today?

3. A portfolio consists of an index mutual fund which represents the overall market and Treasury bills. The fund has a portfolio weight of 60%. The risk-free rate is 3.2% and the market risk premium is 7.6%. What is your best estimate of the portfolio expected rate of return?

4. If MM's proposition II without taxes is true and no bankruptcy risk exists, how much debt will a company prefer if their cost of debt is 6%, cost of equity is 10% and the corporate tax rate is 21%?

5. Assume an unlevered firm changes its capital structure to include $1 million in permanent debt at a 7% interest rate. The tax rate is 21%. According to MM I with taxes, due to this change in its capital structure, what will the value of the firm increase by?

6. Because of its age, your car costs $4,000 annually in maintenance expense. You could replace it with a newer vehicle costing $8,000. Both vehicles would be expected to last 4 more years, at which point they will be valueless. If your opportunity cost is 8%, by how much must maintenance expense decrease on the newer vehicle to justify its purchase?

III. Short Answer Questions (10 points, 5 points for each)

1. What is the slope of the security market line? Draw and explain.

考　試　科　目	財務管理	系 所 別	金融學系/金融管理組	考 試 時 間	2　月　3 日(五) 第 三節

2. Which of the following pair of firms do you think should be more highly levered: A retailing firm with prime downtown real estate, or a social media company whose major assets are its unique software and client loyalty? Briefly explain.

IV. Questions (36 points)

1. Megatrend's stock will generate earnings of $6 per share this year. The discount rate for the stock is 15%, and the rate on reinvested earnings also is 15%.
   a. Find both the growth rate of dividends and the price of the stock if the company reinvests the following fraction of its earnings in the firm: (i) 0%; (ii) 40%; (iii) 60%. (4 points for each)
   b. Redo part (a) now assuming that the rate of return on reinvested earnings is 20%. What is the present value of growth opportunities for each reinvestment rate? (4 points for each)
   c. Considering your answers to parts (a) and (b), can you briefly state the difference between companies experiencing growth and companies with growth opportunities? (4 points)

2. Alpha Inc. currently has EBIT of $25000 and is all-equity-financed. EBIT is expected to stay at this level indefinitely. The firm pays corporate taxes equal to 21% of taxable income. The discount rate for the firm's projects is 10%.

a. Now assume the firm issues $50000 of debt paying interest of 6% per year, using the proceeds to retire equity. The debt is expected to be permanent. What will happen to be the total value of the firm? (3 points)

b. following (a), the debt issue raises the probability of bankruptcy. The firm has a 30% chance of going bankrupt after three years. If it does go bankrupt, it will incur bankruptcy costs of $200000. The discount rate is 10%. Judge if the firm should issue the debt or not. (5 points)

備　　註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	統計學A	系所別	金融學系	考試時間	2 月 3 日(五) 第4 節

1. (25%) The table below is the total number of phone calls arriving to the information center of a bank in 20 randomly chosen working days during its open period (9:00-17:00).

Time	9:00:00-9:59:59	10:00:00-10:59:59	11:00:00-11:59:59	12:00:00-12:59:59	13:00:00-13:59:59	14:00:00-14:59:59	15:00:00-15:59:59	16:00:00-16:59:59
Total number of calls	170	285	314	370	299	122	194	236

The calls occur randomly and independently of one another.

(1) Please estimate the average number of calls in an hour. (5%)

(2) Assume that the hourly number of calls arriving to this information center follows a Poisson distribution. Find the probability that the information center receives more than two calls (including) at 11:00:00-11:10:00 (10 minutes) on a working day. (10%)

(3) Let p represent the ratio of the number of calls arriving to this information center at 12:00:00-12:59:59 to the total number of calls in all working days. At the significance level of 0.01, test whether the proportion of calls, p, exceeds 15%. (10%)

2. (20%) A quality control engineer is interested in the mean length of sheet insulation being cut automatically by machine. The desired length of the sheet insulation is 12 feet.

(1) The engineer randomly chooses 20 sheet insulations and the resulting sample mean length is 12.15 and sample standard deviation in the cutting length is 0.2 feet. Test whether the cutting machine is out of control at the significance level of 0.05. (10%)

(2) Suppose the engineer decided to estimate the mean length to within 0.025 with 99% confidence. What sample size would be needed? (10%)

3. (27%) The following table classifies students by field of study and whether or not they have a loan. The students are randomly selected from a large university.

Field of study	**Student loan**	
	Yes	**No**
Education	20	36
Engineering	94	141
Management	24	51
Science	51	39
Liberal arts	79	125

(1) At the significance level of 0.05, test whether the proportion of students studying in Engineering having a

考試科目	統計學 A	系所別	金融學系	考試時間	2月3日(五) 第4節

loan is greater than that of students studying in Management having a loan. (12%)

(2) Conduct a test to examine whether all the proportions of students having a loan in different fields are equal at the significance level of 0.1. (15%)

4. (28%) A bank manager considers a two-factor experimental design to examine whether different age groups (*Age* with 4 levels) of customers and interface designs (*Interface* with 3 levels) of the automated teller machine (ATM) have effect on the operation time (in seconds) when their customers use an ATM. Each treatment has 3 randomly chosen customers. The manager obtains the following analysis of variance (ANOVA) table.

	Sum of squares	Degrees of freedom	Mean squares	*F*
Age	64.4	(B)	(G)	(K)
Interface	27.6	(C)	(H)	(L)
Age×*Interface*	135.4	(D)	(I)	(M)
Error	(A)	(E)	(J)	
Total	399.5	(F)		

(1) Compute the values of (A) to (M), and describe their required formulas and calculation process in details. (18%)

(2) Describe the null hypothesis and alternative hypothesis for (L) and (M). Draw your conclusions for both tests at a significant level of 0.05. (10%)

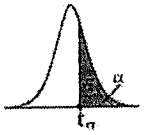
For all questions about statistical tests, please specify null hypothesis and alternative hypothesis in details.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	統計學 A	系所別	金融學系	考試時間	2月3日(五) 第4節

Percentage Points of the *t* Distribution; $t_{\nu,\alpha}$

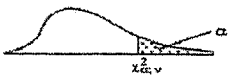
$P(T > t_{\nu,\alpha}) = \alpha$



	α													
ν	0.40	0.30	0.20	0.15	0.10	0.05	0.025	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960	2.054	2.170	2.326	2.432	2.576	2.807	3.291

考試科目	統計學A	系所別	金融學系	考試時間	2月3日(五) 第4節

Table of the Chi-square Distribution



α =	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001	= α
V = 1	0.0000393	0.000157	0.000628	0.000982	0.00393	0.0158	0.0642	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827	V = 1
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815	2
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268	3
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465	4
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517	5
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457	6
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322	7
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125	8
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877	9
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588	10
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264	11
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909	12
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528	13
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123	14
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697	15
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252	16
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790	17
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312	18
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820	19
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315	20
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797	21
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268	22
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728	23
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179	24
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620	25
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052	26
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476	27
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893	28
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302	29
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703	30
40	20.706	22.164	23.838	24.433	26.509	29.051	32.345	47.269	51.805	55.759	59.342	60.436	63.691	66.766	73.402	40
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.661	50
60	35.535	37.485	39.699	40.482	43.188	46.459	50.641	68.972	74.397	79.082	83.298	84.580	88.379	91.952	99.607	60
70	43.275	45.442	47.893	48.758	51.739	55.329	59.898	79.715	85.527	90.531	95.023	96.388	100.425	104.215	112.317	70
80	51.171	53.539	56.213	57.153	60.391	64.278	69.207	90.403	96.578	101.880	106.629	108.069	112.329	116.321	124.839	80
90	59.196	61.754	64.634	65.646	69.126	73.291	78.558	101.054	107.565	113.145	118.136	119.648	124.116	128.299	137.208	90
100	67.327	70.065	73.142	74.222	77.929	82.358	87.945	111.667	118.498	124.342	129.561	131.142	135.807	140.170	149.449	100

Table of Probabilities for the F Distribution

Alpha = 0.05

D/N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	20	24	30	40	60	120
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	242.98	243.91	244.69	245.36	245.95	248.01	249.05	250.10	251.14	252.20	253.25
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41	19.42	19.42	19.43	19.45	19.45	19.46	19.47	19.48	19.49
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.71	8.70	8.66	8.64	8.62	8.59	8.57	8.55
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86	5.80	5.77	5.75	5.72	5.69	5.66
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62	4.56	4.53	4.50	4.46	4.43	4.40
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94	3.87	3.84	3.81	3.77	3.74	3.70
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51	3.44	3.41	3.38	3.34	3.30	3.27
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22	3.15	3.12	3.08	3.04	3.01	2.97
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.05	3.03	3.01	2.94	2.90	2.86	2.83	2.79	2.75
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85	2.77	2.74	2.70	2.66	2.62	2.58
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72	2.65	2.61	2.57	2.53	2.49	2.45
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62	2.54	2.51	2.47	2.43	2.38	2.34
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53	2.46	2.42	2.38	2.34	2.30	2.25
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46	2.39	2.35	2.31	2.27	2.22	2.18
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40	2.33	2.29	2.25	2.20	2.16	2.11
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.40	2.37	2.35	2.28	2.24	2.19	2.15	2.11	2.06
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.35	2.33	2.31	2.23	2.19	2.15	2.10	2.06	2.01
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.31	2.29	2.27	2.19	2.15	2.11	2.06	2.02	1.97
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.28	2.26	2.23	2.16	2.11	2.07	2.03	1.98	1.93
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.22	2.20	2.12	2.08	2.04	1.99	1.95	1.90
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.22	2.20	2.18	2.10	2.05	2.01	1.96	1.92	1.87
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.20	2.17	2.15	2.07	2.03	1.98	1.94	1.89	1.84
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20	2.18	2.15	2.13	2.05	2.01	1.96	1.91	1.86	1.81
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18	2.15	2.13	2.11	2.03	1.98	1.94	1.89	1.84	1.79
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16	2.14	2.11	2.09	2.01	1.96	1.92	1.87	1.82	1.77
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.12	2.09	2.07	1.99	1.95	1.90	1.85	1.80	1.75
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.17	2.13	2.10	2.08	2.06	1.97	1.93	1.88	1.84	1.79	1.73
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12	2.09	2.06	2.04	1.96	1.91	1.87	1.82	1.77	1.71
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.14	2.10	2.08	2.05	2.03	1.94	1.90	1.85	1.81	1.75	1.70
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04	2.01	1.93	1.89	1.84	1.79	1.74	1.68
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95	1.92	1.84	1.79	1.74	1.69	1.64	1.58
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.89	1.86	1.84	1.75	1.70	1.65	1.59	1.53	1.47
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.87	1.83	1.80	1.78	1.75	1.66	1.61	1.55	1.50	1.43	1.35

Right Tailed, D/N = df in denominator = down the rows, df in numerator = across the columns

Note: Table is for an alpha of 0.05

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	統計學 B	系所別	金融學系 財務工程與金融科技組	考試時間	2月3日(五) 第3節

1. (30 points) The probability mass function of Poisson random variable Y is given by
$$f_Y(y) = \frac{\lambda^y e^{-\lambda}}{y!} \qquad y = 0, 1, 2, \ldots$$
   (a) (10 points) Compute the moment generating function of Y.
   (b) (5 points) Compute the mean and variance of Y.
   (c) (5 points) Suppose we have data $\{y_1, \ldots, y_n\}$ which is i.i.d. and has Poisson distribution. Write down the log-likelihood function and find the maximum likelihood estimator for λ.
   (d) (5 points) Suppose we now also have the data on explanatory variable $\{x_1, \ldots, x_n\}$ and specify $\log \lambda_i = \alpha + \beta x_i$. Please write down the log-likelihood function of (α, β).
   (e) (5 points) Let $(\hat{\alpha}, \hat{\beta})$ be the maximum likelihood estimator for (α, β). Please write down the (first order condition) equation that $(\hat{\alpha}, \hat{\beta})$ has to satisfy.

2. (20 points) Let $\{R_1, \ldots, R_n\}$ are the random returns of n assets. The returns are identically distributed with mean μ and σ^2. For any $i \neq j$, the covariance between R_i and R_j is a constant c. That is, $cov(R_i, R_j) = c, \forall i \neq j$.
   (a) (10 points) Find the variance of equally weighted portfolio returns, $Var\left(\frac{1}{n}\sum_{i=1}^n X_i\right)$.
   (b) (10 points) What is the portfolio variance when $n \to \infty$? Please briefly interpret your result.

3. (20 points) Let $\{x_1, \ldots, x_n\}$ be i.i.d. random variable with probability density function
$$f(x; \theta) = \frac{\theta}{x^2}, \qquad x \geq \theta,$$
   where the parameter θ is strictly positive.
   (a) (5 points) Verify that $f(x; \theta)$ satisfies the definition of probability density function.
   (b) (5 points) Evaluate the mean $\mathbb{E}[x]$.
   (c) (10 points) Find the maximum likelihood estimator for θ.

4. (15 points) Let z be standard normal random variable and $g(z)$ be any differentiable function ($g'(z)$ exists). Prove that
$$\mathbb{E}[zg(z)] = \mathbb{E}[g'(z)].$$

5. (15 points) Let $x \overset{iid}{\sim} N(0,1)$, $\varepsilon \overset{iid}{\sim} N(0,1)$, and $\mathbb{E}[\varepsilon|x] = 0$. The random variable y is generated by (the true data generating process)
$$y = \theta_0 + \theta_1 x + \theta_2 x^2 + \varepsilon.$$
   Suppose we estimate the following model with a sample of size n
$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \quad i = 1, \ldots, n.$$
   Is the $\hat{\beta}_1$ (estimated with ordinary least squares) a consistent estimator for θ_1? Please provide the proof for your answer.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。